Triangulating from Optical and SAR Images Using Direct Linear Transformations

Michael H. Brill
Science Applications International Corporation, 803 West Broad Street, Suite 500, Falls Church, VA 22046

ABSTRACT: Given an optical photograph with images of six control points (whose three-space coordinates are known), and the image coordinates of an unknown ground point, the Direct Linear Transformation (DLT) allows a noniterative resection and determination of the line of sight from the camera station to the unknown point. The method requires writing the projective equations for the control points in a form that is linear in functions of the acquisition geometry and (assumed affine) film distortions. When written so that each image coordinate of each control point appears in only one equation, the linear system is solvable for 11 camera variables, which can then be used to determine the line of sight to the unknown point. In the spotlight mode of synthetic-aperture radar (SAR), similar DLT condition equations are developed to determine the projection circle for any unknown ground point given its image coordinates. The Doppler conditions are solved formally for one image coordinate of each control point, and this solution is substituted into the range equations in order to render them nonsingular. The result is a linear system in 13 "camera" variables that can be used to determine the projection circle through a ground point. Eight control points are needed for solution. Methods are discussed for performing intersection using optical lines of sight, SAR projection circles, and combinations thereof. Such methods may be useful for multisensor data fusion from optical and SAR images.

INTRODUCTION

MULTISENSOR DATA FUSION has recently become an active research area for satellite and aerial image acquisition (Himbarger, 1984). In particular, optical imagery and synthetic-aperture radar (SAR) would offer important benefits from data fusion. Optical and SAR images are most useful in distinct environments: SAR has an all-weather day/night capability and offers range-independent range resolution; on the other hand, optical imagery is familiar, is easily acquired, and has a multispectral capability. Fusion of SAR and optical data could enhance the coverage area of both image modalities, and could also enhance the information content in the areas of overlapping coverage.

As an important subclass of information enhancement, SAR and optical images provide literally different points of view of imaged terrain, allowing a generalized triangulation to infer the location of target points from their SAR and optical image coordinates. To provide resection and intersection on such hybrid imagery would require that the corresponding image coordinates of control points be accurately determined, presumably by means of a stereo comparator. The stereo fusability of optical and SAR images is an interesting research question for future study. The present article will not address this question, but rather will discuss an algorithmic problem that besets optical and SAR resection, and whose resolution would greatly benefit the triangulation from hybrid imagery. The problem is that iterative least-squares methods for solving the nonlinear condition equations (see, e.g., Slama (1980)) tend to converge only when the acquisition geometry is rather well known ahead of time; least-squares resection then refines good first estimates of the acquisition geometry.

In the absence of good initial estimates, how can resection be done? The ideal solution to this problem would be to solve the nonlinear condition equations in closed form, so that no iterations (and hence no first estimates) are required. In fact, such a method had been developed for optical images acquired with nonmetric cameras (Abdel-Aziz and Karara, 1971; Williamson, 1972; Karara and Abdel-Aziz, 1974). This method, called Direct Linear Transformation (DLT), has been successful in various applications of close-range photogrammetry (Williamson, 1972, 1986; Karara, 1974; Brandow *et al.*, 1976) where nonmetric cameras are prevalent. The advantages of the DLT method are direct noniterative resection and the ability to proceed without prior interior orientation of the image (subject to the assumption that image distortions are affine). A possible limitation of the method is the difficulty of incorporating into the solution the statistical weightings appropriate to the accuracies of the measured parameters. Such weightings are, of course, an integral part of the iterative least-squares method (Slama, 1980). This limitation notwithstanding, the DLT resection may provide a viable first estimate of the acquisition goemetry that can be used as an input to the traditional least-squares algorithm when better first estimates are unavailable.

As the name would suggest, DLT involves casting the photogrammetric condition equations in a form that is linear in functions of the resection parameters, with coefficients that depend on the object-space coordinates and image coordinates of the control points. In the present article, optical DLT is reviewed in a concise mathematical form. Also, a new method is proposed for performing DLT resection on spotlight-mode SAR images in ground-range presentation. Finally, a method is discussed for hybrid-image "triangulation" using a SAR image and an optical image, predicated on knowledge of a sufficient number of control points and of their image coordinates. This article is based on a paper presented to the Optical Society of America (Brill, 1986).

OPTICAL RESECTION BY DLT

The nominal problem of photogrammetric resection is to determine the position and attitude of a camera with respect to an exterior coordinate system (Slama, 1980). Resection (see Figure 1) requires knowledge of the image coordinates (p_{1i}, p_{2i}) and of the three-dimensional coordinate vectors $\mathbf{X}_i$ of N control points ($i=1, \ldots, N$). To apply the DLT method, the task of resection is expanded somewhat to include interior orientation, subject to the assumption that image distortions are affine. Hence, the known image coordinates (p_{1i}, p_{2i}) will be the coordinates read in by means of a comparator, without reseaux or fiducials. The task of resection in this generalized form will therefore be to determine the camera station **S**, the object-space point **Q** that corresponds to the origin of comparator coordinates, and the object-space 3-vectors **a** and **b** that correspond to the x and y image axes. A

PHOTOGRAMMETRIC ENGINEERING AND REMOTE SENSING,
Vol. 53, No. 8, August 1987, pp. 1097–1102.

0099-1112/87/5308–1097$02.25/0

point on the image plane designated by (p_{1i}, p_{2i}) is therefore expressed in exterior (object-space) coordinates as $Y_i = p_{1i}\,\mathbf{a} + p_{2i}\,\mathbf{b} + \mathbf{Q}$. The point $\mathbf{Y}_i$ is the intersection of the image plane with the line from the camera station $\mathbf{S}$ to the control point $\mathbf{X}_i$. Collinearity of $\mathbf{S}$, $\mathbf{Y}_i$, and $\mathbf{X}_i$ gives rise to the following condition equations:

$$\frac{X_{ik} - S_k}{X_{i3} - S_3} = \frac{Y_{ik} - S_k}{Y_{i3} - S_3} = \frac{p_{1i}\,a_k + p_{2i}\,b_k + (Q_k - S_k)}{p_{1i}\,a_3 + p_{2i}\,b_3 + (Q_3 - S_3)}, \tag{1}$$

where $k=1,2$ denotes the exterior coordinate of a point.

Resection uses the $2N$ equations (Equation 1) to estimate, for known $\mathbf{X}_i$, p_{1i}, and p_{2i}, the 12 numbers that comprise vectors **a, b, Q,** and **S.** It is evident that Equation 1 cannot be solved for all these vector components, for if the equations are satisfied for one set **a, b, Q,** and **S**, they are satisfied for any scaled set $c\mathbf{a}$, $c\mathbf{b}$, $c\mathbf{Q}$, and $c\mathbf{S}$ (for nonzero c). However, enough information can be extracted from the condition equations for multiple images to perform *intersection*: i.e., to determine the coordinates of a target point **X** from its image coordinates. As will be seen, resection determines parametric equations for the line of sight from **S** to **X**, if the image coordinates (p_1, p_2) of **X** are known. This line of sight is precisely what can be determined about point **X** from a single image. Intersection in optical images locates the point common to several such lines of sight, one from each image. The line of sight can also be used to perform an analogue of intersection for an optical and a SAR image, as will be shown later.

The DLT method retrieves the information necessary to construct lines of sight by transforming Equation 1 into a system of equations linear in *functions* of **a, b, Q,** and **S**, with coefficients that depend on the known quantities X_{ik}, p_{1i}, and p_{2i}. A linear system can, of course, be obtained by multiplying Equation 1 by both the denominators, but this produces too many functions of the unknowns to be useful. A sufficient *Ansatz* to reduce this number of functions is first to solve Equation 1 formally for p_{ki} $(k=1,2)$, so that no equation contains both p_{1i} and p_{2i}: i.e.,

$$p_{ki} = \sum_{j=1}^{3} A_{kj}\,(X_{ij} - S_j) \Big/ \sum_{j=1}^{3} A_{3j}\,(X_{ij} - S_j), \tag{2}$$

where $k = 1$ or 2, $i, = 1,\ldots, N$, and

$$\mathbf{A} = \begin{bmatrix} a_1 & b_1 & (Q_1 - S_1) \\ a_2 & b_2 & (Q_2 - S_2) \\ a_3 & b_3 & (Q_3 - S_3) \end{bmatrix}^{-1}. \tag{3}$$

Multiplying Equation 2 by its denominator leads to a homogeneous system of equations linear in functions of **a, b, Q,** and **S**: ie.,

$$\begin{bmatrix} X_{i1}p_{1i} & X_{i2}p_{1i} & X_{i3}p_{1i} & p_{1i} & X_{i1} & X_{i2} & X_{i3} & 1 & 0 & 0 & 0 & 0 \\ \vdots & & & & & & & & & & & \\ X_{i1}p_{2i} & X_{i2}p_{2i} & X_{i3}p_{2i} & p_{2i} & 0 & 0 & 0 & 0 & X_{i1} & X_{i2} & X_{i3} & 1 \\ \vdots & & & & & & & & & & & \end{bmatrix} \begin{bmatrix} \hat{m}_1 \\ \vdots \\ \hat{m}_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{4}$$

where $\hat{m}_k$ are known functions of **a, b, Q,** and **S**, and each vertical ellipsis in the left-hand-side matrix indicates $i=1,\ldots, N$. (The explicit functions are not needed in the algorithm, but are enumerated for completeness in the Appendix.)

To render Equation 4 solvable for functions of the resection values, the equation must be made inhomogeneous. This can be done by dividing the equations by $\hat{m}_{12}$, defining $m_k = -\hat{m}_k/\hat{m}_{12}$, and retrieving a nonzero right-hand side: i.e.,

$$\begin{bmatrix} X_{i1}p_{1i} & X_{i2}p_{1i} & X_{i3}p_{1i} & p_{1i} & X_{i1} & X_{i2} & X_{i3} & 1 & 0 & 0 & 0 \\ \vdots & & & & & & & & & & \\ X_{i1}p_{2i} & X_{i2}p_{2i} & X_{i3}p_{2i} & p_{2i} & 0 & 0 & 0 & 0 & X_{i1} & X_{i2} & X_{i3} \\ \vdots & & & & & & & & & & \end{bmatrix} \begin{bmatrix} m_1 \\ \vdots \\ m_{11} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \end{bmatrix}. \tag{5}$$

At least six control points are required for a solution to these equations; the solution is a least-square best fit to m_k obtained by pseudoinverting the left-hand-side matrix on the right-hand-side vector. In order for the pseudoinverse to exist, the control points must not all be coplanar, for in that case columns 5 to 8 of the left-hand-side matrix would be linear combinations of each other. The eleven *resection parameters* m_k comprise all the retrievable information about **a, b, Q,** and **S**.

It is now readily shown that the line of sight containing an object-space point **X** and the camera station **S** is obtainable from the values m_k and the image coordinates (p_1, p_2) of **X**. This fact emerges from adapting the condition equations (Equation 5) to apply to the point **X**: i.e.,

$$\begin{bmatrix} X_1p_1 & X_2p_1 & X_3p_1 & p_1 & X_1 & X_2 & X_3 & 1 & 0 & 0 & 0 \\ X_1p_2 & X_2p_2 & X_3p_2 & p_2 & 0 & 0 & 0 & 0 & X_1 & X_2 & X_3 \end{bmatrix} \begin{bmatrix} m_1 \\ \vdots \\ m_{11} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{6}$$

where X_1, X_2, X_3 are the coordinates of the point **X**. It can be seen that Equation 6 is linear in the new unknowns X_1, X_2, and X_3, hence the equations define a line (the line of sight) in 3-space.

It is instructive to note the geometrical significance of the resection value that is irretrievable by the DLT algorithm. As noted after Equation 1, multiplying **a, b, Q,** and **S** by a nonzero factor c cannot affect whether these vectors are solutions to the condition equations. The scale factor c corresponds to the scale of the image compared to the distance of the image plane from the camera station. For a given object point **X**, a larger image ($\mathbf{a} \rightarrow c\mathbf{a}$ $\mathbf{b} \rightarrow c\mathbf{b}$) that is farther from the camera station ($\mathbf{Q} - \mathbf{S} \rightarrow c\mathbf{Q} - c\mathbf{S}$) produces the same image coordinates (p_1, p_2). Henceforth, a resection parameter that is irretrievable from a set of condition equations will be referred to as a *cyclic variable*—a term borrowed from Lagrangian mechanics (Goldstein, 1965).

SAR RESECTION BY DLT

In the spotlight mode (Ausherman *et al.*, 1984) and in ground-range presentation, the SAR image is a straightforward geometrical construction (see Rosenfield (1968) and Leberl (1979) for this geometry). As shown in Figure 2, a ground point

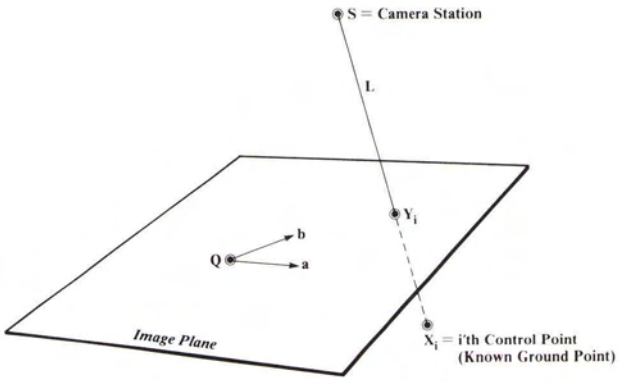

FIG. 1. Optical resection geometry, showing line of sight L between camera station **S** and control point $\mathbf{X}_i$, through image point $\mathbf{Y}_i$.

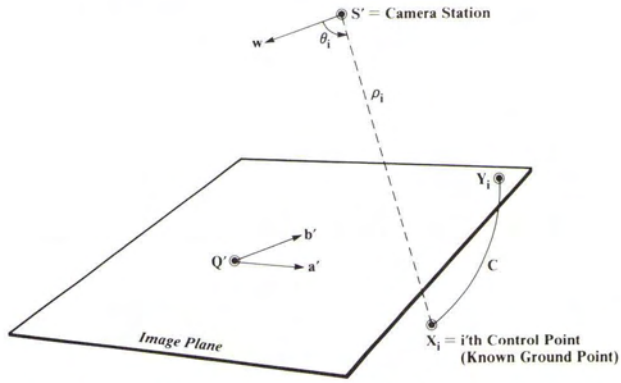

FIG. 2. SAR resection geometry, showing projection circle C that contains control point $\mathbf{X}_i$ and image point $\mathbf{Y}_i$.

$\mathbf{X}_i$ reflects a radar pulse that signals a range ρ_i from "camera station" $\mathbf{S}'$ (determined from the signal round-trip travel time) and a Doppler-cone angle θ_i of the line of sight from the normalized velocity vector $\mathbf{w}$ of the sensor. (The normalization $w_1 = 1$ will be used here.) The numbers P_i and θ_i identify the point $\mathbf{X}_i$ as lysing on a particular range sphere $(\mathbf{X}_i - \mathbf{S}')^2 = \rho_i^2$ and Doppler cone $\cos\theta_i = (\mathbf{X}_i - \mathbf{S}').\mathbf{w}/[\mathbf{X}_i - \mathbf{S}']\,[\mathbf{w}]$. The intersection of these surfaces is called a *projection circle*. A single image of a ground point identifies the projection circle on which a ground point lies, but more than one image is required to determine *where* on that circle that point lies. A radar image of $\mathbf{X}_i$ is obtained by intersecting the projection circle with an arbitrary plane (which must be chosen so that intersections exist for all ground points of interest). As shown in Figure 2, each ground control point $\mathbf{X}_i$ images on the point $\mathbf{Y}_i = \mathbf{Q}' + q_{1i}\,\mathbf{a}' + q_{2i}\,\mathbf{b}'$, where $\mathbf{Q}'$, $\mathbf{a}'$, and $\mathbf{b}'$ are defined in analogy with $\mathbf{Q}$, $\mathbf{a}$, and $\mathbf{b}$. The nominal task of radargrammetric resection is to determine $\mathbf{Q}'$, $\mathbf{a}'$, $\mathbf{b}'$, and $\mathbf{w}$ from N control points $\mathbf{X}_i$ and their image coordinates (q_{1i}, q_{2i}).

The range and Doppler condition equations for this SAR problem are

$$(\mathbf{X}_i - \mathbf{S}')^2 = (\mathbf{Y}_i - \mathbf{S}')^2 = (q_{1i}\mathbf{a}' + q_{2i}\mathbf{b}' + \mathbf{Q}' - \mathbf{S}')^2$$
$$(\mathbf{X}_i - \mathbf{S}')\cdot\mathbf{w} = (\mathbf{Y}_i - \mathbf{S}')\cdot\mathbf{w} = (q_{1i}\mathbf{a}' + q_{2i}\mathbf{b}' + \mathbf{Q}' - \mathbf{S}')\cdot\mathbf{w}. \tag{7}$$

These equations state that the points $\mathbf{X}_i$ and $\mathbf{Y}_i$ (for a particular i) are on the same range sphere and Doppler cone and, hence, on the same projection circle. In analogy with the collinearity equations of photogrammetry, these equations might be called co-circularity relations. The nominal resection problem is to estimate the 14 values w_2, w_3, $\mathbf{a}'$, $\mathbf{b}'$, $\mathbf{Q}'$, and $\mathbf{S}'$ from these condition equations applied to the control points.

As in the previous section, a complete solution for the resection values is in principle impossible, but the reason is more subtle than a simple scaling symmetry. An analogous DLT method, however, allows removal of the cyclic variable, and enough resection parameters can be retrieved so that the projection circle of an arbitrary ground point $\mathbf{X}$ can be unambiguously determined from its image coordinates (q_1, q_2). If two projection circles from two SAR images of the same ground point are intersected, they locate the ground point (sometimes up to a binary decision, because some pairs of circles intersect twice). The point $\mathbf{X}$ can also be located by intersecting a line of sight from an optical image with the projection circle of a SAR image of $\mathbf{X}$. These operations will be discussed in the next section.

Expanding Equation 7 produces a linear system of equations for range and Doppler conditions: i.e.,

$$\begin{bmatrix} 1 & q_{1i}^2 & q_{2i}^2 & q_{1i}q_{2i} & q_{1i} & q_{2i} & X_{i1} & X_{i2} & X_{i3} \\ \vdots & & & & & & & & \vdots \end{bmatrix} \begin{bmatrix} \hat{l}_1 \\ \vdots \\ \hat{l}_9 \end{bmatrix} = \begin{bmatrix} X_i^2 \\ \vdots \end{bmatrix}, \tag{8a}$$

$$\begin{bmatrix} 1 & q_{1i} & q_{2i} & X_{i2} & X_{i3} \\ \vdots & & & & \end{bmatrix} \begin{bmatrix} \hat{l}_{10} \\ \vdots \\ \hat{l}_{14} \end{bmatrix} = \begin{bmatrix} X_{i1} \\ \vdots \end{bmatrix}. \tag{8b}$$

Here, $\hat{l}_k$ are known functions of $\mathbf{a}'$, $\mathbf{b}'$, $\mathbf{Q}'$, $\mathbf{S}'$, and $\mathbf{w}$. These functions are enumerated in the Appendix. Each vertical ellipsis indicates $i = 1, \ldots, N$ in the left-hand-side matrices.

It might appear that these equations can be solved for $\hat{l}_1, \ldots, \hat{l}_{14}$, which can in turn be solved for the 14 resection values. However, the range-equation matrix (in Equation 8a) is always singular, because the Doppler-equation system (Equation 8b) reveals that columns 1, 5, 6, 7, 8, and 9 of the range-equation matrix are linear combinations of each other. This problem can be remedied by solving the Doppler equations formally for q_{1i}, and substituting the result in for the terms q_{1i} in the range-equation system. This yields a new range-equation system with eight unknowns and eight columns in the left-hand-side matrix. (The q_{1i} column has been eliminated.) The condition equations become

$$\begin{bmatrix} 1 & q_{1i}^2 & q_{2i}^2 & q_{i1}q_{i2} & q_{2i} & X_{i1} & X_{i2} & X_{i3} \\ \vdots & & & & & & & \end{bmatrix} \begin{bmatrix} l_1 \\ \vdots \\ l_8 \end{bmatrix} = \begin{bmatrix} X_i^2 \\ \vdots \end{bmatrix} \tag{9a}$$

$$\begin{bmatrix} 1 & q_{1i} & q_{2i} & X_{i2} & X_{i3} \\ \vdots & & & & \end{bmatrix} \begin{bmatrix} l_9 \\ \vdots \\ l \end{bmatrix} = \begin{bmatrix} X_{i1} \\ \vdots \end{bmatrix}, \tag{9b}$$

where $l_1, \ldots, l_{13}$ are readily obtained functions of $\hat{l}_1, \ldots, \hat{l}_{14}$.

At least eight control points are required for a solution to these equations. The solution is a least-square best fit obtained by pseudoinverting the left-hand-side matrices on the right-hand-side vectors. It can be seen that the existence of the pseudoinverse once again depends on the control points not all lying in the same plane—for if they did, the last three columns of the range and Doppler matrices would be linear combinations of each other. The 13 resection parameters comprise all the retrievable resection information, in analogy with the photgrammetry problem of the previous section.

It now can be shown that the projection circle containing a ground point **X** is determined by l_k and the image coordinates (q_1, q_2) of **X**. This can be done by adapting the condition equations (Equation 9) to apply to the point **X** (whose components are now in the "solve state"): i.e.,

$$\begin{bmatrix} 1 & q_1^2 & q_2^2 & q_1q_2 & q_2 & X_1 & X_2 & X_3 \end{bmatrix} \begin{bmatrix} l_1 \\ \vdots \\ l_8 \end{bmatrix} = \mathbf{X}^2 \tag{10a}$$

$$\begin{bmatrix} 1 & q_1 & q_2 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} l_9 \\ \vdots \\ l_{13} \end{bmatrix} = X_1. \tag{10b}$$

Equation 10a defines a sphere in the unknown coordinates X_1, X_2, X_3. Equation 10b defines a plane in terms of these same coordinates. The system of equations (Equation 10) hence define a circle, which is the projection circle containing the point **X**.

As in the optical DLT resection, it is instructive to note the geometrical significance of the cyclic variable removed during SAR resection. From the Appendix, it can be seen that none of the l_k change when the image plane is rotated about an axis through the "camera station" **S′** in the direction of **w**. Such a rotation would effect **Q′ − S′**, **a′**, and **b′**, but clearly does not alter the image coordinates (q_1, q_2) attributed to a point **X**.

INTERSECTION AMONG SAR AND OPTICAL IMAGES

The basic photo/radargrammetric problem addressed in this article is to determine a ground point's three-dimensional location given several two-dimensional images of the ground point and auxiliary information from control points (whose object-space and image-space coordinates are known). The task divides neatly into two parts. The first part, involving single images, consists of using the control points of each image to determine its acquisition geometry (insofar as is possible). In the present article, this resection was augmented by finding parametric expressions for the curves of ambiguity (line of sight for an optical image and projection circle for a SAR image) for each ground point of interest. The second part of the "triangulation" task is to estimate the three-dimensional *intersection* points of corresponding curves of ambiguity from several images. Triangulation from several optical images involves intersecting the corresponding lines of sight. The analogue for several SAR images is to intersect corresponding projection circles. Finally, given an optical and a SAR image, it is possible to intersect an optical line of sight with a SAR projection circle. These intersection procedures are illustrated in Figures 3, 4, and 5.

Although resection by DLT proceeds analogously for optical and SAR images, intersection is a bit more intricate when SAR images are involved. In particular, optical/optical intersection can be done in a single (noniterative) step, whereas SAR/SAR and SAR/optical intersections require iterations over the parameters of the various curves of ambiguity. The reason for this difference is that the curves of ambiguity in optical images are straight lines, whereas those of SAR are circles. The ease of intersecting straight lines can be seen by examining Equation 6 with X_1, X_2, and X_3 in the "solve state" and an implicit image index on all the other parameters in the equations. These equations are easily rewritten as a matrix operating on **X** to give a right-hand-side vector. With as few as two images (yielding four equations), a pseudoinverse operation will produce an estimate of the 3-vector **X**. This is in fact the method used by practitioners of optical DLT (see, e.g., Abdel-Aziz and Karara (1971)). In contrast, the condition equations (Equation 12) for a SAR image combine to yield a system of quadratic equations, which does not have a closed-form solution.

The solution proposed in the present article has been to generate for each point of interest on each image a (one-parameter) curve of ambiguity, and to perform intersection by iterative variation of the parameters of each of the curves. If there are only two images, the intersection process is particularly fast when performed on a computer. For example, in the case of a SAR and an optical image, the intersection can proceed by denoting a point on the line of sight, finding the closest point to it on the projection circle, finding the closest point on the line to the point on the circle, etc. The final estimate of **X** is the average between the final point on the line and the final point on the circle. I have implemented this algorithm (in conjuction with SAR and optical DLT resections) on a VAX 11/750 computer; ten iterations suffices to provide three-decimal-place accuracy

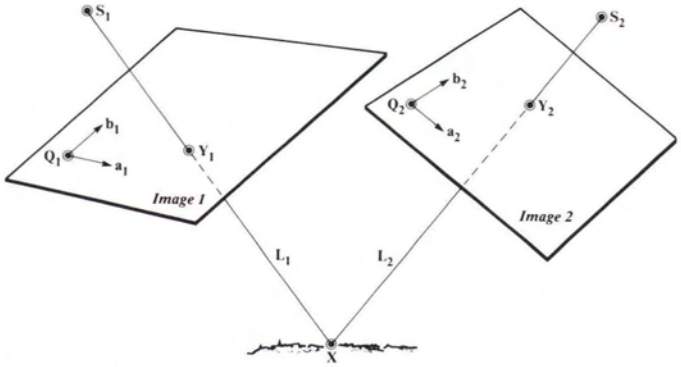

FIG. 3. Optical/optical intersection geometry, by which ground point **X** is estimated following resection.

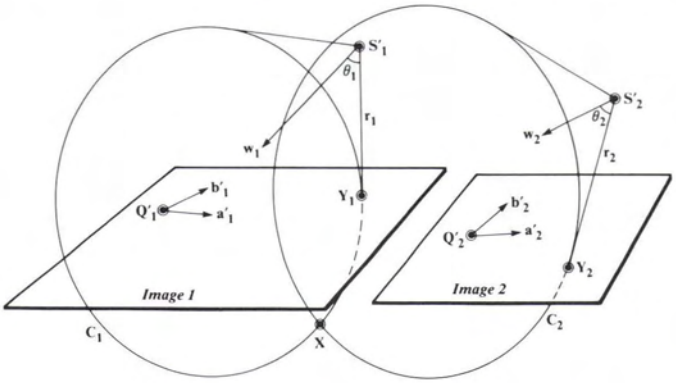

FIG. 4. SAR/SAR intersection geometry.

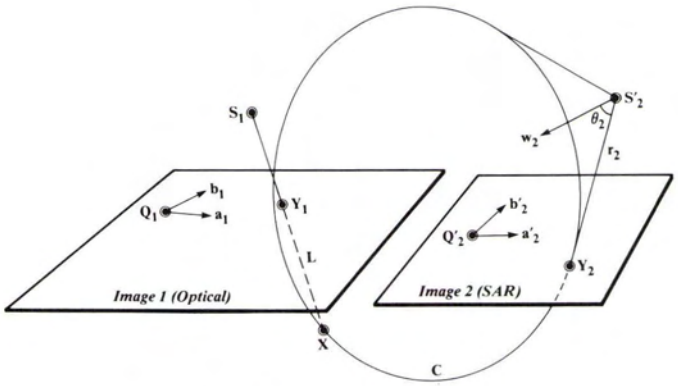

FIG. 5. Optical/SAR intersection geometry.

for a simulated triangulation. A directly analogous algorithm applies to two SAR projection circles, with some caveats for double intersections.

CONCLUSION

The present theoretical study has explained the Direct Linear Transformation (DLT) approach to optical and radar resection, the latter being a completely new approach. The DLT resection methods have been discussed here for the spotlight mode of SAR, but a similar approach can easily be envisioned for the scan-mode condition equations used, for example, in SIR-B (Leberl *et al.*, 1986). The accuracy of the optical DLT has been found to be at least as great as for the conventional solution (Abdel-Aziz and Karara, 1971; Williamson, 1972). An assessment of accuracy for SAR DLT remains to be done. Whereas DLT methods are noniterative for SAR and optical resection, intersection involving SAR images requires one parameter of iteration for a pair of images. Efficient means of intersecting more than two images involving SAR will be the subject of future investigation.

APPENDIX
OPTICAL AND SAR RESECTION PARAMETERS

Below are enumerated the respective optical and SAR parameters $\hat{m}_k$ and $\hat{t}_k$ in terms of the quantities **A, S, w, a′, b′, Q′**, and **S′** which are related in the text to the acquisition geometries.

Optical parameters:

$$\hat{m}_1 = A_{31}$$
$$\hat{m}_2 = A_{32}$$
$$\hat{m}_3 = A_{33}$$
$$\hat{m}_4 = -(A_{31} S_1 + A_{32}S_2 + A_{33} S_3)$$
$$\hat{m}_5 = -A_{11}$$
$$\hat{m}_6 = -A_{12}$$
$$\hat{m}_7 = -A_{13}$$
$$\hat{m}_8 = (A_{11}S_1 + A_{12}S_2 + A_{13}S_3)$$
$$\hat{m}_9 = -A_{21}$$
$$\hat{m}_{10} = -A_{22}$$
$$\hat{m}_{11} = -A_{23}$$
$$\hat{m}_{12} = (A_{21}S_1 + A_{22}S_2 + A_{23}S_3)$$

SAR parameters:

$$\hat{t}_1 = \mathbf{Q}'^2 - 2\,\mathbf{Q}'\cdot\mathbf{S}'$$
$$\hat{t}_2 = \mathbf{a}'^2$$
$$\hat{t}_3 = \mathbf{b}'^2$$
$$\hat{t}_4 = 2\,\mathbf{a}'\cdot\mathbf{b}'$$
$$\hat{t}_5 = 2\,(\mathbf{Q}' - \mathbf{S}')\cdot\mathbf{a}'$$
$$\hat{t}_6 = 2\,(\mathbf{Q}' - \mathbf{S}')\cdot\mathbf{b}'$$
$$\hat{t}_7 = 2\,S_1'$$
$$\hat{t}_8 = 2\,S_2'$$
$$\hat{t}_9 = 2\,S_3'$$
$$\hat{t}_{10} = \mathbf{Q}'\cdot\mathbf{w}$$
$$\hat{t}_{11} = \mathbf{a}'\cdot\mathbf{w}$$
$$\hat{t}_{12} = \mathbf{b}'\cdot\mathbf{w}$$
$$\hat{t}_{13} = w_2$$
$$\hat{t}_{14} = w_3$$

ACKNOWLEDGMENT

It is a pleasure to thank Dr. James R. Williamson of Science Applications International Corporation for introducing the DLT method to me, and for subsequent valuable discussions. Also, thanks to Sandra Peterson and Paulette Rosner (at SAIC) for preparation of the figures.

REFERENCES

Abdel-Aziz, Y., and H. M. Karara, 1971. Direct Linear Transformation from Comparator Coordinates into Object-Space Coordinates, *ASP Symposium on Close-Range Photogrammetry*, Urbana, Illinois, Jan., 1971.

Ausherman, D. A., A. Kozma, J. L. Walker, H. M. Jones, and E. C. Poggio, 1984. Developments in Radar Imaging, *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-20, pp. 363–400.

Brandow, V. D., H. M. Karara, H. H. Damberger, and H.-F. Krausse, 1976. A Non-Metric Close-Range Photogrammetric System for Mapping Geologic Structures in Mines, *Photogrammetric Engineering*, Vol. 42, pp. 637–648.

Brill, M. H., 1986. Direct Linear Transformation Methods of Triangulating from Optical and SAR Images, *J. Opt. Soc. Am. A*, Vol 3, No. 13, p. P116.

Goldstein, H., 1965. *Classical Mechanics*, Addison-Wesley, Reading, Mass, pp. 48–49.

Himbarger, R., 1984. Fundamental Concepts of Multisensor Fusion for Estimation and Detection Problems, *Applications of Estimation Techniques in Remote Sensing*, workshop sponsored by Science Applications International Corporation, Berkeley, Calif., December, 1984. SAIC Document ACD/467-100-84-1.

Karara, H. M., 1974. Aortic Heart Valve Geometry, *Photogrammetric Engineering*, Vol. 40, pp. 1393–1402.

Karara, H. M., and Y. I. Abdel-Aziz, 1974. Accuracy Aspects of Non-Metric Images, *Photogrammetric Engineering*, Vol. 40, pp. 1107–1117.

Leberl, F. W., 1979. Accuracy Aspects of Stereo Side-Looking Radar, *Photogrammetric Engineering and Remote Sensing*, Vol. 45, pp. 1083–1096.

Leberl, F. W., G. Domik, J. Raggam, and M. Kobrick, 1986. Radar Stereomapping Techniques and Applications to SIR-B Images of Mt. Shasta, *IEEE Transactions on Geoscience and Remote Sensing*, Vol. GE-24, pp. 473–481.

Rosenfield, G. H., 1968. Stereo Radar Techniques, *Photogrammetric Engineering*, Vol. 24, pp. 586–594.

Slama, C. C. (Ed.), 1980. *Manual of Photogrammetry*, Fourth Edition, American Society of Photogrammetry, Falls Church, Virginia.

Williamson, J. R., 1972. *Analytical Reduction of Underwater Photography*, M.S. Thesis, University of Illinois.

———, 1986. *An Investigation of Close-Range Forensic Photogrammetry Using Direct Linear Transformations and Rear-Screen Projection Mensuration*, Ph.D. Dissertation, Kennedy-Western University.

(Received 6 February 1987; accepted 26 March 1987)

Optical Science & Engineering Short Course

Doubletree Hotel, Tucson, Arizona
5–16 October 1987

The purpose of this course is to acquaint both the specialist and the non-specialist engineer or scientist with the latest techniques in the design and the engineering of optical systems. The course comprises 18 three-hour lectures; detailed notes will be supplied. The wide range of topics covered includes geometrical optics, optical-system layout and design, Fourier methods, diffraction, polarized light, radiometry, image quality, interferometry and optical testing, image recording, thin films, laser systems and applications, photodetectors, microcomputers in the laboratory, and visible and infrared systems.

For further information please contact

Philip N. Slater
Optical Systems & Engineering Short Courses Inc.
P. O. Box 18667
Tucson, AZ 85731
Tele. (602) 885-3798

INVITATION AND CALL FOR PAPERS

Symposium on the Applications of Close-Range Photogrammetry

Melbourne, Australia
17–18 November 1987

This Symposium — sponsored by the Australian Photogrammetric and Remote Sensing Society and the Institution of Surveyors, Australia — will concentrate on the themes of

- High Precision Metrology
- Digital Imagery
- Architectural and Heritage Recording

Those wishing to present a paper should submit an abstract by *1 August 1987*. The full text of the paper will be required by *1 October 1987*. Abstracts and general inquiries should be directed to

Dr. Mark R. Shortis
Department of Surveying and Land Information
The University of Melbourne
Parkville, Victoria 3052
Australia