Limiting Error Propagation in Network Design

Clive S. Fraser

Geodetic Services, Inc., Melbourne, FL 32901

ABSTRACT: Analysis of the covariance matrix of object point coordinates forms an integral part of photogrammetric network optimization, especially at the zero- and first-order design stages. Yet, computation of full covariance matrices can impose a considerable time burden in interactive network design systems which are implemented on microcomputers. In this paper an examination is made of the applicability of the so-called limiting error propagation to see just how representative of the true covariance structure the covariance matrix from limiting error propagation really is. The method, if found to yield satisfactory variance estimates, offers the potential of immediately solving the zero-order design problem while providing a simpler approach to covariance matrix computation in network design.

INTRODUCTION

THROUGH THE PROCESS of network design the photogrammetrist seeks to reach a certain overall quality of the photogrammetric network. This quality can be expressed in terms of a number of objective functions. Perhaps the one most often heard in practice is along the lines of "I seek maximum accuracy at minimum cost." From such a broadly stated requirement, specific objectives related usually to precision, reliability, and economy can be formulated, either explicitly or implicitly. The design process then proceeds until such time that the user-specified objectives are met, ideally in an optimum fashion.

In the case of precision, the covariance matrix of the unknown parameters, specifically the object point XYZ coordinates, provides the measure of network quality. Precision is influenced by both the geometric configuration of the network and the distribution of observational work, i.e., the variances of the image coordinate observations. The selection of a datum also impacts on network precision. In designing a minimally controlled network to yield a covariance matrix that is optimal in some sense, two design problems need to be addressed, namely, Zero-Order design (ZOD) and First-Order design (FOD). ZOD involves the search for an optimum reference coordinate system or datum for the network, and FOD comprises the configuration problem which basically entails the selection of an optimal imaging geometry for a given set of object target points. For a more comprehensive account of the role of ZOD and FOD in photogrammetric network design, the reader is referred to Fraser (1984).

There are both direct and indirect solution approaches possible for these design problems. With the direct approach, FOD is solved for mathematically. That is, given an object point field, the precision of the image coordinate measurements, and an ideal covariance matrix for the object point coordinates, a solution is obtained for the optimum geometry and number of camera stations. The direct approach is, however, not without its problems, and to date no practical analytical FOD solution scheme for close-range photogrammetry has been developed. The general applicability of such an approach also remains in question.

Under the indirect approach, the FOD is solved in an iterative or trial-and-error manner through the process of network simulation. The designer formulates an initial network geometry and then evaluates the resulting precision. If the precision meets user-specified requirements and the reliability and economy are deemed to be satisfactory, the network is adopted. If, on the other hand, the design is found to be deficient, the camera station configuration is updated and the precision is again analyzed. This process is continued until a satisfactory, though not necessarily mathematically "optimal," design is achieved. Although the indirect or simulation method of photogrammetric network design has been successfully employed for some time, the development of powerful personal computers with graphics capabilities has recently given a considerable boost to this approach. Networks can now be designed and analyzed in a short session at the computer with the aid of interactive computer graphics techniques. Simulation software packages based on interactive design principles are presently commercially available (e.g., Gustafson and Brown, 1985) and these offer a powerful tool to the user of non-topograhic photogrammetry.

Following the formulation or updating of a camera configuration scheme, the designer seeks to analyze the precision of the network. This entails the computation of the covariance matrix of the network parameters. For a self-calibrating bundle adjustment these parameters include the exterior orientation elements for each camera station, the additional parameters (APs), and the parameters of primary concern, the *XYZ* coordinates of triangulated object points. Unfortunately, this computation is no trivial matter. For a network of a hundred or so points and half a dozen camera stations, the required covariance matrix is obtained through the formation and inversion of a matrix of rank 350 or so. On a personal computer the computation time for such a process extends to minutes and can reach 10 minutes and more for large networks. Moreover, ZOD may warrant the computation of more than one covariance matrix.

Generally speaking, the necessity to compute the full covariance matrix of XYZ object point coordinates during the interactive network design phase can impose a considerable time burden, especially on personal computers. The standard practice of computing this covariance matrix through the formation and inversion of the normal equation system for the bundle adjustment will be referred to here as Total Error Propagation (TEP). An alternative to TEP, designated in Brown (1980) as Limiting Error Propagation (LEP), can be applied to drastically reduce computation time. This non-rigorous method, however, takes no account of ZOD and also does not consider either the propagation of errors in projective parameters into the object space coordinates or the correlations between object points. The aim of this paper is to investigate the applicability of LEP for interactive network design. If LEP can produce a covariance matrix of object point coordinates which is sufficiently representative of the true covariance structure, it offers the possibility of a simpler and faster computational scheme for the determination of object point precision in interactive network design.

COVARIANCE MATRIX OPTIONS

The starting point for the discussion of covariance matrix determination is given by the general system of normal equations for the self-calibrating bundle adjustment:

PHOTOGRAMMETRIC ENGINEERING AND REMOTE SENSING, Vol. 53, No. 5, May 1987, pp. 487–493. 0099-1112/87/5305–487\$02.25/0 ©1987 American Society for Photogrammetry and Remote Sensing PHOTOGRAMMETRIC ENGINEERING & REMOTE SENSING, 1987

$$\begin{bmatrix} \mathbf{A}_{1}^{\mathrm{T}} \mathbf{P} \mathbf{A}_{1} & \mathbf{A}_{1}^{\mathrm{T}} \mathbf{P} \mathbf{A}_{2} \\ \mathbf{A}_{2}^{\mathrm{T}} \mathbf{P} \mathbf{A}_{1} & \mathbf{A}_{2}^{\mathrm{T}} \mathbf{P} \mathbf{A}_{2} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{1} \\ \mathbf{x}_{2} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{1}^{\mathrm{T}} \mathbf{P} \ell \\ \mathbf{A}_{2}^{\mathrm{T}} \mathbf{P} \ell \end{bmatrix}$$
(1)
or
$$\begin{bmatrix} \mathbf{N}_{1} & \overline{\mathbf{N}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{1} \\ \mathbf{x}_{1} \end{bmatrix} = \begin{bmatrix} \mathbf{c}_{1} \end{bmatrix}$$
(2)

N₂ x₂ c₂ In Equation 1, x_1 is the vector of corrections to the exterior orientation and additional parameters, X2 is the vector of corrections to object space XYZ coordinates, A1 and A2 are the corresponding configuration matrices, ℓ is the discrepancy vector, and P is the weight matrix of the image coordinate observations.

In the model indicated by Equation 1, X_1 and X_2 are treated as free unknowns since no prior information regarding the means and variances of parameters has been incorporated into the normal equations. In network design the inclusion of prior information on the parameters is certainly not precluded, though a few straightforward restrictions on the structure of the appropriate weight matrices do apply when considering ZOD (Fraser, 1982). For simplicity, the parameters will be treated only as free unknowns in the present discussion. In addition, only minimally constrained networks are considered. For networks with redundant object space coordinate control, there is no datum problem. As an example, consider periodic inspection surveys of assembly tooling in industry where the photogrammetric network often includes a number of truly stable points against which movements are referenced. At the design stage the influence of this object space control on network precision must be taken into account through a TEP which incorporates the zero variances in the coordinates of the reference points. In such situations both LEP and TEP by means of inner or minimal constraints are inappropriate.

EXPLICIT MINIMAL CONSTRAINTS

Through the process of ZOD a datum is selected which yields a covariance matrix which is "best" in some sense. The datum or zero-variance computational base is defined through the imposition of constraints which establish the fixed origin, orientation, and scale of the XYZ reference coordinate system. There are essentially two practical options for ZOD. Both represent rigorous treatments of TEP. The first involves the explicit removal of the datum defect of the normal equations by "fixing" seven appropriate coordinate values (e.g., two points fixed in XYZ and a further non-collinear point fixed in the coordinate axis which is most nearly normal to the plane containing the three points). The covariance matrix of the parameters is then obtained as follows:

$$Q_{x} = \begin{bmatrix} Q_{x}^{(1)} & Q_{x}^{(1,2)} \\ Q_{x}^{(2,1)} & Q_{x}^{(2)} \end{bmatrix} = \begin{bmatrix} N_{1} & \overline{N} \\ & \\ \overline{N}^{T} & N_{2} \end{bmatrix}^{-1}$$
(3)

In selecting which minimal control configuration of seven coordinates to assign zero variance, it is normal practice to seek a "best" form for only $Q_x^{(2)}$ because the object point coordinates are typically the paramters of interest, the exterior orientation and additional parameters being essentially nuisance parameters. As has been shown, for example by Fraser (1984), the magnitude of $Q_x^{(2)}$ varies significantly with changes in the minimal control.

From the partitioned normal equations in Equation 1, the following expression for $\mathbf{Q}_{x}^{(2)}$ is obtained:

$$Q_x^{(2)} = N_2^{-1} + K_c \tag{4}$$

where

$$\mathbf{K}_{c} = \mathbf{N}_{2}^{-1} \,\overline{\mathbf{N}}^{\mathrm{T}} \, \mathbf{Q}_{x}^{(1)} \,\overline{\mathbf{N}} \, \mathbf{N}_{2}^{-1}.$$
(5)

As written, Equation 4 provides the means to compute the full covariance matrix $Q_x^{(2)}$ of object point coordinates from the set of reduced normal equations which are formed through the elimination of the parameters x_2 . The equation is, however, not commonly applied in this form. Rather, the 3 by 3 covariance matrices $Q_{ij}^{(2)}$ for each object point *j* are computed sequentially from the expression (Brown, 1958)

$$\mathbf{Q}_{x_{j}}^{(2)} = \mathbf{N}_{2j}^{-1} + \mathbf{N}_{2j}^{-1} \mathbf{N}_{j}^{\mathrm{T}} \mathbf{Q}_{x}^{(1)} \mathbf{N}_{j} \mathbf{N}_{2j}^{-1}.$$
(6)

The computation time for the covariance matrix $Q_x^{(2)}$ formed by the individual 3 by 3 matrices $Q_{xi}^{(2)}$ is considerably less than that required by Equation 4. Recall, however, that no point-to-point correlation information is being produced in this approach. For the present it is assumed that a full $Q_{x}^{(2)}$ matrix is being computed by Equation 4.

INNER CONSTRAINTS

The second ZOD option in photogrammetric network design involves the implicit assignment of a datum through the freenetwork approach of inner constraints (e.g., Blaha, 1971; Fraser, 1982; Papo and Perelmuter, 1982). Here, the required covariance matrix $\mathbf{Q}_x^{(2)}$ is obtained from the Cayley inverse

$$\mathbf{Q}_{x} = \begin{bmatrix} \mathbf{Q}_{x}^{(1)} & \mathbf{Q}_{x}^{(1,2)} & \cdot \\ \mathbf{Q}_{x}^{(2,1)} & \mathbf{Q}_{x}^{(2)} & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} \mathbf{N}_{1} & \mathbf{N} & 0 \\ \mathbf{N}^{\mathrm{T}} & \mathbf{N}_{2} & \mathbf{G} \\ 0 & \mathbf{G}^{\mathrm{T}} & 0 \end{bmatrix}^{-1}$$
(7)

where G is a Helmert transformation matrix. Each "." represents a submatrix of Q_x which is not pertinent to the present discussion. After a rearrangement of rows and columns of the bordered matrix shown in Equation 7, an expression corresponding to Equation 4 can be derived for the inner constraint solution

$$Q_x^{(2)} = N_2^{-1} + K_i \tag{8}$$

where

$$\mathbf{K}_{i} = \overline{\mathbf{N}}_{2}^{-1} \left(\overline{\mathbf{N}}^{\mathrm{T}} \mathbf{G} \right) \mathbf{Q}_{x}^{(1)} \begin{pmatrix} \overline{\mathbf{N}} \\ \mathbf{G}^{\mathrm{T}} \end{pmatrix} \mathbf{N}_{2}^{-1}.$$
(9)

Here, $\hat{\mathbf{Q}}_{x}^{(1)}$ includes seven extra columns corresponding to the Lagrangian multipliers of the inner constraint solution. In much the same way that covariance matrices $Q_{x_i}^{(2)}$ can be computed sequentially by means of Equation 6, so can they be computed from Equation 8.

The inner constraint approach provides the datum which yields the minimum mean variance for the object point coordinates, i.e.,

$$\sigma_c^2 = \operatorname{tr} \mathbf{Q}_x^{(2)} / 3n \to \operatorname{minimum}$$
(10)

where *n* is the number of points and tr is the trace operator. In the sense that it yields maximum precision, the inner constraint approach provides the optimum solution to the ZOD problem. Chances are that, if the inner constraint solution does not yield the required level of precision, the FOD will need to be readdressed.

LIMITING ERROR PROPAGATION

Under the assumption that projective parameters are error free (i.e., $Q_x^{(1)} = 0$), and that variances in the object point coordinates arise solely from the propagation of random error in the image coordinate measurements, it is readily apparent from Equations 4 and 8 that the covariance matrix equation for Q⁽²⁾ reduces to

$$Q_{y}^{(2)} = N_{2}^{-1} \tag{11}$$

and, because N2 is block-diagonal,

$$\mathbf{Q}_{xj}^{(2)} = \mathbf{N}_{2j}^{-1}.$$

NT

The complete covariance matrix $Q_x^{(2)}$ from LEP is therefore block diagonal, being constructed of all the uncorrelated sub-matrices $Q_{xj}^{(2)}$. In relation to the time required for a full inversion of the normal equation matrix (even in its reduced form, as will be later detailed), the time needed to compute all $Q_{xj}^{(2)}$ matrices is miniscule, being seconds instead of minutes for most small to moderate sized networks. Thus, the approach is attractive for interactive network design. But, the real question, which will now be addressed, is whether the $Q_x^{(2)}$ matrix from LEP produces a sufficiently accurate representation of the true covariance matrix structure obtained from TEP for the object points.

THE RELATIONSHIP BETWEEN LEP AND TEP

As a firm which is routinely involved in high-precision nontopographic photogrammetry, Geodetic Services, Inc. (GSI) is called upon to carry out numerous and varied photogrammetric measuring tasks. Standard practice dictates that all potential measurements are first subjected to a comprehensive network design process. Both LEP and TEP form an integral part of this phase. Typically, as part of the interactive network design, the planner seeks to optimize $Q_x^{(2)}$ utilizing LEP. Once this is done the network is subjected to a TEP to yield the more rigorous estimate for the precision.

One consequence of the use of both LEP and TEP is the network designs carried out is that there is often a reasonably predictable discrepancy between the corresponding values of the mean standard error $\overline{\sigma}_c$ obtained from the two error propagations. For "strong" photogrammetric networks, i.e., those exhibiting high internal reliability, it is common for the magnitude of $\overline{\sigma}_{c}$ obtained from Equation 4 with a favorable minimal control configuration to be some 20 to 40 percent higher than the corresponding value from LEP. For weaker networks, e.g., normal stereo geometry with relatively few object points or networks with unfavorable minimal control geometry, the discrepancy may readily exceed 50 percent. Moreover, situations can arise where overparameterization in self-calibration or network configuration defects (insufficient information to perform a relative orientation) lead to an LEP which gives a grossly optimistic and therefore misleading indication of both the structure of $Q_x^{(2)}$ and the magnitude of $\overline{\sigma}_c$. By referring back to Equation 4, it can be seen that LEP will only yield a representative estimate of TEP for a minimally controlled network when $K_c \simeq 0$. In the author's experience, this condition is never sufficiently realized.

Turning now to the inner constraint case, a remarkable feature is noticed in network design, that being that K_i is very often near zero in magnitude. This implies that the optimum solution to ZOD, the datum that yields a minimum mean standard error for the object point coordinates, is closely approximated by the simple computation of LEP. Recent experience at GSI is that the assumption of $K_i \approx 0$ holds for the majority of networks which are designed for industrial photogrammetric measurement tasks.

With the considerable numerical evidence of the essential equivalence between the $Q_x^{(2)}$ obtained from a TEP by means of inner constraint adjustment and that obtained in an LEP, one is tempted to seek a mathematical explanation for $K_i = 0$. To date, the author has not had any success in this regard. Rather, it is instructive to turn to relative orientation for some measure of explanation. For networks exhibiting reasonable internal reliability, the inherent strength of the relative orientation is invariably quite high. This suggests that, for all practical purposes, the projective parameters can be treated as "fixed" and that the only significant error to be considered is in the spatial direction of the imaging rays, this error being indicated by the variance of the image coordinate measurements. This also implies that, in the relatively oriented network, point-to-point correlation should be at a very low level. In many ways it would be beneficial to specify network quality in terms of relative orientation. For example, measures of precision of parameters of shape because these are invariant with changes in the applied minimum constraint in a TEP. Yet measures of the accuracy of form elements of shape do not seem to readily lend themselves to simple and easily understood one-number indicators of precision such as the mean standard error of object point coordinates.

PRACTICAL EXAMPLES

To demonstrate the practical equivalence of LEP and TEP by inner constraints, some examples are now presented. Three categories of network have been chosen to represent a broad coverage of typically encountered imaging geometries. With each of the cases considered both the variances, as represented by $\overline{\sigma}_c$, and the covarance elements of $Q_x^{(2)}$ are examined. Recall that LEP yields a covariance matrix with zero correlation between object points. If a similar structure is produced by the inner constraint method, the implications may be of practical significance because it means that analysis of functions of x_2 could be greatly simplied, especially in fields such as deformation analysis, and propagation of errors into functions of the triangulated object points (e.g., volumes, distances, etc.).

EXAMPLE 1

One of the imaging geometries considered in the first network design example is shown in Figure 1. Although the object depicted is an antenna of 10-m diameter having 60 target points, it could well be any shallow object of similar dimensions, e.g., an aircraft wing panel or a large industrial mold. In the figure, three camera stations are symmetrically disposed about the antenna at a radial distance of 13 m, and the convergence angle is 80°. For the design simulations conducted, a camera of 23 by 23-cm format and 240-mm focal length has been assumed, thus giving rise to an image scale of about 1:54. A standard error value of 2 micrometres has been adopted for the image coordinates.

For this first design example, five exposure station configurations are examined. Four of these are obtained by simply adopting differing numbers of equally spaced camera positions around the circle shown in Figure 1. The numbers considered will be eight, four, three, and two. In addition, a "normal" stereo geometry of two camera stations at a base/distance ratio of 0.5 has been included. Thus, the range of network geometries is from a strong eight-station network, to the relatively poor reliability case of a two-station stereopair. In normal practice the two- and three-station configurations would not be recommended because they do not exhibit sufficient internal



 F_{IG} 1. Network geometry for three-station configuration of Example 1, an antenna.

reliability for high-accuracy work (e.g., Förstner, 1985). Nevertheless, they are included here so that the relationship between LEP and TEP can be examined for such network geometries.

With the exception of the two two-station geometries, the networks include self-calibration parameters, namely the principal distance c, and the coordinates x_{pr} y_{pr} of the principal point. More APs could have been considered, but it was decided to include only these three interior orientation elements because the precision of their recovery, and indeed the degree of their projective coupling with other exterior orientation and object point parameters, is significantly influenced by network geometry.

One of the features of a network with a geometry similar to that shown in Figure 1 is that it generally displays a very homogeneous object point precision. Because all target points are imaged in all photographs, and the geometry of ray intersections is similar from point to point, little variability should be expected in the *XYZ* coordinate standard errors. Naturally, this homogeneity of triangulation accuracy does not necessarily extend to the individual precision for the *X*, *Y*, and *Z* coordinates. With the different camera station configurations considered in this first example, however, similar values for the standard errors in *X* and *Y* can be anticipated, and so precision in "planimetry" (*XY*) and "height" (*Z*) will be distinguished.

Shown in Table 1 is the precision obtained for the five imaging geometries by means of both TEP (the inner constraint approach) and LEP. The table lists the mean standard error $\overline{\sigma}_c$ of the 60 object points, the corresponding standard errors $\overline{\sigma}_{XY}$ and $\overline{\sigma}_Z$ for the *XY* plane and *Z* direction, the proportional accuracies in relation to the size of the object, a summary of the correlation coefficient values in $Q_x^{(2)}$ as obtained in the TEP, and the value of the design factor *q* which is derived from the expression (Fraser, 1984)

$$\sigma_c \simeq q S \sigma$$
 (13)

where *S* is the scale number and σ is the standard error of image coordinate observations.

The most striking feature of Table 1 is that LEP yields essentially the same standard error values as the inner constraint approach of TEP. Even for the weaker geometries the difference in the corresponding values does not exceed a few percentage points. This level of agreement is certainly encouraging for the proposition of utilizing LEP in interactive network design. A further feature of Table 1, which will be discussed in a later section, is that the correlation coefficient values are very small. Note also in the table that the $\overline{\sigma}_c$ values produced in a TEP with minimal control range in magnitude from 1.2 to 1.4 times the corresponding values obtained with inner constraints (see also Fraser (1982; 1984)).

EXAMPLE 2

A second example, one in which the network geometry is not expected to yield a high level of homogeneity for the object point precision, is now considered. From the point of view of both geometric strength and internal reliability, this second network is certainly not optimal, but is included here so as to further examine the level of correspondence between LEP and TEP for a network with a less than optimal FOD. It is not unusual in practice to experience physical constraints which necessitate the adoption of a less than desirable camera station configuration.

The network is illustrated in Figure 2. The object in this case is rectangular in shape and has dimensions of 4 m by 1.5 m by 2 m (high). Such a shape is not unrepresentative of large assembly fixtures which are periodically inspected by photogrammetry in the aerospace industry (e.g., Fraser and Brown, 1986). The imaging geometry comprises eight camera stations. At four



FIG. 2. Network geometry for the four- and eight-station configurations of Example 2.

TABLE 1. OBJECT POINT PRECISION OBTAINED FOR THE FIVE NETWORKS OF EXAMPLE 1 BY MEANS OF BOTH LEP AND TEP (INNER CONSTRAINTS). D IS THE DIAMETER OF THE OBJECT. THE σ_c VALUES SHOWN IN PARENTHESES ARE THOSE OBTAINED FOR A FAVORABLE MINIMAL CONTROL CONFIGURATION IN A TEP BY MEANS OF THE MINIMAL CONTROL APPROACH (Eq. 4).

No. of Camera Stations	LEP or TEP	q	$\overline{\sigma_c}$ (mm)	$\overline{\sigma}_{xy}$ (mm)	$\overline{\sigma}_{z}$ (mm)	$\frac{D}{\overline{\sigma}_c}$	$\frac{D}{\overline{\sigma}_{xy}}$	$\frac{D}{\overline{\sigma}_z}$	Correlation Coefficients from TEP
8 3 APs	TEP LEP	0.42 0.41	0.045 0.044 (0.056)	0.040 0.039	0.052 0.053	222,000 227,000	250,000 256,000	192,000 189,000	$\begin{array}{l} 99.1\% < 0.15 \\ 0.9\% < 0.3 \end{array}$
4 3 APs	TEP LEP	0.59 0.59	0.064 0.063 (0.086)	0.058 0.056	0.074 0.074	156,000 159,000	172,000 179,000	135,000 135,000	$\begin{array}{l} 96.1\% < 0.15 \\ 3.9\% < 0.3 \end{array}$
3 3 APS	TEP LEP	0.69 0.68	0.074 0.073 (0.100)	0.067 0.065	0.085 0.087	135,000 137,000	149,000 154,000	118,000 115,000	$\begin{array}{l} 97.2\% < 0.15 \\ 2.8\% < 0.3 \end{array}$
2 Converg. No APs	TEP LEP	0.88 0.86	0.094 0.093 (0.127)	0.084 0.082	$0.110 \\ 0.111$	106,000 108,000	119,000 122,000	91,000 90,000	98.8% < 0.15 1.1% < 0.3 0.1% < 0.45
2 'Normal' No APs	TEP LEP	1.6 1.6	0.174 0.175 (0.239)	0.090 0.091	0.273 0.274	57,000 57,000	111,000 110,000	37,000 36,000	$\begin{array}{l} 99.0\% < 0.15 \\ 0.5\% < 0.45 \\ 0.5\% < 0.99 \end{array}$

positions an exposure is to be taken at heights of both a metre and 3 m above the floor. A camera of 23 by 23-cm format and 240-mm lens is again to be used, the imaging distance being approximately 4.7 m. On the object are 65 target points, 15 distributed on each of the sides and 35 evenly spread points in the front surface area. With the adopted geometry, points on one end of the object are not imaged from camera stations on the other side of the centerline. Thus, it is apparent that these points will display lower precision than points on the front because they are imaged with poorer geometry by fewer camera stations. Furthermore, with the fairly acute intersection angles for the 30 points on the two sides, moderately high correlations should be expected between the *X* and *Y* coordinates of these triangulated targets.

In addition to considering the eight-station geometry, the network formed by just the lower four stations will also be examined. This geometry displays extremely poor internal reliability for points on the sides of the object and would be avoided in practice.

The precision obtained for each of these two networks is listed in Table 2, which has a similar format to the previous table, but differs in that $\overline{\sigma}_x$ and $\overline{\sigma}_y$ are listed instead of $\overline{\sigma}_{xy}$. On examining the table, it is again seen that for the stronger network geometry LEP yields standard error estimates which are closely representative of those obtained in a TEP by means of inner constraints. Note here also that the bulk of the correlation coefficients are again very small in magnitude. Those that are not express the high correlation between X and Y for the 30 points on the sides of the object; on a point-to-point basis $Q_x^{(2)}$ still has a near-orthogonal structure.

For the four-station network a different picture is seen. Here there is a relatively large discrepancy between the precision obtained by means of the two approaches. The $\overline{\sigma}_{c}$ value obtained in the TEP is some 30 pecent higher than that in the LEP and the difference for $\overline{\sigma}_x$ is 65 pecent. In addition, the correlation coefficients are markedly higher than previously encountered. If the photogrammetrist were to base his estimates of precision for the network on LEP in this case, the results would be too optimistic. In searching for the answer as to why LEP produces an unrepresentative mean standard error, one need only look at the precision of the recovered APs in the TEP. Although the object is three-dimensional, those points not on the front surface exhibit very poor two-ray geometry. Thus, the planar front of the object has to provide the geometry which will afford a recovery of the elements of interior orientation, especially the principal distance, c. For this network the recovery of c is weak, the standard error being about 40 micrometres. Hence the projective coupling between *c* and the object point coordinates is exhibited by both inflated variances and covariances. (If the APs are suppressed, TEP and LEP regain their equivalence.) This case represents one shortcoming of utilizing only LEP in a network design: the planner will gain no indication from his FOD that the network may be deficient from the point of view of self-



FIG. 3. Camera station configuration for Example 3, a cylinder.

calibration. The resulting overparameterization could lead to instability in the final bundle adjustment.

EXAMPLE 3

As a final network design example, an examination is made of a second, less obvious, situation in which the addition of self-calibration parameters can lead to a near singular normal equation matrix, thus yielding again an LEP which conceals the adverse influences of overparameterization. The object in this case is a cylinder of 10-m diameter and 9-m length. Around this cylinder are three rings of targets at 15° intervals, one ring at each end and one in the middle. This cylinder may well be a section of a pressure vessel or submarine, and photogrammetry is to be applied to determine the degree of circularity. Such surveys are carried out frequently by GSI, albeit with a very different network geometry to the one considered here.

The imaging geometry for this example is shown in Figure 3. Six camera stations are positioned at 60° intervals around the cylinder at a radial distance of 16.5 m out from the center. All points are imaged in either two or three photographs, the three-

TABLE 2. OBJECT POINT PRECISION OBTAINED FOR THE TWO NETWORKS OF EXAMPLE 2 BY MEANS OF BOTH LEP AND TEP (INNER CONSTRAINTS). D IS THE DIAGONAL DISTANCE ACROSS THE OBJECT.

No. of Camera Stations	LEP or TEP	9	_ σ _c (mm)	$\frac{-}{\sigma_x}$ (mm)	$\overline{\sigma}_{y}$ (mm)	$\overline{\sigma_z}$ (mm)	$\frac{D}{\sigma_c}$	$\frac{D}{\sigma_x}$	$\frac{D}{\sigma_y}$	$\frac{D}{\sigma_z}$	Correlation Coefficients from TEP
8	TEP	0.71	0.027	0.025	0.034	0.019	174,000	188,000	138,000	247,000	98% < 0.15 1.6% < 0.3
3 APS	LEP	0.71	0.027	0.024	0.035	0.018	174,000	196,000	134,000	261,000	0.4% <0.75
4	TEP	1.8	0.070	0.079	0.085	0.032	67,000	59,000	55,000	147,000	49% < 0.15 24% < 0.3 14% < 0.45
3 APS	LEP	1.4	0.054	0.048	0.076	0.028	87,000	98,000	62,000	168,000	$\begin{array}{ccc} 11\% & <0.19\\ 10\% & <0.60\\ 3\% & <0.99 \end{array}$

TABLE 3. OBJECT POINT PRECISION OBTAINED FOR THE CYLINDER NETWORK, EXAMPLE 3, VIA BOTH LEP AND TEP (INNER CONSTRAINTS). D IS THE CYLINDER DIAMETER.

No. of Camera Stations	LEP or TEP	9	$\overline{\sigma_c}$ (mm)	$\overline{\sigma}_{xy}$ (mm)	$\bar{\sigma}_z$ (mm)	$\frac{D}{\overline{\sigma}_c}$	$\frac{D}{\overline{\sigma}_{xy}}$	$\frac{D}{\vec{\sigma}_z}$	Correlation Coefficients from TEP
	TEP	1.2	0.119	0.128	0.099	84,000	78,000	101,000	77% <0.15
6									21% <0.3
	LEP	0.94	0.093	0.100	0.077	108,000	100,000	130,000	2% <0.45
3 APS									

ray points lying at 60° intervals. The camera to be employed is again of 23 by 23-cm format with a 240-mm lens, and the image coordinate measuring standard error is assumed to be two micrometres.

It is well known that to enhance, and in some cases make possible, the recovery of the coordinates x_p and y_p of the camera's principal point, a diversity of camera roll angles is necessary. With this in mind, one practical approach would be to simulate the network in Figure 3 with alternating roll angles of 0° and 90°. This, however, leads to a singular system of normal equations for the self-calibrating bundle adjustment, a fact not reflected at all in an LEP. To overcome this singularity, one need only change the camera roll angle sequence to 0°, 180°, 90°, 270°, 0°, 90°, for example. But to arrive at this conclusion TEP is needed.

As shown in Table 3, even when there is an appropriate FOD to enable the recovery of interior orientation elements to an acceptable level of precision, there is still a 20 percent discrepancy between the results of LEP and TEP, and this difference persists when APs are suppressed. From the point of view of reliability, the geometry is very weak (too many two-ray points) and it is unlikely that an experienced photogrammetrist would settle for such a poor design in practice. If he did, he could ill afford to rely on LEP to yield a satisfactory measure of object point precision.

COVARIANCE STRUCTURE

Generally speaking, in network design most of the attention is focused on optimizing the variances of object point coordinates. For example, the inner constraint approach is preferred because it yields minimum mean variance. But what of the covariances? If point-to-point or coordinate-to-coordinate correlations are high, simply emphasizing variances can lead to a misleading interpretation of the structure of $Q_x^{(2)}$. One area in which covariances play a role is in determining the precision of functions of the *XYZ* object point coordinates, for example, volumes, distances, or the determination of changes of object shape. What needs to be examined then is the correlation structure of $Q_x^{(2)}$ and the degree to which LEP with its zero point-to-point covariances represents this structure.

Based on the examples covered in this paper, it seems that in this regard also we may be able to substitute LEP for TEP with little loss of covariance information, at least for strong networks. In Example 1 (see Table 1), networks comprising three or more camera stations did not exhibit any correlation coefficient values exceeding 0.3, and 96 to 99 percent were under 0.15. The $Q_x^{(2)}$ matrices were homogeneous and near isotropic, a very desirable covariance structure! In the eight-station network of Example 2, no point-to-point correlation coefficients exceeded 0.3, and the high coordinate-to-coordinate values for points on the sides of the object were reflected in the LEP. Even in the weaker network of Example 3 the covariances were small in magnitude. These results support the contention that the $Q_{y}^{(2)}$ matrix from LEP, with its block-diagonal structure, will usually be a sufficiently accurate representation of the corresponding matrix obtained in a TEP by means of inner constraints.

Unfortunately, the same cannot be said of LEP and the covariance matrix obtained in a TEP by means of the minimal control approach. Here, point-to-point correlations are typically more significant. In the eight-station minimally controlled network in Example 1, for instance, 58 percent of correlation coefficient values were less than 0.15, 35 percent were between 0.15 and 0.45, and the remaining 7 percent exceeded 0.45. By some measures these correlations are still modestly low, but they do not match the free-network results. The near-orthogonal covariance structure produced in photogrammetric networks offers the potential of simplifying some procedures which utilize $Q_x^{(2)}$. One prominent example is in the analysis of deformation measurement networks of relative type, which requires datum transformations of the covariance matrix of object points.

In a photogrammetric network the magnitude of the correlation coefficients in $Q_x^{(2)}$ is influenced by the number of object points. As the target density increases, the degree of point-topoint correlation can be expected to decrease. Conversely, accompanying a reduction in the number of object points is an increase in the level of correlation. Experience suggests, however, that this inflation of correlation coefficient values is of no practical significance provided there are some 25 or more object points in the network, with around 20 or more points being imaged on each photograph.

CONCLUDING REMARKS

The network design examples covered in this paper have ranged from the geometrically strong to the weak, but still only represent a sample of the many and varied design configurations encountered in practice. Nevertheless, the results presented strongly support the use of LEP in network design as a substitute for TEP by means of inner constraints, at least for networks displaying a high level of both internal reliability and recoverability for APs. TEP should, however, be applied in cases were uncertainties remain.

In interactive network design, LEP solves the datum problem (ZOD) and provides a means to very rapidly evaluate the precision of a network configuration (FOD). As an example of the computation times involved, consider the eight-station network of Example 1. Here, there are 231 unknowns in the bundle adjustment. For the LEP a time of 21 seconds is required on an IBM PC/AT to compute and list on the screen the individual coordinate standard errors for each point. Utilizing the inner constraint approach on a point-by-point basis (Equation 9) in which correlations are not computed, the corresponding time required is around 190 seconds. For a full TEP in which all elements of $Q_x^{(2)}$ are computed, some 12 minutes of computer time is required to form and invert the normal equation system, Equation 7.

REFERENCES

- Blaha, G., 1971. Inner Adjustment Constraints with Emphasis on Range Observations. Dept. of Geodetic Science Report No. 148, Ohio State University, Columbus, 85 p.
- Brown, D. C., 1958. A Solution to the General Problem of Multiple Station Analytical Stereotriangulation. RCA-MTP Data Reduction Technical Report No. 43, Patrick Air Force Base, Florida, 53 p.
 - -----, 1980. Application of Close-Range Photogrammetry to Measure-

ments of Structures in Orbit. GSI Technical Report No. 80-012, Geodetic Services, Inc., Melbourne, Florida, Vol. 1, 131 p.

- Förstner, W., 1985. The Reliability of Block Triangulation, Photogrammetric Engineering and Remote Sensing, Vol. 51, pp. 1137–1149.
- Fraser, C. S., 1982. Optimization of Precision in Close-Range Photogrammetry. *Photogrammetric Engineering and Remote Sensing*, Vol. 48, pp. 561–570.
 - —, 1984. Network Design Considerations for Non-Topographic Photogrammetry. *Photogrammetric Engineering and Remote Sensing*, Vol. 50, No. 8, pp. 1115–1126.
- Fraser, C. S., and D. C. Brown, 1986. Industrial Photogrammetry: New Developments and Recent Applications. *The Photogrammetric Record*, Vol. 12, No. 68, pp. 197–217.
- Gustafson, P. C., and J. D. Brown, 1985. Interactive Photogrammetric Design on a Microcomputer. *Presented paper, ACSM-ASP Annual Convention*, Washington, D.C., 9p.
- Papo, H., and A. Perelmuter, 1982. Free Net Analysis in Close-Range Photogrammetry. *Photogrammetric Engineering and Remote Sensing*, Vol. 48, pp. 571–576.

(Received 18 December 1986; accepted 6 January 1987)

Engineering Summer Conferences The University of Michigan

Ann Arbor, Michigan

15–19 June 1987 — Infrared Technology Fundamentals and System Applications

Presentations cover radiation theory, radiative properties of matter, atmospheric propagation, optics, and detectors. System design and the interpretation of target and background signals are emphasized.

22–26 June 1987 — Advanced Infrared Technology

Presentations cover atmospheric propagation, detectors and focal plane array technology, discrimination characteristics of targets and backgrounds, and system designs. The prerequisite is familiarity with the fundamentals of infrared/electro-optics.

20-24 July 1987 — Synthetic Aperture Radar Technology and Applications

The design, operation, and application of synthetic aperture radar (SAR) are presented. Topics covered include range-doppler imaging of rotating objects, spotlight radar concepts, bistatic radar, and the technology used in optical and digital processing of SAR data for terrain mapping.

20-24 July 1987 — Computer Vision

A computer vision system recovers some useful information from images of a scene. This course introduces basic techniques in computer vision with emphasis on solving problems having a variety of industry, government, and scientific applications. For further information please contact

> Engineering Summer Conferences 300 Chrysler Center, North Campus The University of Michigan Ann Arbor, MI 48109 Tele. (313) 764-8490

The Second Industrial and Engineering Survey Conference

University College London 2–4 September 1987

The 2nd Industrial and Engineering Survey Conference has been arranged under the joint auspices of the International Society for Photogrammetry and Remote Sensing (ISPRS) Commission V and the International Federation of Surveyors (FIG) Commission 6. The time is now appropriate to bring together interests in large-scale metrology, industrial and engineering surveying, and close-range photogrammetry in order to assess recent advances in this area. All authors of papers at the Conference have been specially invited. A wide range of industrial, commercial, and academic backgrounds will be represented.

Organization of many aspects of the conference has been shared with colleagues at The City University, University of Surrey, Imperial College of Science and Technology, and South Bank, North East London, and Portsmouth Polytechnics.

For further information please contact

Dr. J. C. Iliffe University College London Gower Street London WC1E 6BT United Kingdom Tele. 01-387 7050, ext. 2733