Accuracy Estimation of Digital Elevation Data Banks

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ABSTRACT: Digital elevation data banks are today used for an increasing number of applications. For the estimation of the standard error of the elevations and functions thereof, the covariance function of the elevation errors has to be known. In this paper, four methods for the estimation of this covariance function are described and compared.

It is concluded that biased estimates of variances will in general be achieved if the correlations among the errors are neglected. Properly applied experience-based correlation functions are shown to be useful, but the approach is so far limited to photogrammetrically sampled elevation data. The MINQUE method was in general very successful. But, due to its lack of robustness, it should not be used alone. In combination with for instance direct computation of the autocovariance function, it seems to be justified.

INTRODUCTION

A DIGITAL ELEVATION DATA BANK is a resource which can be used for many different purposes. The change of data storage technology from graphical contour maps to digital elevation data banks has provided an efficient use of elevation data for a large number of applications. The exchange of geographical information between different organizations is steadily increasing, and public geographical data banks are today delivering data for many different purposes.

During the latter decades several digital elevation data banks have been established at national as well as local mapping organizations. They are usually established with respect to a limited set of predefined applications, such as the production of orthophoto maps, contour maps of a certain scale and contour interval, volume calculation for highway design, etc. The method for data acquisition and the density of the stored data are chosen with respect to these predefined applications.

One common problem arises when elevation data are to be used for purposes other than the predefined applications. A digital elevation data bank, intended for orthophoto production, might be needed for geometric correction of satellite imagery or for physical planning purposes. An important question here is, whether or not a particular data set of elevations is accurate enough for a certain application. In order to answer this question, the accuracy of the data set and its influence on the final result have to be estimated.

Many countries have today a specified map accuracy standard for contour lines and spot elevations (see, for example, Slama (1980) and Leatherdale (1980)). Although the specifications differ somewhat among the countries, they are in general formulated from a producer's point of view and designed for contour maps and not for digital elevation data. The map can be checked and found to be within tolerance, but a specific user still does not know whether or not the product fits his needs.

Instead of specifying tolerances as map accuracy standards, it should be possible to specify the accuracy of a certain data set in such a way that a user can estimate whether the data set is accurate enough for a specific application (Moellering, 1985). This approach leads to other questions to be answered, namely,

- Which quantities have to be known in order to estimate the geometric accuracy of an arbitrary application of elevation data?
- Which methods can be used for the estimation of these quantities?
- How can they be specified in a quality specification?

The purpose of this paper is mainly to study different methods for accuracy estimation of digital elevation data. A basic idea of this work is that experience, or *a priori* information, can be used to improve the estimates of the accuracy. On the basis of the result of the evaluation, the problem of quality specification will be discussed. Although the maximum error and the distribution of the errors are of great interest in many applications, this paper is restricted to the estimation of the standard error of functions of the elevations.

THE STANDARD ERROR OF FUNCTIONS OF THE ELEVATION

A digital elevation data bank consists usually of a limited set of discrete points. To obtain a continuous surface similar to the real terrain, surface elements are constructed by interpolation. These surface elements are used in the further processing of the elevation data.

Let us first consider some very simple applications, for instance, slope determination and volume computation. They can be expressed as the convolution

$$a(x,y) = w(x,y) * z(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w(u,v)\, z(x-u,y-v)\, du dv \quad (1)$$

where $a(x,y)$ is the final result (application),
$w(x,y)$ is a weight function, independent of the elevations, and
$z(x,y)$ are the true elevations of the terrain.

Due to the errors in the stored elevations and the inadequacy of the interpolator used, the interpolated terrain surface is burdened with errors $e_z(x,y)$. Its influence on the final result can then be expressed as

$$e_a(x,y) = w(x,y) * e_z(x,y) \quad (2)$$

where $e_a(x,y)$ is the error in the application and
$e_z(x,y)$ are the errors in the interpolated terrain surface.

In other, more complicated, applications such as the geometric rectification of satellite imagery, a limited set of discrete elevations is generally used in the process. The application can here be expressed as a multidimensional function of the elevations

$$a(x,y) = f(x,y,z_1,z_2,z_3,\ldots,z_n)$$

For the estimation of the error in the application (e_a), the function f can be expanded into a Taylor series. When neglecting terms of second and higher order, a linear expression such as Equation 2 is obtained. This Taylor expansion is very similar to the linearization of non-linear observation equations in a least-

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squares adjustment. Necessary conditions for this are that a limited set of discrete elevations is used for the application and that the approximate values are fairly correct, providing the opportunity to neglect second and higher order terms in the Taylor expansion.

From Equation 2, the covariance function of the errors in the result can be computed as

$$\mathrm{Cov}(e_a) = e_a(x,y) * e_a(-x,-y)$$

and the corresponding variance is obtained as

$$\mathrm{Var}(e_a) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\cdot \mathrm{Cov}(e_z)\,dxdy \qquad (3)$$

where $g(x,y) = w(x,y) * w(-x,y)$

It follows from Equation 3 that the information needed to estimate the standard error of a function of the elevations is the weight function $w(x,y)$, given by the application and the covariance function of the errors in interpolated elevations ($\mathrm{Cov}(e_z)$). Apparently, the latter function needs to be estimated and specified.

Two problems when specifying the covariance function of the errors are the questions of stationarity and isotropy. For stationary random functions, the covariance function depends only on the difference between the coordinates. According to Torlegård *et al.* (1984) and Frederiksen *et al.* (1984), the geometric accuracy of photogrammetrically sampled digital elevation models is mainly dependent on the terrain type and the sampling procedure. For a certain map sheet, which is a very common unit for data bank organization, the point density is usually homogeneous while the terrain type can be very heterogeneous. This means that one cannot assume the errors in interpolated elevations to be a stationary random function over the entire map sheet (data storage unit). To solve this problem, the area can be divided into smaller homogeneous parts, but this puts strong requirements on the organization of the data banks. An alternative is instead to specify an average covariance function, a solution which might be preferred from a data storage point of view. The drawback is that the "resolution" of the accuracy specification is reduced and that only average estimates of the variance can be made. In the remaining part of this paper, only average covariance functions will be considered.

The terrain surface is here supposed to be a non-isotropic two-dimensional surface in a three-dimensional space. It can, therefore, also be assumed that the errors in interpolated elevations are non-isotropic. As a consequence, two-dimensional covariance functions should be used for proper quality specification.

ESTIMATION OF THE COVARIANCE FUNCTIONS OF INTERPOLATED ELEVATION ERRORS

The estimation of the errors in digital terrain data can be based on experience and/or on check measurements. Experience-based error estimation is less time consuming but the estimates may, on the other hand, be less accurate. Check measurements should of course be of considerably higher quality than the data being checked, in this case interpolated terrain elevations. The check measurements and the interpolated elevations should also be as independent as possible. One should, for example, avoid using the same photographs and orientations for the check measurements as for the original digital elevation matrix (DEM) measurements. If the two data sets are highly correlated, the systematic part of the errors in the data bank is difficult to estimate.

In this paper, four different methods for the estimation of the covariance of errors in interpolated elevations will be described and compared.

UNCORRELATED ERRORS ASSUMED

Usually, the accuracy of digital elevation data banks is specified by the standard error of stored elevations. If only this information is used when estimating the accuracy of functions of the elevations, the covariances among the errors are neglected. Although the method yields biased estimates, it is often used because of its simplicity and because the covariance of the errors is unknown. When neglecting the covariance of the errors, the standard error, σ_a, of an arbitrary application is estimated by

$$\sigma_a^2 = \sigma_z^2 \cdot g(0,0)$$

where σ_z is the standard error of the elevation data.

The standard error (σ_z) can be estimated using rules of thumb or from check measurements.

A Priori KNOWN CORRELATION FUNCTIONS

Instead of neglecting the covariance function, *a priori* known correlation functions can be used to improve the estimates. Figure 1 shows a sample of correlation functions of interpolated errors. A correlation function is a covariance function normalized with the variance. The functions in Figure 1 are derived from the result of an international test of DEMs, conducted by the International Society for Photogrammetry and Remote Sensing (ISPRS). The material is described in detail by Torlegård *et al.* (1984).

The total data set of the ISPRS DEM test consists of 66 DEMs, covering six different areas. When constructing the *a priori* correlation function, 11 DEMs were used, covering three of the six test areas (Figure 1). The three test areas were chosen randomly.

The function was constructed graphically from Figure 1. The average correlation then obtained is

$$K(d) = k_1(d) + k_2(d) + k_3(d) \qquad (4)$$

where

$$k_1(d) = 0.20 \cdot \begin{cases} 1 & \text{if } d = 0 \\ 0 & \text{otherwise;} \end{cases}$$

$$k_2(d) = 0.55 \cdot \begin{cases} 1 - \dfrac{d}{50} & \text{if abs}(d) < 50 \text{ m; and} \\ 0 & \text{otherwise} \end{cases}$$

$$k_3(d) = 0.25 \cdot \begin{cases} 1 - \dfrac{d}{450} & \text{if abs}(d) < 450 \text{ m} \\ 0 & \text{otherwise} \end{cases}$$

When using *a priori* known correlation functions, the standard error of an arbitrary application σ_a is estimated by

$$\sigma_a^2 = \sigma_z^2 \cdot \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\cdot K(d)\,dxdy$$

where σ_z is the standard error of the elevations and

$$d^2 = x^2 + y^2.$$

As in the previous case, the standard error, σ_z, can be estimated either by using rules of thumb or from check measurements.

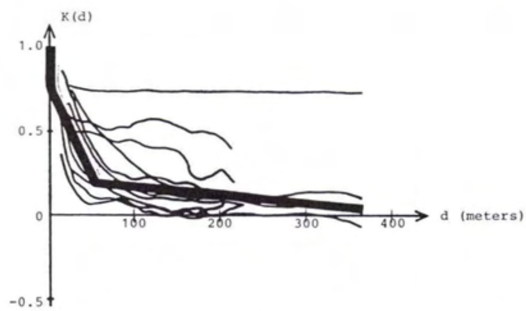

FIG. 1. Sample of correlation functions of interpolated errors, area A, C, and E. The thick solid line shows the average correlation function $K(d)$, Equation 4.

Autocovariance Computation

The term autocovariance is used for expressing the covariance within one single set of data. In the one-dimensional case, the k^{th} lag autocovariance, c_k, is estimated as

$$c_k = \frac{1}{N-k} \sum_{t=1}^{N-k} z_t \, z_{t+k}, \; k << N \, .$$

The method requires in this case that check measurements be performed along profiles in the one-dimensional case or in patches in the two-dimensional case. The check measurements are then compared with elevations interpolated from the data bank.

According to Frederiksen *et al.* (1984), the estimation of the covariance of the *terrain* is difficult in practice, mainly due to the fact that

- the elevations are not normally distributed and individual extremes have undue influence on the estimates, and
- long trends or semi-systematic fluctuations seriously distort the estimates.

To obtain good estimates, Frederiksen *et al.* (1984) suggest that the correlation be estimated by rank correlation methods. They also suggest that large regional terrain forms be described by a separate stochastic process, a proposal also made by Schagen (1980).

There is no reason to believe that the estimation of the covariance of the *errors* will be relieved of these problems. Because the errors in interpolated elevations are assumed not to be a stationary stochastic process, several profiles have to be selected to achieve a good estimate of the average covariance function.

The MINQUE Method

The problems in the estimation of the autocovariance functions, as mentioned in the previous section, can be reduced by applying some kind of restriction on the behavior of the functions. There are several filters for smoothing a function. Some of the filters, which preserve the characteristics of a correlation function, are described in Groten *et al.* (1979). Those methods are, in general, more or less numerical methods for obtaining good looking curves. Another approach is to try to model the underlying processes for the errors in interpolated terrain elevations. The MINQUE method is an attempt to incorporate some additional information in the procedure.

The MINQUE (Minimum Norm Quadratic Unbiased Estimator) method is a method for the estimation of variance-covariance (V-C) components within a least-squares adjustment. Its theoretical background can be found in, for example, Persson (1981) and Sjöberg (1983). In this paper, the method will be used with respect to the determination of V-C components for a set of elevation data, checked by accurate check measurements.

Let $\hat{z}_i$ be a set of interpolated elevations and let z_i be the corresponding check measurements (true values). These two sets of data form a general condition equation system

$$\mathbf{B} \cdot \boldsymbol{\epsilon} = \mathbf{W}$$

where $\mathbf{B}$ = $\mathbf{I}$ (unit matrix),
$\mathbf{W}$ = a vector of discrepancies $\hat{z}_i - z_i$,
$\boldsymbol{\epsilon}$ = a vector of normally distributed errors,
$E(\boldsymbol{\epsilon})$ = 0,
$E\{\boldsymbol{\epsilon}\boldsymbol{\epsilon}^T\}$ = $\mathbf{Q}$, and
$\mathbf{Q}$ = a positive definite covariance matrix.

Assume that the covariance matrix $\mathbf{Q}$ is a sum of a limited set of V-C components, γ_i, multiplied by the corresponding correlation matrices, $\mathbf{K}_i$, such that

$$\mathbf{Q} = \sum_{i=1}^{n} \mathbf{Q}_i$$

where $\mathbf{Q}_i$ = $\gamma_i \mathbf{K}_i$ is the i^{th} covariance matrix and γ_i is the corresponding covariance coefficient

According to Sjöberg (1983), the Best Quadratic Unbiased Estimator (BQUE) or γ, if γ estimable, is

$$\hat{\gamma} = \mathbf{S}^{-1}\mathbf{u} \tag{7}$$

$$\text{where } \mathbf{u}_i = \mathbf{W}^T\mathbf{Q}^{-1}\mathbf{Q}_i\mathbf{Q}^{-1}\mathbf{W}$$

$$\mathbf{S}_{ij} = \text{trace}\,\{\mathbf{Q}^{-1}\mathbf{Q}_i\mathbf{Q}^{-1}\mathbf{Q}_j\}$$

The matix $\mathbf{S}$ is assumed to be nonsingular.

The procedure is iterative. From initial values of the V-C components γ_i, the matrices $\mathbf{Q}_i$ are formed and used in the computations of new γ values. The procedure is iterated until convergency. It should be noted that the estimated γ may come out negative, which appears to be in conflict with common restrictions on variance components. Reasons for this may be an inadequate stochastic model and/or too few degrees of freedom in the adjustment.

Consider Equation 4 when designing a V-C model for MINQUE estimation of errors in digital elevation data. The average correlation function $K(d)$ is here expressed as the sum of a regional, a local, and an uncorrelated function. It can be shown that some systematic errors, such as improper orientation of the stereomodel or image deformation, give regional covariance functions of the errors, while errors in the height settings usually give local covariance functions. A V-C model based on the three subfunctions in Equation 4 therefore seems justified.

In accordance with Equation 4, the V-C model being used for the estimation of the covariance function is

$$\mathbf{Q} = \gamma_1\mathbf{K}_1 + \gamma_2\mathbf{K}_2 + \gamma_3\mathbf{K}_3.$$

If check measurements are carried out along a profile with a point spacing of 25 metres, the $\mathbf{K}_i$ matrices are

$$\mathbf{K}_1 = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix},$$

$$\mathbf{K}_2 = \begin{pmatrix} 1 & 0.5 & 0 & \cdots & 0 \\ 0.5 & 1 & 0.5 & \cdots & 0 \\ 0 & 0.5 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}, \text{ and}$$

$$\mathbf{K}_3 = \begin{pmatrix} 1 & 0.95 & 0.89 & \cdots & 0 \\ 0.95 & 1 & 0.95 & \cdots & 0 \\ 0.89 & 0.95 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}.$$

COMPARATIVE TEST OF METHODS FOR ACCURACY ESTIMATION

Description of the Test Data

The methods for error estimation are evaluated by using the data sets from the ISPRS DEM test. The material is described by Torlegård *et al.* (1984) and a brief review only will be given here.

Fourteen different organizations participated in the ISPRS test. They were asked to derive digital elevation models covering six different areas, using photogrammetric methods. The six test areas are located in the central and northern parts of Europe. The aerial photographs varied in scale between 1:4,000 and 1:30,000. After returning the photographs to the organizers of the test, coordinates of check points were delivered to the participants. The check points are ordered on a regular grid, randomly translated and rotated with respect to the map coordinate system. Elevations of the check points were interpolated by the participants and compared with the corresponding true values by the organizers of the test. The "true" elevations were obtained by direct measurements on aerial photographs with considerably larger scales (1:1,500 to 1:10,000). The data sets consist of 66 error surfaces, representing the differences between interpolated elevations and the corresponding "true" elevations.

Design of the Test for the Comparison of the Methods for Covariance Estimation

One purpose of this paper is to compare methods for the estimation of the covariance function of errors in interpolated elevations, as described earlier. Profiles from the areas A, C, and E have been used when deriving the *a priori* correlation function. Therefore, error surfaces from the other three areas B, D, and F of the ISPRS test have been used in the comparative test. The data are located in regular grids, providing a sample of profiles in two perpendicular directions. In a real situation, these profiles correspond to precise check measurements which can be carried out by field surveys or by photogrammetric surveys. For each error surface, a sample of profiles has been examined and the covariance functions have been estimated by using the four methods described earlier. Due to limited computer resources, only one-dimensional covariance functions have been estimated. By assuming isotropy, the standard errors in functions of the elevations have been estimated. In the evaluation, the error estimates have been compared with the true errors as computed directly from the two-dimensional non-isotropic error surfaces. This means that also the assumption of isotropy to some extent has been tested.

RESULTS

In the evaluation of the methods for covariance estimation, four different estimates have been used, namely,

- the errors in the estimation of point accuracy,
- the errors in the estimation of height difference (slope) accuracy,
- the errors in the estimation of volume accuracy (local area, 0.04 to 1.4 hectare), and
- the errors in the estimation of volume accuracy (regional area, 0.5 to 16 hectare).

For each profile, the standard errors in elevation, elevation difference, and volume have been estimated. By comparing these estimates with the corresponding "true" standard errors, as computed directly from the entire error surface, the root mean square (RMS) of the estimates has been computed. For the detection of any possible bias in the estimates, the mean value and standard deviations of the errors in the estimates have also been computed. Table 1 shows an example of the results obtained for participant 3, area F, when uncorrelated errors were assumed.

The result shown in Table 1 is fairly representative of the results obtained when neglecting the covariance. A high t-value, which is the absolute value of the ratio between the mean value and its standard deviation, indicates a biased estimate. The different profiles, from which the estimates are derived, are located at such distances that the correlations between them in most cases are close to zero. This means that the bias of the estimates (its difference from zero) can be tested by a Student test. Table 2 shows the result of such a test.

When reading Table 2, one should consider that the t-test is

Table 1. Errors in Accuracy Estimation by Neglecting the Covariances. Participant 3, Area F.

	Point	Coord. diff.	Volume-1	Volume-2
Mean error	0.03	0.17	−1065	−1002
Std. deviation	0.08	0.11	158	53
RMS error	0.08	0.20	1076	1004
t-value	0.38	1.51	6.73	19.02

Table 2. Percentage of Biased Estimates According to a Student Test. Area B, D, and F. $\alpha = 0.05$.

	Estimation of errors in		
Error estim. method	Elevation (%)	Elev. diff. (%)	Volume (%)
Neglecting correlation	0	5	91
A priori correlation	0	5	7
Autocovariance computation	0	0	0
MINQUE	5	5	3

Table 3. RMS of erros in the Volume Estimates (global area) by using only one Profile of 50 Check Measurements (m^3/ha).

	Test Area		
	Bohuslän	Drivdalen	Spitze
Neglecting correlation	5840	16070	1050
A priori correlation	3580	11520	1310
Autocovariance comp.	3680	10520	1280
MINQUE	9560	8330	760

carried out under a risk of 5 percent. This means that 5 percent of the unbiased estimates will here be classified as biased. One should also consider that the different profiles are not fully independent. Although the correlations among the profiles are very limited, one should be careful when drawing conclusions from Table 2. It is, however, obvious that neglecting the correlation will give biased estimates, especially for applications such as volume calculations where a fairly large part of the covariance function should be used. It should also be noticed that the method of using *a priori* known correlation functions gives nearly unbiased estimates. This means that the covariance function in Equation 4 worked very well also for the three areas B, D, and F.

The RMS of the errors in the volume estimates are shown in Table 3. Here one can notice that, in general, the worst results were obtained when neglecting the correlation. The method also has the disadvantage that it gives biased estimates (see Table 2). The benefit of using more check measurements is, therefore, very limited. This is of great importance because it clearly shows that this computational method should be avoided unless one has strong reasons to assume that the errors are uncorrelated.

The average length of the profiles is 50 check measurements, while the length of the correlation function is covered by 16 check measurements. Due to limited computer resources, the profile length for the MINQUE method was decreased to 20 check measurements.

One problem of the MINQUE method is that some estimated V-C components may come out negative. Because the number of degrees of freedom is large (20 conditions and three unknown V-C components), the presence of negative estimates is assumed to be dependent on an insufficient V-C model. As a consequence, these estimates have been neglected. This happened for almost 64 percent of the profiles. The problem might have been reduced by using longer profiles for the MINQUE estimation. The result shown in Table 3 is normalized in such a way that all results correspond to one measured profile with 50 check measurements.

The *a priori* known correlation function seems to give nearly the same results as the method of computing the autocovariance function. This means that, if only the standard error of the elevations is known, one could use the correlation function described in Equation 4. However, it should be pointed out that this correlation function is only tested on photogrammetrically measured digital elevation models of terrain in northern Europe. If other data acquisition methods are used or if other terrain types are measured, its usefulness may be limited.

The MINQUE method gives, in general, the best result, with two important exceptions. For two of the data sets of the Bohuslän area, the MINQUE estimation gave very poor estimates. The data sets differ somewhat from the other data sets used, in that one of the sets was obtained from digitized contour lines and the other data set was derived from a DEM with an extremely low density of measurements.

These exceptions indicate, however, that the MINQUE method has to be used carefully and preferably as a complement to some other more robust method. The selection of a proper set of correlation functions is obviously essential for a successful MINQUE estimation.

The lack of robustness of the MINQUE method also shows the needs of methods for accuracy estimation of the error estimates. By using several profiles, average estimates and standard deviations can be computed. For all data sets studied, a large standard deviation indicated poor estimates. The opposite relation, a small standard deviation and accurate estimates, holds only in cases when the estimates are unbiased. For a specific case, this relation has, as a consequence, to be used with care.

DISCUSSION

In the introduction, three fundamental questions concerning the specification of the accuracy in digital elevation data were raised, namely,

- Which quantities have to be known in order to estimate the accuracy in functions of the terrain elevations?
- How can they be estimated? and
- How can they be specified?

If we restrict ourselves to considering the standard errors in functions of the elevations, it has been shown that the covariance function of the interpolated errors has to be known and specified (Equation 3). The estimation of this covariance function can by performed either from check measurements or by using rules of thumb. If the data are sampled by photogrammetric methods in stereo instruments, the correlation function described in Equation 4 seems to be suitable as a rule of thumb. Otherwise, the covariance function could be estimated by performing check measurements and using the MINQUE method in combination with autocovariance computations. If the results obtained by the two methods are nearly in agreement, the MINQUE estimate will probably be more accurate.

The specification of the accuracy of a digital elevation data bank can be expressed in several ways. One way is to specify the *source* of the data, for example, data aquisition method, sampling point density, image scale of aerial photographs, etc. For data banks where the covariance function of the errors is unknown, this might be a solution. Even if the *standard error* of the stored elevations is known and specified, the source of the data has to be specified for a proper selection of correlation function.

If several variance-covariance components are estimated by, for example, the MINQUE technique, the quality of the specification will be improved. One central problem here is to decide on which correlation functions the MINQUE estimate and the quality specification should be based. In this paper, Equation 4 has been used. A more rigorous approach might be to estimate the correlation functions by using the ISPRS test material in a Karhunen-Loeve expansion. In this approach, principal correlation functions are derived numerically. The benefits of such an approach are left for further studies.

There are, of course, several other topics which are of interest for further studies, for example,

- This study is restricted mainly to digital elevation data sampled in photogrammetric stereo instruments. There are many other types of measurements which are of interest, for example, digitized contour maps, field surveys, and automatic correlated digital images. It is of great interest to derive suitable covariance functions for different data acquisition methods, which can be used either in the quality specification or by the user when performing error estimation.
- This study is also limited to the standard errors in functions of the elevations. Another urgent research task is to study other accuracy estimates, for example, maximum errors. In this case, the size and distribution of the gross errors will play an important role.
- The mathematical models used in this study can, of course, be extended. Using two-dimensional covariance functions will probably increase the accuracy of the estimates. It would also be of interest to study methods of incorporating additional information into the estimation procedure.
- It can be assumed that different data sets and different terrain types cause different kinds of covariance functions. For data banks being updated, the data also have to be specified in the fourth dimension (time). This puts strong demands on the organization of the data banks, which yet is unsolved. Nevertheless, the increasing distribution and reuse of terrain information requires a proper specification of its quality.

Today, data banks containing terrain information are already used for various applications on a commercial basis. It can be assumed, that a proper quality specification of the information will increase the interest for its use. A good quality specification can probably be a tool for broadening the market for the producers of terrain information. The user definitely wishes to be able to estimate the accuracy of his product. If he can evaluate the effects of using a certain data set, he will probably choose that solution instead of using a, perhaps, cheaper data set but with unknown quality. But before the quality of the data sets can be specified, guidelines for such a specification have to be worked out. It is hoped that this paper has contribued to some extent to this very important task.

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