

The Importance of Measurement Error for Certain Procedures in Remote Sensing at Optical Wavelengths

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ABSTRACT: Two problems are identified in the use of linear regression to relate remotely sensed data to ground variables: a specification problem and an errors problem. The extent of the errors is examined for commonly measured remotely sensed variables and ground variables. Three alternative methods of line fitting are examined: Wald's grouping methods, the reduced major axis, and a least-squares procedure. The least-squares method is recommended if the data are available.

INTRODUCTION

THE USE OF remotely sensed data to describe ground conditions often depends upon the statistical association between remotely sensed variables and ground variables. The methods of relating such variables are loosely called "correlation methods" and the statistical techniques most commonly employed are the *correlation coefficient* (being used to measure the covariation of the two data sets) and *regression* (being used to fit an equation which can then be inverted to provide conversion of the remotely sensed data into expected ground variables). The application of inverted regression equations in remote sensing is discussed in Twomey (1977) and the current *Manual of Remote Sensing* (Colwell (1983), pp. 87, 978, 1095, 1349, 1421, 1461, and 2359), with recent examples being given in Curran (1983b) and Gower *et al.* (1984). The argument of this paper is that although the *correlation coefficient* is an acceptable statistic to describe the goodness of fit, the *simple regression* model is inappropriate for calibration in many cases and other techniques should be employed. There are two related reasons for this claim: the first depends upon a question of specification and can be dealt with briefly, the second depends upon assumptions about the nature of the data sets which can be dealt with in general terms but is illustrated by specific examples in the next section. The third section then describes some possible alternative methods and shows, with a worked example, that differences in the methods may be far from trivial.

The specification problem is introduced in Figure 1. If two variables (x and y) are correlated on a scatter diagram, *two* lines may be fitted by regression methods—the regression of x on y and the regression of y on x . The divergence of these two lines is related to the correlation coefficient in that, when

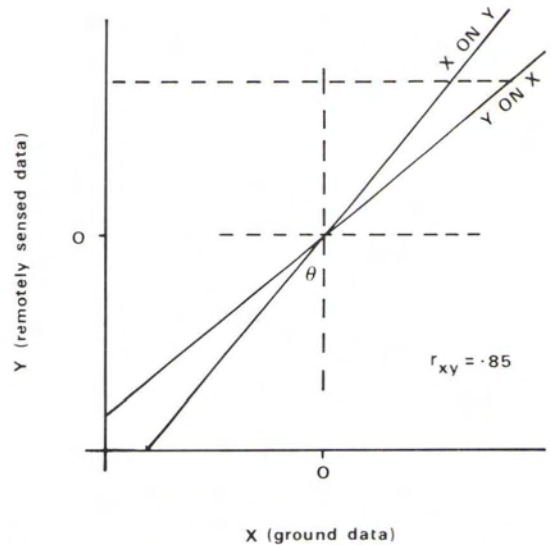


FIG.1. The use of simple regression to calibrate remotely sensed data with ground data.

data are normalized, the tangent of the angle θ equals the correlation coefficient r_{xy} (e.g., Loveday (1961), p. 104). In conventional applications the regression of y on x would be used to estimate y given x , while the regression of x on y would be used to estimate x given y . It is usually argued that the equation should use the independent (or causal) variable to predict the dependent (or resultant) variable. However, in remote sensing applications this interpretation cannot be maintained because the causal variable in the physical sense is the Earth's surface which reflects or emits a signal that is recorded by the aircraft or satellite mounted sensor. Therefore, the direction of estimation is from the dependent

measurement back to the independent measurement.

The reasoning behind this categorization is closely linked to certain assumptions about the form of the errors in the relationship being fitted. The regression model has the form

$$y_i = \beta x_i + \alpha + \epsilon_i$$

where α and β are the coefficients of the regression line and ϵ is a disturbance or error term which includes the effect of uncontrolled exogenous variables and any errors in the measurement of y_i . It is assumed that the x values are free of any measurement errors, and that measurement errors in y , are independent of the value of x_i .

It can be shown (Johnston (1972), pp. 282-283) that, if there are measurement errors in the x_i values, the estimate of β will be biased.

$$\hat{\beta} = \frac{\beta}{1 + \frac{\sigma_v^2}{\sigma_x^2}}$$

where $\hat{\beta}$ indicates the estimate of β ,
 σ_v^2 denotes the variance arising from measurement errors in x , and
 σ_x^2 denotes the variance arising from the 'true' variation in x .

Inspection of the expression reveals that any measurement error in x_i values will result in an underestimate of the β . In calibration of remotely sensed data when the β are usually positive, an underestimate of β will lead to overestimates of the ground value above the mean and underestimates below the mean.

Although some authors have recognized the presence of errors in their data sets and commented on their implications for results (e.g., Tucker, 1977; Ahlrichs *et al.*, 1979; Milton, 1982, Curran and Williamson, 1985a, 1985b), there is no review of the incidence and magnitude of such errors. To date, no attempt has been made to identify an alternative methodology for estimating the equation of the best-fit line, although the problem has a long history in the statistical literature which is reviewed in articles by Madansky (1959), Moran (1971), and Anderson (1976).

ERRORS OF MEASUREMENT

ERRORS IN THE MEASUREMENT OF GROUND VARIABLES

Errors in the measurement of ground variables arise in two main ways. First, there is an error which is built into the design and operation of the measuring device. Such errors can be estimated quite accurately, for example, by repeated measurement in controlled conditions (e.g., Curran and Williamson, 1985b), although specific operators may contribute their own unique variances. Secondly, there

is error involved in sampling a spatial variable. It must be remembered that, while at a 2-m spatial resolution the ground resolution cell of an optical sensor is only 4 m², this rapidly increases to 6,400m² at an 80-m spatial resolution. Furthermore, there is evidence that, due to problems of location and blurring, the ground area represented by a nominal ground resolution cell is even larger. Justice and Townshend (1981) suggest that ground plots of 16m² and 25,600m², respectively, are needed to describe the contents of such ground resolution cells. For many ground variables, plots of such size cover a wide range of values, and a complete enumeration even if technically feasible would be extremely costly. As a consequence, most field surveys involve sampling of points or small areas from which a mean value is derived for the whole ground resolution cell. The errors in such a mean are dependent upon both the number of observations (Hatfield, 1979; Robinson, 1984) and the area utilized (see Figure 2, for example). Such random sampling errors are handled more easily than the errors which arise from purposive sampling (e.g., Myers, 1975) or transect sampling (e.g., Sørensen, 1979; 1980; Cracknell, 1980). In an attempt to present an overall picture of errors from both these sources, data from a number of studies have been reworked for 1-m² plots. The figures in Table 1 indicate the maximum deviation of a single observation above or below the median for a small sample of observations, both as an absolute value and as a percentage of the median. The figures presented, therefore, combine both sources of measurement error and are indicative of the errors which might be expected to occur if a single observation is used to represent a ground resolution cell. The sample sizes are insufficiently large to estimate true standard errors or variances. For further discussion of this source of error, refer to Curran and Williamson (1985a, 1986).

ERRORS IN THE MEASUREMENT OF REMOTELY SENSED VARIABLES

There are seven sources of error in remotely sensed measurements by optical sensing devices: irradiance variation, sensor calibration error, sensor radiometric resolution, sensor drift, signal digitization, atmospheric attenuation, and atmospheric path radiance. Together, the first five of these result in a skewed distribution of absolute errors of around ± 15 percent and a normal distribution of inter-band errors of under ± 1 percent (Table 2).

Irradiance variation. Irradiance (E) has two noise components: Low frequency noise with periods of minutes or hours and high frequency noise with periods of 0 to a thousandth of a second. If radiance (L) is recorded when low frequency noise is present, usually as a result of patchy cloud, then the absolute error of an irradiance measurement $3\sigma E/E$ is likely to be in the order of ± 180 percent (Duggin, 1981;

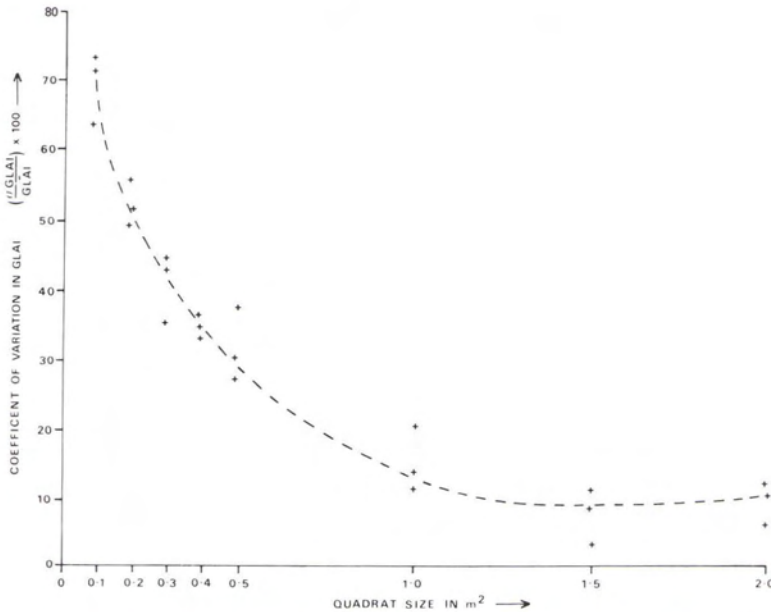


FIG.2. The relationship between the coefficient of variation for green leaf area index (GLAI) measurements and quadrat size for *Pteridium* at Snelsmore Common, Berkshire, U.K. For further details of the experiment associated with these data, refer to Curran (1983b).

TABLE 1. EXAMPLES OF ERROR ASSOCIATED WITH THE MEASUREMENT OF GROUND DATA FOR 1-M² AREA.

Ground data (independent variable x)	Units	Particular example	Absolute error	Absolute error as % of median	Reference to origin of data set
Surface soil moisture	% by volume	Thin humus podsol	9	18	Curran (1981a)
Soil temperature	°C	Ploughed clay	2	16	Curran (1985)
Surface snow moisture	% by volume	Deep snow (January 1984)	10	19	Unpublished
Suspended sediment concentration	mg l ⁻¹	Coastal water	14	50	Curran <i>et al.</i> (1986)
Crop height	m	<i>Hordeum vulgare</i> (barley)	0.1	13	Curran (1981b)
Total biomass	g/m ²	<i>Phragmites communis</i> (reed)	12	23	Curran (1982)
Green leaf area index	unitless	<i>Calluna vulgaris</i> (heather)	0.11	28	Curran (1983a)
Green leaf area index	unitless	<i>Festuca ovina</i> and <i>Carex flacca</i> (grass)	0.7	35	Curran and Williamson (1985a)
Canopy cover	%	<i>Betula pubescens</i> (birch)	5	9	Curran (1982)
Chlorophyll concentration	mg/m ²	<i>Lepidium sativum</i> (cress)	5	26	Curran and Milton (1983)

Duggin *et al.*, 1975). If irradiance is measured on one of those rare days when the sky is haze-free and low frequency noise is absent, then the absolute error of an irradiance measurement can amount to little more than ± 3 percent (Slater, 1980). This 3 percent error is likely to manifest itself in variation in the radiance data recorded by sensors with short integration times. For example, the radiance recorded

by the Landsat MSS, which has an integration time of 9.958 sec (NASA, 1976), is more likely to be affected by this high frequency noise than are the radiance data recorded by the Spectron ground radiometer, which has an integration time of 1/60th to 1 second (Spectron Engineering, 1982). Because remotely sensed radiance data are rarely collected on either cloudy or perfectly haze-free days, a more

TABLE 2. ESTIMATES OF THE ERROR, BOTH ABSOLUTE AND INTERBAND, FOR A GROUND BASED RADIOMETER, AIRBORNE MULTISPECTRAL SCANNER, AND THE SATELLITE BORNE THEMATIC MAPPER. ERRORS RESULTING FROM ATMOSPHERIC ATTENUATION AND ATMOSPHERIC PATH RADIANCE ARE NOT INCLUDED.

	Ground based radiometer			Airborne multi-spectral scanner		Satellite borne Thematic Mapper*	
	Absolute	Interband		Absolute	Interband	Absolute	Interband
	%	One radiometer %	Two radiometers %	%	%	%	%
Irradiance variation	15	5	0	15	0	15	0
Sensor calibration error	3.4	0	0	3.4	0	3.4	0
Sensor radiometric resolution	1	1	1	0.1	0.1	0.3	0.3
Sensor drift	0	0	0	0	0	5.8	0
Signal digitization	0	0	0	0.3	0.3	0.3	0.3
Total (root-sum-square)	15.4	5.1	1.0	15.4	0.3	16.4	0.4

*(Norwood and Lansing, 1983).

representative irradiance measurement is that made under apparently clear skies. Such a measurement is reported to have an absolute error of around ± 15 percent (Duggin, 1974; Slater, 1975) and as such is often regarded as the precision limit of radiance measurements when using the sun as a source (Slater, 1980).

If these variations in irradiance are not suppressed, then they increase considerably the absolute error of the radiance measurements (Duggin, 1974; Slater, 1980). The methods for suppressing irradiance variations are ratioing and the calculation of reflectance. Ratioing involves the division of the radiance recorded in one waveband, e.g., near infrared (NIR) with the radiance recorded in another, e.g., red (R), on the assumption that the irradiance error (E_e) is the same for both wavebands (Maxwell, 1976; Curran, 1980). That is,

$$\frac{L_{\text{NIR}} - L_{\text{R}}}{L_{\text{NIR}} + L_{\text{R}}} = \frac{L_{\text{NIR}} \pm E_e (\%) - L_{\text{R}} \pm E_e (\%)}{L_{\text{NIR}} \pm E_e (\%) + L_{\text{R}} \pm E_e (\%)}$$

As the similar but non-ratio waveband combinations, e.g., the perpendicular vegetation index (PVI), do not suppress irradiance variations, they are calculated using reflectance (ρ) and not radiance (L) data. That is,

$$\text{PVI} = \sqrt{(\rho_{\text{soil}_R} - \rho_{\text{vegetation}_R})^2 + (\rho_{\text{soil}_{\text{NIR}}} - \rho_{\text{vegetation}_{\text{NIR}}})^2}$$

The assumption upon which ratioing is based is not tenable for radiance measurements made separately in each waveband. For this reason, data collected by a ground radiometer are customarily converted from radiance (L) to reflectance (ρ) values before ratioing. This can be achieved by measuring irradiance (E) simultaneously with radiance. That is, $\rho = E/L$. Such a methodology is only possible when two radiometers are available, one pointing upwards and one pointing downwards. On apparently clear

days this approach has been reported to remove both low and high frequency irradiance variations (Duggin and Philipson, 1981; Duggin and Piwinski, 1982; Duggin, 1983). The approach fails only in that simple measurements of irradiance do not differentiate between direct and diffuse irradiance, the proportion of which have a large effect on the presence of shadow and therefore radiance. For example, Milton (1981) has indicated that the passage of a thin cloud in front of the sun could, without a change in irradiance, result in a fourfold increase in red as opposed to near infrared radiance.

For some applications two sensors may not be available and so the irradiance portion of the reflectance formula must be estimated by means of the radiance (L) of a scene standard (ss). That is, $\rho = L_{ss}/L$. This scene standard may be a grey card when using a ground radiometer or a large feature such as an airport runway when using airborne or spaceborne sensor data. This formula is based upon two assumptions: first, the intensity and spectral distribution of irradiance on the Earth's surface is invariant during the measurement of radiance from both the Earth's surface and the scene standard; and second, the scene standard is a perfectly reflecting Lambertian surface that is not being influenced by the method of measurement.

As these assumptions are not valid in the majority of cases (Kimes and Kirchner, 1982; Kimes *et al.*, 1983), the effects of low but not high frequency irradiance variations are reduced but not removed by using a scene standard (Duggin, *et al.*, 1975; Slater, 1975; Duggin, 1981).

Sensor calibration error. If a remote sensor is to be used to measure electromagnetic radiation in the physical units of radiance (L), then it must be calibrated to a primary standard. This procedure has three sources of error: first, measurement of the primary standards, which has an absolute error of around ± 1 percent (Wolfe and Zissis, 1978); second,

calibration of a laboratory integrating sphere to a natural standard, which has an absolute error of around ± 3 percent; and third, calibrating the sensor from the laboratory integrating sphere, which has an absolute error of around ± 2 percent. Therefore, the root-sum-square (RSS) absolute error of such a calibration is around ± 4 percent (Norwood and Lansing, 1983). Some users require measurements in these physical units (e.g., Robinove, 1982) for the estimation of sea surface temperature (Harries *et al.*, 1983). However, for the majority of applications where relative values of radiance are required, it is only necessary to ratio the signal or to convert it to a reflectance value in order to remove this large source of error.

Sensor radiometric resolution. The radiometric sensitivity or resolution of a sensor is dependent upon two types of noise: signal independent noise and signal dependent noise (Billingsley, 1982). The signal independent noise, which is the larger source of noise, is the result of a number of factors including the thermal noise of the feedback resistor, thermal noise of the detector resistance, shot noise due to leakage currents, input voltage noise, and noise arising from the lossy dielectric of the preamplifier input capacitance (Engel, 1980). The signal dependent noise, which is proportional to the square root of the signal current, results from several interrelated factors (Billingsley, 1982). The most important factor is fluctuation in the generation of conducting electrons. The former occurs even when the radiation falling on the detector is considered to be constant and is a result of the statistical nature of radiation flow. The latter occurs because the probability of freeing an electron following the interaction of radiation with matter is also stochastic (Lowe, 1976). Both of these sources of noise vary between sensors. Their magnitude can be assessed by reference to the Thematic Mapper sensor on board Landsats 4 and 5 (Table 3).

These two sources of noise can be described in terms of the signal (S) to noise (N) ratio (S/N) of the signal. This index is useful in obtaining a feel of what a noise level has on the error of data values, as an S/N ratio of 4 is required for the reliable detection of objects (Welch *et al.*, 1981) and an S/N ratio of 50 is required for the production of apparently "sharp" images. For example, in visible wavelengths the typical minimum S/N ratio of the AVHRR on board the NOAA satellites is 3 (Schwalb, 1979; Foote and Draper, 1980); the RBV on board the Landsat satellites is 32 (Freden and Gordon, 1983); the MSS on board the Landsat satellites is 98 (Freden and Gordon, 1983); the Thematic Mapper on board the Landsat satellites varies between 152 for band 1 and 341 for band 4 (Salmonson and Koffler, 1983); the radiometer on board the Meteosat satellite is 200 (Morgan, 1981); as is both a good quality ground radiometer (Spectrascan, 1982) and an aerial photograph (Slater 1980).

TABLE 3. THE NOISE LEVELS ASSOCIATED WITH THE THEMATIC MAPPER SENSOR. CALCULATED FROM SIGNAL-TO-NOISE RATIO DATA (USGS, 1982).

Waveband	Noise level (%)	
	Scene of low reflectance of around 10%. Noise primarily fixed.	Scene of high reflectance of around 90%. Noise fixed and signal generated
Visible	0.2	0.4
Near infrared	0.3	0.4
Middle infrared	0.5	0.6

As these S/N figures are not very useful when trying to estimate the error of a particular radiance measurement, the sensitivity of a sensor is often expressed in terms of the detectable noise equivalent difference ($NE\Delta$) (Robinson and DeWitt, 1983). This is the minimum variation in reflectance (ρ) or temperature (T) that can be detected by the sensor, under specified conditions of measurement (Gibbons and Richard, 1979; Lowe, 1980). The error of any measurement (which is $(NE\Delta/2)$) varies between sensors (Table 4) with a maximum rarely exceeding 1 percent.

Signal independent noise is a problem with which a user must live (Duggin, 1985), but signal dependent noise can be suppressed, although not removed, by logarithmically transforming the signal during the digitization procedure. This not only compresses higher signal levels where the noise is greatest and stretches the lower signal levels where the noise is least (Slater, 1980) but also makes the noise additive and therefore easier to model out (Billingsley, 1982).

Sensor drift. The sensitivity of sensors changes with time. In order to maintain a calibration between radiance and signal, satellite borne sensors are usually corrected to a standard which is viewed once per scan and the sun which is viewed once per orbit, as was the case for the MSS on Landsats 1 to 3 (Lowe, 1980). The subsequent recalibration of the sensor is made on the assumption that the calibration equipment and the environment of the sensor do not deteriorate. Unfortunately, for satellite borne sensors there are three changes occurring: first, the calibration of the sensor is often disturbed on launch, typically resulting in ± 3 percent absolute error; second, the internal calibration standards change by around ± 1 percent; and third, the measurements made in the vacuum of space can be up to ± 5 percent different from those made on Earth. Therefore, the root-sum-square of these temporal change errors amounts to around ± 6 percent, which is ± 0.31 K for the AVHRR on board the NOAA satellites (ITT Aerospace, 1979). Fortunately, these errors do not apply to satellite sensor data that are ratioed or converted to reflectance data and are

TABLE 4. THE RADIOMETRIC RESOLUTION OF A RANGE OF OPTICAL SENSORS.

Sensor/Platform	Visible NE $\Delta\rho$ (%)	Noise Equivalent Difference (NE Δ)		
		Infrared		thermal
		near NE $\Delta\rho$ (%)	middle NE $\Delta\rho$ (%)	NE ΔT (K)
S192/Skylab Satellite ¹	1.73	1.16	2.10	2.2
MSS/Landsat Satellites 1, 2, 3, 4, and 5 ¹	0.57	0.68	NR	1.4
TM/Landsat Satellites 4 and 5 ¹	0.60	1.00	2.00	0.5
Radiometer/HCMM Satellite ¹	NR	NR	NR	0.4
HRV/SPOT Satellite ¹	0.50	0.50	NR	NR
OCM/ERS 1 Satellite ²	0.05	0.05	0.05	0.2
AVHRR/NOAA Satellite ³	NR	NR	NR	0.05
Daedalus AADS 1268/Aircraft ⁴	0.15	0.21	0.50	0.2
Daedlus DEI-100 and DS-1200/Aircraft ⁵	NR	NR	NR	0.2
Bendix T/M LN - 3/Aircraft ⁵	NR	NR	NR	0.2
Texas Inst. RS - 18/Aircraft ⁵	NR	NR	NR	0.1
Infratrace 801/Ground ⁶	NR	NR	NR	1.4
Exotech 100AX/Ground ⁷	0.04	0.04	NR	NR
Spectrascan MSP 1000/Ground ⁸	0.15	0.15	NR	NR

NR Not relevant

¹Reference next to sensor name in Sensor Radiometric Resolution section

²RSS (1981)

³Robinson *et al.* (1984)

⁴Hunting Geology and Geophysics (1982) for 2.5 mrad resolution

⁵Lowe (1980) for 2.5 mrad resolution

⁶Kane-May Ltd. (1983)

⁷Exotech (Personal Communication)

⁸Spectrascan (1982)

minimal for airborne and ground radiometric data where the sensor is calibrated regularly.

Signal digitization. Before the sensor signals are used, they are usually digitized to digital numbers (DN). This process involves error as a result of fitting a straight line to the curvilinear signal/DN response curve (Gillespie, 1980; Billingsley, 1984). If the signal is well digitized, the error is likely to be less than 1 DN in the central part of the signal range. For example, band 3 of the Thematic Mapper sensor on board Landsat satellites 4 and 5 has an error of around ± 0.3 percent (± 0.77 DN), (J.R.G. Townshend, personal communication) and the HRVs on board the satellite SPOT have an error of around ± 0.2 percent (± 0.5 DN) in the central part of the signal range and ± 0.35 percent (± 0.9 DN) over the whole range (Chevrel *et al.*, 1981; Begni, 1982). Unfortunately, as these errors are waveband dependent, the errors remain after both ratioing and the calculation of reflectance.

Atmospheric attenuation and atmospheric path radiance. Atmospheric attenuation and atmospheric path radiance increase considerably the error of a remotely sensed signal (Curran, 1985) by an amount that is extremely variable (Chahine, 1983). If the remotely sensed signal is required in physical units, then a complex atmospheric correction model must be applied to the data (Turner *et al.*, 1975). For example, when measuring sea surface temperature from

spaceborne sensors, the application of an atmospheric correction model can reduce error to ± 0.5 K (Robinson *et al.*, 1984). For the majority of applications and where the atmospheric path radiance is known, ratioing can be used successfully to suppress the effects of atmospheric attenuation on the same principle that it suppresses the effects of irradiance variations. For further discussion, refer to Slater (1980) and Robinson (1985).

ERRORS IN THE PHYSICAL CORRELATION OF GROUND VARIABLES AND REMOTELY SENSED VARIABLES

The two most important sources of error in the physical correlation of ground variables and remotely sensed variables are misregistration in space and misregistration in time. Together these represent a sizeable but so far intractable source of error.

Misregistration in space. The data from all airborne and spaceborne sensors have errors of location (Orti, 1981). For example, this is around ± 160 metres for the Landsat MSS and ± 118 metres for the Landsat RBV, with a within-band registration error of around ± 50 metres for both systems (Lintz and Simonett, 1976; USGS, 1979). If an adequate number of ground control points are used, it is possible to decrease this error, in the case of Landsat MSS, to ± 40 metres (± 0.5 pixel) 99 percent of the time (Slater, 1980; Engel, 1980).

2%	4%	1%	
20%	52%	14%	→ Direction of scan
2%	4%	1%	

FIG.3. The contribution of surrounding ground resolution cells to the radiance record of a central Landsat MSS pixel (modified from Forster, 1980).

As a result of the point spread function of the sensor, velocity smear, and atmospheric scatter (Curran, 1985), only about half of the radiance value for each pixel can be attributed to the ground resolution cell it represents. This results in image blur and a decrease in the accuracy with which a ground point can be related to a remotely sensed signal (Figure 3).

The combined effects of pixel locational error and image blur conspire to increase the error of radiance measurements away from the average radiance value for the scene. These problems are not as serious as they first appear, because the pixels of a remotely sensed scene are spatially autocorrelated (Steiner and Salerno, 1975) and as a result the radiance value of any one pixel is likely to be the same as its neighbors. It only becomes a serious problem when the spatial frequency of the terrain is high in relation to the spatial resolution of the sensor. This is the case for urban areas on Landsat MSS imagery and field boundaries on airborne scanner imagery.

An indication of the magnitude of this error is given by Forster (1980). He reports that water bodies in Sydney, Australia have near infrared radiance values on Landsat MSS data that are up to 9.5 times higher than their true values as a result of pixel location error and image blur. A similar effect can be noted when extracting near infrared radiance data from Landsat MSS imagery over sharp environmental boundaries. For example, the 7 February 1977 Landsat MSS imagery of Ladybower reservoir to the west of Sheffield, United Kingdom has a DN in the near infrared waveband (band 4) of 4 in the center of the water body, rising to 7 near unvegetated fields and 12 near vegetated fields! The message is clear; investigators need to make their measurements in homogeneous areas that cover an area of at least 3 by 3 pixels.

Misregistration in time. Remotely sensed data are correlated with ground data on the often incorrect assumption that they were both measured at the

same time. By way of example, Table 5 gives an indication of the likely scale of errors associated with the recording of green leaf area index and surface soil moisture one hour, one day, and one week after the collection of remotely sensed data. It can be seen that, while the timing of ground data acquisition makes little difference to the recording of slowly changing green leaf area index, it can result in considerable error when recording rapidly changing surface soil moisture.

POSSIBLE SOLUTIONS

The material reviewed above indicates that, in addition to the specification problem, there are substantial problems arising from measurement error. In this section three possible solutions are reviewed. They may be used in two ways, *either* to replace regression methods *or* to give a sensitivity test of the results obtained by regression.

WALD AND BARTLETT - THE METHOD OF GROUPS

In econometrics a number of authors have advocated the methods devised by Wald (1940) and amended by Bartlett (1949). Wald advocated that the regression line should be fitted by dividing the observations into two halves on the basis of their x_i values (name these sample *A* and sample *B*; if there is an uneven number of observations the median value should be discarded). The line is then fitted from

$$\hat{\beta} = \frac{\bar{y}_B - \bar{y}_A}{\bar{x}_B - \bar{x}_A} \text{ and}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x}.$$

The intuitive rationale for this approach is that the errors in the x_i (assumed to sum to zero) will be mutually-compensating within the two subsets of data. Bartlett (1949) argued that a more efficient estimator is achieved by partitioning the original data into three equal subsets, discarding the central subset, and using the highest (*A*) and lowest subsets (*C*):

$$\hat{\beta} = \frac{\bar{y}_C - \bar{y}_A}{\bar{x}_C - \bar{x}_A} \text{ and}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x} \text{ as before.}$$

There are two objections to such a procedure in this context. First, the estimates ($\hat{\beta}$ and $\hat{\alpha}$) will have fairly large sampling variances, so although unbiased in general, a specific calibration may be subject to greater error than that introduced in the conventional regression model. Second, the Wald-Bartlett method does nothing to resolve the specification problem.

THE REDUCED MAJOR AXIS

The reduced major axis is used in the biological and geological sciences, having been devised apparently by Jones (1937) and applied by Teissier

TABLE 5. COEFFICIENT OF VARIATION OF FIVE MEASUREMENTS OF GREEN LEAF AREA INDEX (GLAI) AND SURFACE SOIL MOISTURE (SSM) FOR FOUR SITES ON SNELSMORE COMMON, BERKSHIRE, U.K. MEASUREMENTS FOR THE CALCULATION OF GLAI AND σ SSM WERE MADE ONE HOUR, ONE DAY, AND ONE WEEK AFTER THE MEASUREMENT OF GLAI AND SSM AT 1115 ON 26 JUNE 1981. FOR FURTHER DETAILS OF THE EXPERIMENTS ASSOCIATED WITH THESE DATA, REFER TO CURRAN (1981A, 1983B).

	Coefficient of variation of green leaf area index (GLAI)			Coefficient of variation of surface soil moisture (SSM)		
	$\left(\frac{\sigma \text{ GLAI}}{\text{GLAI}} \times 100\right)$	$\left(\frac{\sigma \text{ GLAI}}{\text{GLAI}} \times 100\right)$	$\left(\frac{\sigma \text{ GLAI}}{\text{GLAI}} \times 100\right)$	$\left(\frac{\sigma \text{ SSM}}{\text{SSM}} \times 100\right)$	$\left(\frac{\sigma \text{ SSM}}{\text{SSM}} \times 100\right)$	$\left(\frac{\sigma \text{ SSM}}{\text{SSM}} \times 100\right)$
	+ 1 hour	+ 1 day	+ 1 week	+ 1 hour	+ 1 day	+ 1 week
Young <i>Calluna</i>	11	9	14	16	164	143
Mature <i>Calluna</i>	7	4	6	9	142	174
<i>Pteridium/Calluna</i>	9	6	7	9	205	112
<i>Pteridium</i>	10	10	11	11	191	294
Mean	9	7	10	11	176	181

(1948) and Imbrie (1956). An attempt to draw it to the attention of geographers was made by Till (1973), and it appears in a standard text on statistics for geology (Miller and Kahn, 1962).

The main principle of the method is illustrated in Figure 4. Whereas conventional regression seeks to minimize the sum of the squares of the differences vertically (i.e., $y_e - y_o$) in the case of y on x and horizontally ($x_e - x_o$) in the case of x on y , the reduced major axis minimizes the sum of the cross products ($y_e - y_o$) ($x_e - x_o$) of the differences on both axes. The resulting line is independent of the units of measurement and invariant with the rotation of the axes. The line can be fitted from

$$\hat{\beta} = \frac{\sigma_y}{\sigma_x} \text{ and}$$

$$\hat{\alpha} = \bar{y} - \frac{\sigma_y}{\sigma_x} \bar{x}.$$

It will always bisect the angle between the regression lines, passing (like them) through the mean of the array. It is designed to handle those problems in which errors of similar magnitude are present in x 's and the y 's and in which "the terms 'dependent variate' and 'independent variate' have no real meaning" (Kermack and Haldane (1950) p. 30).

The equation fitted in this manner can be treated in the same way as a regression line. The correlation coefficient can still be used as a measure of covariation, and somewhat surprisingly the standard error of the slope coefficient is the same as the standard error of the regression coefficient: i.e.,

$$SE_{\beta} = \sqrt{\frac{\sigma_y^2 (1 - r^2)}{\sigma_x^2 n}}$$

This standard error of the regression coefficient can be used with the SE of the mean of y to give a standard error of the forecast which increases with distance from the mean of the array.

A LEAST-SQUARES PROCEDURE

The two preceding solutions assume that there is no knowledge about the relative magnitudes of the errors in the x and y measurements. If such knowledge does exist, much better estimates of β can be obtained. Accounts of these methods appear in Madansky (1959) and Sprent (1969). The basic starting point is that it is possible to partition the variance in the x 's and y 's as measured into two parts; a true variance and an error variance. So

$$\hat{\sigma}_x^2 = \sigma_x^2 + \sigma_u^2 \text{ and}$$

$$\hat{\sigma}_y^2 = \beta^2 \sigma_x^2 + \sigma_v^2$$

where σ_x^2 is the true variance of the x values and σ_u^2 and σ_v^2 are the error variance in the measurement of x and y , respectively. If the u_i and the v_i (the measurement errors) are uncorrelated (i.e., $\text{COV}(u_i, v_i) = 0$), then $\text{COV}(x, y) = \hat{\beta} \hat{\alpha}_x^2$. The three equations can be used to yield estimates of β as follows:

(i) if σ_v^2 is known,

$$\hat{\beta} = \frac{\hat{\sigma}_y^2 - \sigma_v^2}{\text{COV}(x, y)};$$

(ii) if σ_u^2 is known,

$$\hat{\beta} = \frac{\text{COV}(x, y)}{\hat{\sigma}_x^2 - \sigma_u^2}; \text{ and}$$

(iii) if both σ_u^2 and σ_v^2 are known,

$$\hat{\beta} = \sqrt{\frac{\hat{\sigma}_y^2 - \sigma_v^2}{\hat{\sigma}_x^2 - \sigma_u^2}}.$$

It will be clear from inspection of the third case that, if the errors in y (i.e., the σ_v^2) are larger than those in x (i.e., the σ_u^2), then the estimate of β will fall below that for the reduced major axis; if the position is reversed, then the $\hat{\beta}$ will be greater than that for the reduced major axis. If the errors are very small,

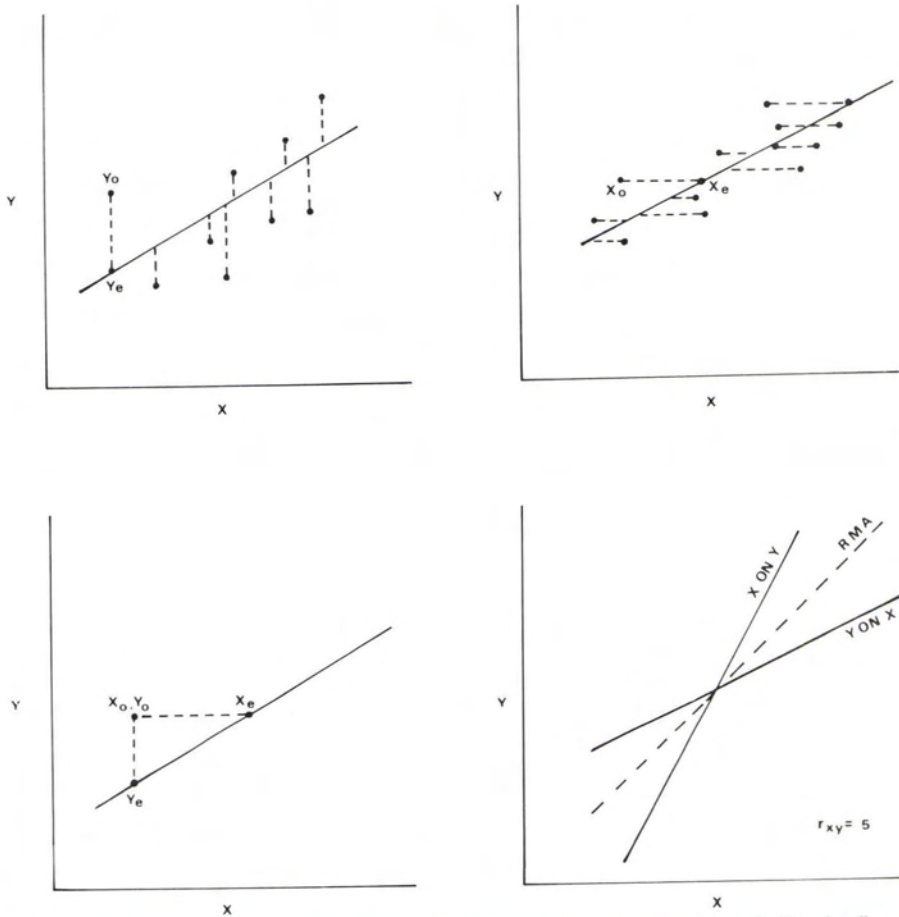


FIG.4. The minimization problem in fitting regression lines and the reduced major axis (for details see text).

the estimate of β will approach the major axis, but in the case where the errors are zero for one variable but large for the other (for x or for y), the $\hat{\beta}$ will converge on one of the regression estimates.

This purely statistical procedure gives no indication as to how the measurement errors might themselves be recorded. In the worst case technical data of the kind given in Tables 1 and 2 above may be used to give estimates of the likely error variances. In other cases it may be possible to estimate errors by replicating measurements of the same signal or the same ground conditions. If this were to become standard practice, it might be possible to arrive at estimates of measurement errors which could be routinely employed in remote sensing applications.

A WORKED EXAMPLE

The data in Table 6 refer to 26 ground measurements of the green leaf area index (GLAI) and a near infrared/red index made in Lathkill Dale, United Kingdom in the spring and summer of 1983.

The data have been used to calculate best-fit lines by regression (x on y and y on x), by Bartlett's version of Wald's method, by the reduced major axis, and by the least-squares method using error estimates, derived from Tables 1 and 2, of 0.03 in GLAI and 0.01 in the near infrared/red ratio. The results are given in Table 7 and the lines are fitted in Figure 5. It is evident from the diagram that, for values of GLAI well below or well above the mean, there are important differences in the estimates arrived at by different methods.

CONCLUSIONS

The evidence in this paper makes it clear that conventional regression methods are seldom appropriate in calibration of remotely sensed variables with ground variables. It is also clear that regression can lead to marked mis-estimates of the ground conditions, especially if the correlation coefficient is low. The positive recommendations are therefore two-fold:

TABLE 6. A TEST DATA SET—LATHKILL DALE, UK, 1983.

	GLAI	NIR/ R		GLAI	NIR/ R
1	0.57	1.65	14	1.09	3.99
2	0.62	1.99	15	1.60	3.56
3	1.14	3.00	16	2.77	3.85
4	2.43	3.66	17	2.16	4.45
5	2.27	3.20	18	2.18	4.87
6	1.23	2.85	19	2.19	4.28
7	1.08	2.60	20	1.77	4.24
8	0	1.45	21	0.97	2.38
9	2.04	4.01	22	1.17	2.56
10	1.42	3.43	23	1.37	2.54
11	1.98	4.11	24	1.16	2.35
12	1.83	4.30	25	1.34	2.68
13	2.97	3.68	26	1.43	2.79

Data courtesy N.W. Wardley (University of Sheffield); reflectance in the near infrared (NIR) and red (R) wavebands was measured using a radiometer; green leaf area index (GLAI) was measured in the laboratory.

TABLE 7. RESULTS OF LINES FITTED TO THE DATA IN TABLE 6.

Method	Fitted form	Form used for prediction
Regression of y on x	$y = 1.023x + 1.645$	$x = 0.977y - 1.607$
Regression of x on y	$x = 0.593y - 0.358$	$x = 0.593y - 0.358$
Wald's method	$y = 1.093x + 1.534$	$x = 0.915y - 1.403$
Reduced major axis	$y = 1.314x + 1.187$	$x = 0.761y - 0.903$
Least squares with error estimates	$y = 1.3496x + 1.131$	$x = 0.741y - 0.838$

- (a) Where no estimates of measurement variances are available Wald's method or the reduced major axis is to be preferred to regression; and
- (b) wherever possible estimates of measurement variances should be made, and the least squares methods outlined in the previous section should be adopted.

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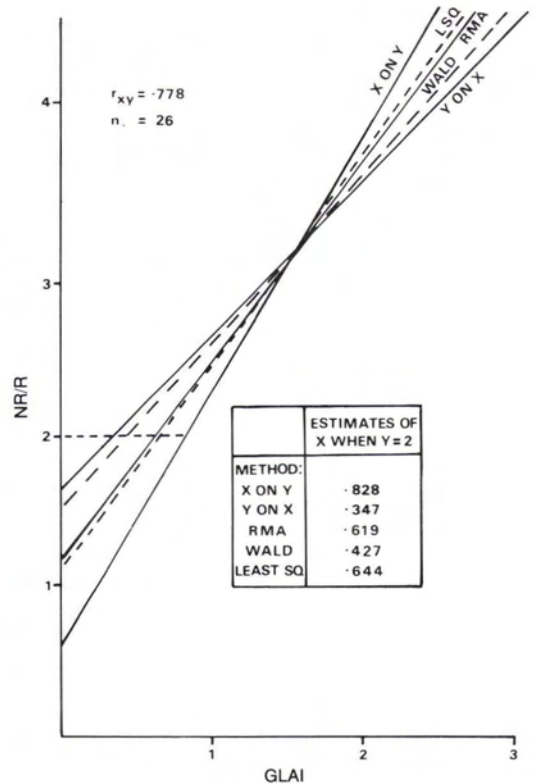


FIG.5. The application of five calibration methods to the data in Table 6.

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