Aerotriangulation without Ground Control

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ABSTRACT: Optimum accuracy in conventional aerotriangulation requires ground control around the perimeter of the area at intervals of seven airbases or less, and, if precise elevations are to be determined, there must also be elevation control in the center of the area. Recent investigations indicate that it may be possible to derive observations of the exposure station positions with submetre accuracy from a technique that uses one Navstar Global Positioning System (GPS) receiver in the aircraft and another on the ground. A method for employing these additional observation data in an aerotriangulation adjustment is presented, along with results of simulations which indicate that accurate aerotriangulation may be achievable without any ground control. Attempts at experimental verification have been hindered so far by weather, equipment problems, the limited satellite constellation, and competition for the use of available receivers. More experiments are planned for the fall of 1986.

INTRODUCTION

WHEN THE FULL CONSTELLATION of 18 satellites is operational, real-time aircraft navigation will be an important function of the Global Positioning System (GPS). The accuracies that can be expected from real-time navigation will be on the order of tens of metres and may be adequate for many mapping applications. In remote areas, where control is sparse or nonexistent, aerotriangulation at this level of accuracy may serve an important need. However, this paper addresses the potential of GPS-controlled aerotriangulation to attain accuracies in the submetre, or even decimetre, range.

The capability of GPS to achieve relative positioning to centimetre accuracy has been documented by Goad and Remondi (1984) and Remondi (1984). Remondi (1985) has also dealt with the case of a receiver on a moving platform. He has shown that fixed receiver accuracies are not so much the result of solving a simpler problem as they are of the solution methods that are employed. When real-time fixes are not required, the precise ephemeris and more sophisticated data gathering and data processing techniques can be employed. Remondi obtained centimetre level accuracies in relative positioning with a receiver moving at the relatively slow speeds of a terrain vehicle.

Mader *et al*. (1986) used one fixed receiver on the ground observing simultaneously with another in a moving airplane. They found only a slight degradation in relative positioning accuracy resulting from the much higher velocity of the moving receiver. In this experiment, the geocentric Cartesian coordinates obtained from GPS reductions were transformed into horizontal and local vertical coordinates. Because the experiment was flown over Chincoteague Bay, Virginia, the derived elevation profile could be compared directly with simultaneously acquired laser altimeter observations. The results from this experiment indicate that aircraft positioning to decimetre accuracy is obtainable, at least in elevation, and presumably in all three coordinates.

If a GPS receiver on board the aircraft and another fixed receiver on the ground can provide exposure station positions to submetre accuracy in each coordinate, no additional ground control is needed. Each exposure station would become a virtual control point, and the combined effect of all of them would provide greater geometric strength than any practical configuration of ground control.

OBSERVATION EQUATIONS

It is nearly always possible, but seldom practical, to combine raw observation data of two or more very different types in a single grand adjustment. At the present time, the processing of dynamically acquired GPS observations to obtain precise positions seems to be an art better left to the experts. Hence, the following discussion will assume that GPS phase observations have been reduced separately to provide a set of "derived observations," i.e., the positions of the antenna at the time of each photographic exposure. These "derived observations" will, of course, be accompanied by weight matrices obtained from the separate GPS phase data adjustment.

The GPS cannot provide a direct observation of the exposure station position because the phase center of the antenna and the rear nodal point of the aerial camera lens cannot occupy the same point in space. It appears that GPS observations can determine a network of antenna positions with decimetre level accuracy. It has been shown (Brown 1977; Lucas 1984) that photogrammetry can, in the presence of sufficient ground control, determine a network of camera positions to about the same level of accuracy. For the GPS network to provide independent observations of the exposure stations, it must be incorporated into the photogrammetric adjustment through an appropriate observation equation.

In a conventional installation, the position of the camera's perspective center, with respect to the GPS antenna or any other fixed point on the aircraft, is continually changing as the camera is maneuvered in its gimbaled mount by the photographer. The magnitude of this relative motion is small, but is unknown and inconvenient to measure. It may be practical to equip the camera mount with some means of sensing and recording the rotations about each axis, but until the feasibility of GPS-controlled photogrammetry has been established, a simpler installation is adequate.

The camera-antenna separation problem can be simplified by operating the camera in a locked-down mode and planning for more overlap in order to compensate for variations in coverage due to roll, pitch, and yaw of the aircraft. As a result, the separation of the camera and antenna will be reduced to a vector that is constant in any coordinate system fixed with respect to the aircraft. By resolving this vector into components parallel to the camera axes, which are now fixed with respect to the aircraft, the orientation matrix that relates image space to object space can also be used to relate the GPS-determined antenna positions to the photogrammetrically determined camera positions.

Let $\overline{\mathbf{A}} = [X_A \; Y_A \; Z_A]^T$ and $\overline{\mathbf{C}} = [X_C \; Y_C \; Z_C]^T$ be the geodetic position vectors to the phase center of the GPS antenna and the perspective center of the aerial camera, respectively, as shown in Figure 1. By convention, the perspective center is also the origin of the camera coordinate system, and for each exposure there is an orientation matrix, $\mathbf{M}$, that will transform geodetic vectors into the camera coordinate system.

There is a separation in time as well as space, because the GPS receiver and camera will be acquiring their observations independently. Therefore, there must be a means for recording the exact time at which all observations of either type were made, and there must be a means for interpolating between

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GPS fixes to obtain the antenna position at the time of each exposure. These operational requirements can be satisfied by relatively inexpensive off-the-shelf hardware, and need not be of concern in the data adjustment phase.

Therefore, in addition to the conventional image coordinate data, assume that the geodetic positions of the GPS antenna at the time of exposure of each photograph are available from a separate adjustment of the GPS phase data. These positional data will be accompanied by covariance data, also obtained from the separate adjustment, and can be treated as directly observed quantities without loss of rigor.

The separation between the camera and antenna is the difference between two geodetic vectors that are changing rapidly with time. The vector difference can be transformed into the camera coordinate system, however, where it is invariant as long as the camera is in a locked-down mode. This transformation is accomplished with the orientation matrix, **M**. The antenna offset in the camera coordinate system, then, is the vector

$$\begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix} = \mathbf{M} \begin{bmatrix} X_A - X_C \\ Y_A - Y_C \\ Z_A - Z_C \end{bmatrix} \qquad (1)$$

Because the observables are the components of the geodetic position of the antenna, it is convenient to rearrange Equation 1 into the form

$$\begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} = \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix} + \mathbf{M}^T \begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix} \qquad (2)$$

so that this vector appears on the left-hand side.

The unknown parameters in Equation 2 are the three components of the camera position, the three orientation angles implicit in the matrix **M**, and the three components of the antenna offset vector. In linearized form

$$\underset{\text{computed}}{\begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix}} - \underset{\text{observed}}{\begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix}} = \begin{bmatrix} \Delta X_C \\ \Delta Y_C \\ \Delta Z_C \end{bmatrix} + \frac{\partial\, X_A Y_A Z_A}{\partial\, \omega\phi\kappa} \begin{bmatrix} \Delta\omega \\ \Delta\phi \\ \Delta\kappa \end{bmatrix} + \mathbf{M}^T \begin{bmatrix} \Delta x_A \\ \Delta y_A \\ \Delta z_A \end{bmatrix} \qquad (3)$$

Equation 3 relates three observables to nine unknowns. Six of these unknowns are the position and orientation parameters of the photograph, which are already over-determined from the conventional observations of the bundle adjustment. The three new parameters are the elements of the camera-antenna offset vector, which can be assumed known from preflight measurements or solved for as a part of the adjustment.

Aerotriangulation without ground control requires that this offset vector be known in advance to approximately the same accuracy expected from the ground points being adjusted. While the magnitude of this vector should be easy to measure to the required accuracy, the angles needed to resolve it into components parallel to the camera axes will be more difficult. However, over the 2-metre distance separating the camera and antenna in a typical installation, these orientation angles do not need to be known to better than about 1.5 degrees.

If the camera-antenna offset vector is unknown, it can be solved for in the bundle adjustment, but only if there is sufficient ground control. The GPS-determined antenna positions and the photogrammetrically determined camera positions form two separate networks that can be related to one another only through the camera orientations and the components of this offset vector. The requirement for ground control when the offset vector is unknown is mentioned because it may be desirable to verify the offset vector by including its components as parameters in the adjustment, and one should be aware that this option is available.

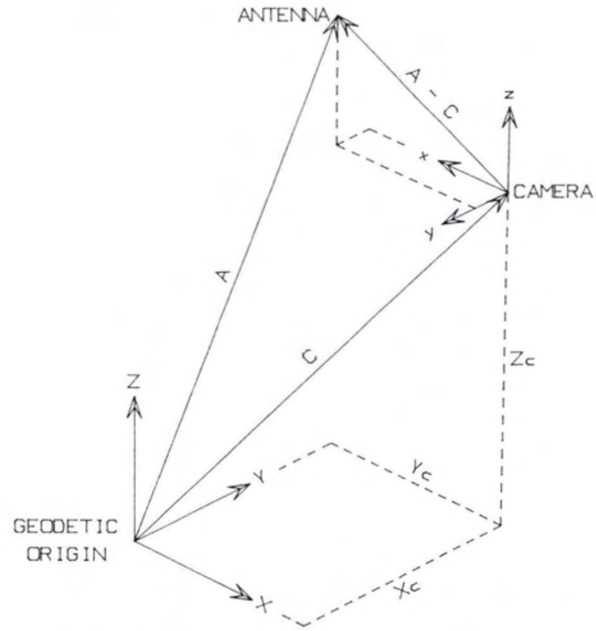

FIG. 1. Relative positions of camera and GPS antenna.

SIMULATION STUDIES

As a first step in verifying the theoretical advantages of including airborne GPS observations in an aerotriangulation adjustment, a test was made with simulated data. A fictitious photogrammetric network was constructed consisting of 49 ground points and an equal number of photographs. The ground points were arranged in seven rows by seven columns and were spaced at one mile intervals in each direction. Each photo was placed at an altitude of 12,000 feet directly above one of the ground points. With a standard 6-inch focal length mapping camera, this arrangement provides 1:24,000 scale photography in which both forward and side overlap is 67 percent. This simulated network is the configuration that the National Ocean Service tries to attain in photogeodesy projects.

The first simulation adjustment consisted of conventional photogrammetric image coordinate observations, assumed to have standard errors of 3 micrometres, and employed five ground control points. The four corner points were assigned standard errors of 5 cm in each coordinate and the center point was assigned a standard error of 5 cm in elevation only. This configuration is the minimum network that provides an ideal separation of seven photos between ground control points (Slama, 1980), and was used as a baseline for comparing the GPS-assisted adjustment.

The other data set consisted of the same image data with the same standard errors, but with no constraints on any of the ground points. Instead, a realistic camera-antenna offset vector was used to generate simulated GPS observations. The antenna position associated with each exposure was obtained using Equation 2 with the simulated camera position and orientation and the assumed offset vector. These simulated GPS observations were assigned standard errors of 10 cm in each coordinate and a 10-cm standard error was assigned to each component of the offset vector.

The results of these two simulation adjustments are shown in Tables 1 and 2. Both tables display the standard errors in rows and columns that are intended to convey the spatial distribution of the ground points. In each cell the first number is the standard error obtained from the ground-controlled adjust-

TABLE 1. STANDARD ERRORS IN CENTIMETRES OF HORIZONTAL POSITION AS A FUNCTION OF POSITION WITHIN THE NETWORK. TOP ENTRY IN EACH CELL OBTAINED USING GROUND CONTROL; BOTTOM ENTRY FROM INCLUSION OF GPS OBSERVATIONS WITHOUT GROUND CONTROL.

4.9 14.7	8.5 12.2	10.0 11.1	10.5 10.9	10.0 11.1	8.5 12.2	4.9 14.7
8.5 12.2	7.4 9.7	8.1 8.7	7.8 8.5	8.1 8.7	7.4 9.7	8.5 12.2
10.0 11.1	7.9 8.7	7.3 7.6	7.3 7.3	7.3 7.6	7.9 8.7	10.0 11.1
10.5 10.9	8.1 8.5	7.3 7.3	7.2 7.0	7.3 7.3	8.1 8.5	10.5 10.9
10.0 11.1	7.9 8.7	7.3 7.6	7.3 7.3	7.3 7.6	7.9 8.7	10.0 11.1
8.5 12.2	7.4 9.7	8.1 8.7	7.8 8.5	8.1 8.7	7.4 9.7	8.5 12.2
4.9 14.7	8.5 12.2	10.0 11.1	10.5 10.9	10.0 11.1	8.5 12.2	4.9 14.7

TABLE 2. STANDARD ERRORS IN CM OF ELEVATION AS A FUNCTION OF POSITION WITHIN THE NETWORK. TOP ENTRY IN EACH CELL OBTAINED USING GROUND CONTROL; BOTTOM ENTRY FROM INCLUSION OF GPS OBSERVATIONS WITHOUT GROUND CONTROL.

5.0 19.5	21.9 16.2	25.5 15.5	27.1 15.2	25.5 15.5	21.9 16.2	5.0 19.5
21.9 16.2	17.6 13.5	18.1 12.6	18.1 12.3	18.1 12.6	17.6 13.5	21.9 16.2
25.5 15.5	18.1 12.6	14.7 11.5	12.6 11.1	14.7 11.5	18.1 12.6	25.5 15.5
27.1 15.2	18.1 12.3	12.6 11.1	5.0 10.8	12.6 11.1	18.1 12.3	27.1 15.2
25.5 15.5	18.1 12.6	14.7 11.5	12.6 11.1	14.7 11.5	18.1 12.6	25.5 15.5
21.9 16.2	17.6 13.5	18.1 12.6	18.1 12.3	18.1 12.6	17.6 13.5	21.9 16.2
5.0 19.5	21.9 16.2	25.5 15.5	27.1 15.2	25.5 15.5	21.9 16.2	5.0 19.5

ment and the number below it is the standard error that results from using GPS observations in place of ground control. Table 1 shows the circular standard errors in horizontal positions and Table 2 shows the linear standard errors in computed elevations.

The tables provide the numerical values of the standard errors to the nearest tenth of a centimetre for those concerned with their magnitudes. For quick relative comparisons, however, Figures 2 and 3, which are graphical representations of the data from Tables 1 and 2, are more useful. In these figures, the ground-controlled case is on the left and the GPS-assisted case on the right.

Edge effects, larger errors in ground points along the edge of the network caused by fewer rays intersecting these points, are obvious in both horizontal and vertical errors of both adjustments. These effects are greatest in the corners as seen in the GPS results, and provide the largest differences because the corner ground points were constrained to 5 cm in the ground-controlled case.

The only significant differences in horizontal errors between the two data sets are in the vicinity of the corners. The two or three closest neighbors of the corner points show the influence of the constraints placed on the corners in the ground-controlled case. In contrast, edge effects are most severe at the corners and adjacent ground points when only GPS control is used. At all other points, the GPS-controlled adjustment produced standard errors that are nearly the same as those obtained with ground control.

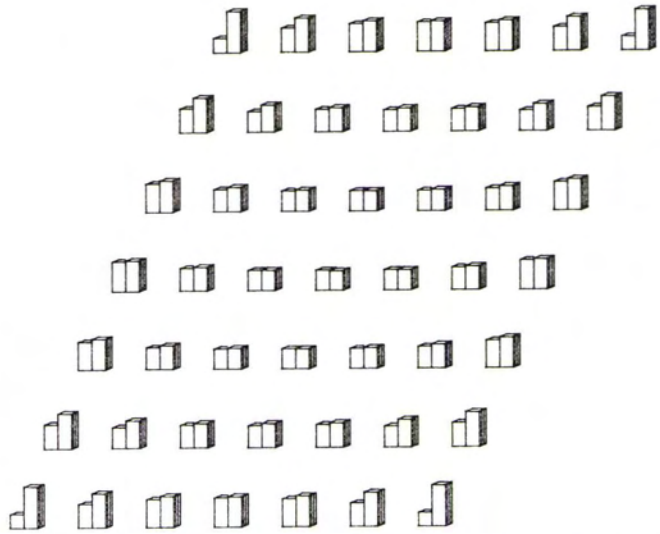

FIG. 2. Graphic representation of the data of Table 1.

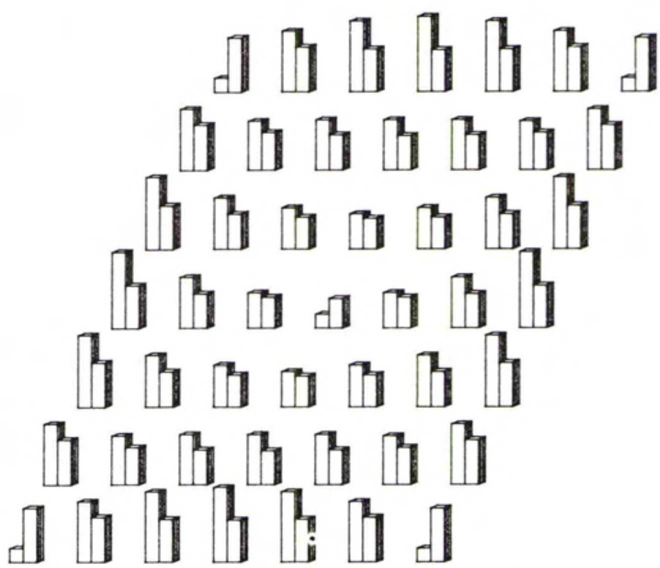

FIG. 3. Graphic representation of the data of Table 2.

Table 2 and Figure 3 show that GPS has a definite advantage over ground control in precision of elevation determination. All except the five constrained points are less precisely determined in the ground-controlled case. There are two factors at work here. First, edge effects degrade the precision of elevation determination more severely than they do horizontal precision (compare Tables 1 and 2). This is due, in part, to the greater uncertainty in the elevations of the exposure stations along the edges of the network, which are resected by fewer ground points. The GPS observations of these exposure station positions play an important role in counteracting this component of edge effects. Secondly, it has been pointed out by Slama (1980) that elevation errors in aerotriangulation tend to grow with distance from control at a much greater rate than do horizontal errors. Because each exposure station acts as a control point when GPS observations are included, distance from control is no longer a factor.

EXPERIMENTAL VERIFICATION

The first experiment to verify the above simulation results was scheduled in late July 1985, but the limited time in which the receivers and aircraft were simultaneously available was too short and the weather uncooperative. A test of the feasibility of determining aircraft elevations by GPS for altimetry was completed at that time (Mader *et al.*, 1986). These results, elevations determined to approximately 10-cm standard errors, were very encouraging, but it was disappointing that there was no opportunity to acquire photography.

Due to the unavailability of GPS receivers, a second experiment could not be scheduled until December 1985. The satellite window, the period of time during which four of the satellites of the present limited constellation would be simultaneously above 20 degrees elevation angle, had moved to early morning by that time and would soon be too early for aerial photography. Nonetheless, apparently adequate photography and accompanying GPS observations were acquired on four of the five days scheduled for the experiment. Unfortunately, the week chosen for this experiment had also been chosen by the Air Force for moving one of the satellites, and its position was found to be totally unreliable during this period.

The satellite window will not coincide with photographic hours again until August through December, 1986. During that time period, at lest two more attempts to complete a verification experiment are scheduled. The first will be conducted in Texas in August in cooperation with the Texas State Department of Highways and Transportation, and the second is presently planned in cooperation with the Washington State Department of Highways for early October. A third experiment, to be conducted near Dulles Airport, Virginia, is contingent on the availability of GPS receivers.

CONCLUSIONS

Simulation is the first step in a feasibility study and must be followed by experimental verification. At this writing, experimental verification is lacking, and one must be careful of conclusions based on incomplete evidence. However, the potential indicated by the simulations described above and the preliminary comparisons of GPS-determined aircraft positions with altimetry are very encouraging.

GPS-controlled photogrammetry appears to have the potential to facilitate the mapping of inadequately controlled or inaccessible areas, and to provide better elevation data with, or without, ground control. While it has been shown to be theoretically possible to accomplish very accurate aerotriangulation without any ground control whatsoever, a more practical application will be the opportunity to rely on the existing control in a project area without having to supplement it with new control.

As frustrating as it has been, trying to use the limited number of GPS satellites that are now available, there is an advantage to having several years lead time before the full constellation becomes operational. This new technology will have to be proven and then refined, and new hardware and software will need to be developed to facilitate its implementation, but when it becomes operational, it will change the way we do photogrammetry.

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