Quantitative Description and Classification of Drainage Patterns

Demetre P. Argialas
Department of Civil Engineering and Remote Sensing and Image Processing Laboratory, Louisiana State University, Baton Rouge, LA 70803
John G. Lyon
Department of Civil Engineering, The Ohio State University, Columbus, OH 43210
Olin W. Mintzer
U. S. Army Engineer Topographic Laboratories, Fort Belvoir, VA 22060

ABSTRACT: A structural pattern recognition methodology is presented for computer-assisted description and classification of drainage patterns. Eight patterns types were quantitatively described and classified. The methodology to classify these patterns consisted of drainage pattern models, hierarchical and relational models, attribute extraction, and classification strategies. The results indicated that a digital computer can assist human interpretation of such patterns.

INTRODUCTION

DRAINAGE NETWORKS are formed by aggregation of natural drainage ways. The configuration in which a given set of tributaries arrange themselves within a drainage network has been called the pattern of the drainage network or drainage pattern (Parvis, 1950). Comprehensive empirical descriptions of more than thirty pattern configurations were discussed by Parvis (1950) and Howard (1967).

Drainage patterns result from variations in the conditions of topography, porosity, permeability, geologic structure, and chemical composition of soils and rocks. Table 1 demonstrates the significance of drainage patterns as key indicators of terrain factors. Drainage patterns have been employed in water resources assessment and hydrogeomorphic analysis (Salomonson, 1983), engineering applications of remote sensing (Mintzer and Messmore, 1984), and photogeology for the interpretation of soils and rocks from remotely-sensed images (Way, 1978; Estes *et al.*, 1983; Mintzer, 1983). The usefulness of computer classification of drainage patterns for formalizing and automating the classification of textures and structures appearing on remotely-sensed images has been suggested by Gudilin (1973) and Estes *et al.* (1983).

Drainage networks have been described by their topology and geometry. The majority of research has focused on methods of characterizing, coding, and storing the topologic aspects of drainage patterns (Jarvis and Woldenberg, 1984). Coding procedures for determining drainage network geometry as well as topology have been investigated by Coffman and Turner (1971) and Jarvis (1984). However, limited work has been reported on the quantitative classification of the patterns of drainage networks. Two-dimensional histograms and vectorial rosettes have been employed for drainage pattern analysis by Morisawa (1963), Jarvis (1976), Argialas (1979), and Scheidegger (1979). Rayner (1971) employed two-dimensional Fourier analysis for description of dendritic and rectangular patterns.

TABLE 1. SIGNIFICANCE OF DRAINAGE PATTERNS.

Drainage Pattern	Significance
Dendritic	Horizontally Bedded Sedimentary Strata, Volcanic Tuff, Old Dissected Coastal Plains
Parallel	Uniformly Sloping Sedimentary Strata, Large Basalt Flows, Tilted Coastal/Lake Plains
Rectangular/Angular	Slate, Schist, Gneiss, Sandstone
Pinnate	Loess, High Silt Content Materials
Trellis	Folded/Faulted Sedimentary Strata, Maturely Dissected Coastal Plains
Radial	Domelike Hills
Annular	Domelike Hills on Granitic or Sedimentary Rocks

The subject of automated delineation of river networks from digital terrain models and satellite images was investigated by the theory of critical numbers (Johnston and Rosenfeld, 1975; Peuker and Douglas, 1975; Carrol, 1983), by a tree grammar approach (Fu, 1976), and by spatial relational models (Wang et al., 1983).

Identification of drainage patterns by photointerpreters has been accomplished for years. However, it is costly and time consuming, as photointerpreters must have training and experience. A computer-assisted approach for pattern classification can alleviate the problem and at the same time render a formal, objective, and repeatable approach. Hence, the interest in quantitative classification and this effort to develop a methodology.

METHODOLOGY

This research effort classified eight "basic" (Parvis, 1950; Howard, 1967) drainage patterns, whose characteristics were such that most photointerpreters could readily distinguish them. They were the dendritic, pinnate, parallel, trellis, rectangular, angular, radial, and annular drainage patterns. Figure 1 shows some typical representations of these patterns.

The premise of this research was that drainage patterns can be described and classified by their distinct topologic, geometric, and structural relationships among their constituent streams. In particular, the research was aimed toward the description and classification of these patterns through a structural pattern recognition approach.

Structural pattern recognition refers to digital image analysis methods that represent patterns in terms of their intrinsic structure (Fu, 1982). Often this intrinsic structure has been represented by rules expressing the juxtaposition of identifiable pattern elements, their attributes, and their interrelationships. Gelsema and Kanal (1980), Kanal and Rosenfeld (1981), and Fu (1982) have summarized applications of this approach for recognition of images of alpha-numeric characters, speech pat-

PHOTOGRAMMETRIC ENGINEERING AND REMOTE SENSING,
Vol. 54, No. 4, April, 1988, pp. 505–509.

0099-1112/88/5404–505$02.25/0
©1988 American Society for Photogrammetry and Remote Sensing

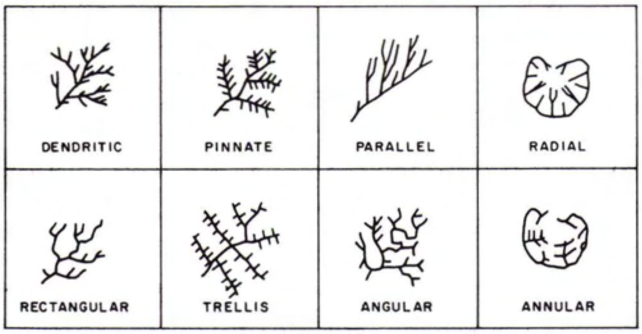

FIG. 1. Samples of the eight drainage pattern types.

terns, geometric figures, chromosomes, and fingerprints. Implementation of the structural approach required the design of drainage pattern models, hierarchical and relational models, extraction of pattern attributes, and design of a classification strategy (Thomanson and Gonzalez, 1981; Argialas, 1985).

DRAINAGE PATTERN MODELS

A structural model requires the representation of patterns in terms of their constituent objects, object attributes, and object relationships. Unfortunately, there exists no general approach for the selection or generation of the most useful and appropriate objects and attributes for a given pattern (Fu, 1974; Thomanson and Gonzalez, 1981). Selection of component objects, and attributes has often relied on the past experience, engineering intuition and domain-specific knowledge of the system designer (Gelsema and Kanal, 1980; Kanal and Rosenfeld, 1981; Fu, 1982).

In the present effort, object and attribute selection was based on efficient use of the structure and constraints of patterns as those were embedded in their descriptions (Parvis, 1950; Howard, 1967). These are

- The *dendritic pattern* has a treelike branching system where tributaries join a gently curving mainstream at acute angles.
- The *pinnate pattern* resembles the dendritic, but its secondary tributaries are evenly and closely spaced, and parallel. They generally join the primary tributaries at acute angles.
- The *parallel pattern* has tributaries flowing nearly parallel to each other.
- The *trellis pattern* contains primary tributaries that are long, straight, and parallel to each other, but perpendicular to the main stream. Its secondary tributaries are numerous, short, perpendicular to the primary ones, parallel to the main stream, and essentially the same size.
- The *rectangular pattern* has distinguishable right angle bends in both the main stream and its tributaries.
- The *angular pattern* consists of a mixture of acute, right, and obtuse angles.
- The *radial pattern* contains streams radiating from a central area, like the spokes of a wheel.
- The *annular pattern* has circular streams and a few radial streams.

According to these descriptions, a drainage pattern can be identified by recognizing that

- It is composed of a main stream, from which issue primary tributaries, and from which main stream and primary tributaries issue secondary tributaries.
- It is characterized by specific attributes of and relationships among the main stream and the primary and secondary tributaries.

The main stream and the primary and secondary tributaries have been named the trunk, branches, and leaves, respectively. Collectively they are called semantic objects (SO) because they expressed the meaningful components of the patterns according to their descriptions.

Drainage pattern attributes were designed to describe the semantic objects (e.g., elongation of branches, straightness of trunk, uniformity of leaves), the object relationships (e.g., perpendicularity, bifurcation ratio), and the overall pattern (e.g., average of all junction angles). In particular, attributes were designed for the trunk, branches, leaves, trunk-branch relations, branch-leaf relations, trunk-leaf relations, leaf-leaf relations, and the overall pattern (Tables 2 and 3).

The semantic objects (main stream, primary and secondary tributaries), their attributes, and their interrelationships constituted the "stereotype" structural drainage pattern model (Table 3). This model represented a method by which the information embedded in the patterns can be organized so that attributes of and relationships among their semantic objects were identifiable.

Models of the eight pattern classes were obtained by selecting and inserting appropriate values for the attributes of each pattern according to its description. Thus, each pattern model contained

TABLE 2. COMPUTED DRAINAGE PATTERN ATTRIBUTES.

TSHAPE	Shape of the Trunk
BRTYPE	Type of Branching
MAOT	Mean Intermediate Angle on the Trunk
BSHAPE	Branch Shape
BRELON	Branch Elongation
MAOB	Mean Intermediate Angle on the Branches
UNLEAF	Uniformity of Leaves
MAOL	Mean Intermediate Angle on the Leaves
MATB	Mean Junction Angle between the Trunk and the Branches
RBRBL	Ranked Bifurcation Ratio between Branches and Leaves
BLAZDIF	Azimuthal Difference between Branches and Leaves
MABL	Mean Junction Angle between the Branches and Leaves
MALL	Mean Junction Angle among Leaves
RJAW	Ranked Junction Angle
RANMT	Ranked Angle with Vertex on the Center of Gravity of the Nodes and Sides Diverging to the Mouth of the Pattern and to Its most Distant Node

TABLE 3. THE STEREOTYPE DRAINAGE PATTERN MODEL.

OBJECT = TRUNK		
	TSHAPE =	(STRAIGHT, CIRCULAR)
	BRTYPE =	(ONESIDED, TWOSIDED)
	MAOT =	(ACUTE, RIGHT, OBTUSE)
OBJECT = BRANCH		
	BSHAPE =	(STRAIGHT, BENT)
	BRELON =	(SHORT, MEDIUM, LONG)
	MAOB =	(ACUTE, RIGHT, OBTUSE)
OBJECT = LEAF		
	UNLEAF =	(UNIFORM, NONUNIFORM)
	MAOL =	(ACUTE, RIGHT, OBTUSE)
TRUNK – BRANCH RELATIONS		
	MATB =	(ACUTE, RIGHT, OBTUSE)
BRANCH – LEAF RELATIONS		
	RBRBL =	(SMALL, MEDIUM, LARGE)
	BLAZDIF =	(ACUTE, RIGHT)
	MABL =	(ACUTE, RIGHT, OBTUSE)
	RMA12 =	(ACUTE, RIGHT, OBTUSE)
TRUNK – LEAK RELATIONS		
	NONE	
LEAF – LEAF RELATIONS		
	MALL =	(ACUTE, RIGHT, OBTUSE)
OVERALL PATTERN ATTRIBUTES		
	RJAW =	(SACUTE, ACUTE, RIGHT)
	RANMT =	(SMALL, LARGE)

its significant attributes and their values necessary for describing that pattern class. According to these models, each drainage pattern class is a category determined by the chosen common attributes and their values. Therefore, a drainage pattern class encompassed all possible pattern instances that were characterized by the stated common attributes and their values. Table 4 shows the structural model of the trellis pattern type.

Table 5 shows the correspondence of the visual (qualitative) and computed (quantitative) attributes for the trellis pattern. The qualitative descriptions of the patterns did not always contain explicit reference to all the attributes of each pattern (Parvis, 1950; Howard, 1967). However, quantitative models require explicit reference to all significant attributes and their values. This has contributed to having more computed than visual attributes. For example, for the trellis model (Table 4), five additional attributes—TSHAPE, BRTYPE, RJAW, RANMT, MAOT—were computed which were not explicitly used in the qualitative description of the pattern.

Moreover, for certain visual attributes, it was necessary to use more than one computed attribute in order to appropriately capture their meaning and therefore describe them better. For example, to express the straightness of the branches, both BSHAPE and MAOB were employed (Table 5).

Table 4. The Trellis Drainage Pattern Model.

OBJECT = TRUNK		
	TSHAPE =	STRAIGHT
	BRTYPE =	TWOSIDED
	MAOT =	OBTUSE
OBJECT = BRANCH		
	BSHAPE =	STRAIGHT
	BRELON =	MEDIUM OR LONG
	MAOB =	OBTUSE
OBJECT = LEAF		
	UNLEAF =	UNIFORM
	MAOL =	OBTUSE
TRUNK – BRANCH RELATIONS		
	MATB =	RIGHT
BRANCH – LEAF RELATIONS		
	RBRBL =	MEDIUM OR LARGE
	BLAZDIF =	RIGHT
	MABL =	RIGHT
TRUNK – LEAF RELATIONS		
	NONE	
LEAF – LEAF RELATIONS		
	MALL =	RIGHT
OVERALL PATTERN ATTRIBUTES		
	RJAW =	RIGHT
	RANMT =	LARGE

Hierarchical and Relational Models

Building a drainage pattern model requires that each of the semantic objects be defined in terms of its lower-level objects; at the lowest level of description, the objects are regarded as primitives where each part above the level of a primitive has its own hierarchical description (Thomanson and Gonzalez, 1981). In the present effort, a drainage pattern object hierarchy was designed by employing the following lower-level objects: Strahler segments, reaches, and nodes. The semantic objects (trunk, branches, leaves) were decomposed into Strahler segments (SS) of three orders (Strahler, 1964). The Strahler segments were decomposed into reaches (Coffman and Turner, 1971) and the reaches were defined by their nodes (Figure 2).

The actual procedure for generating the hierarchical model started at the bottom of the hierarchy. First, nodes were aggregated to reaches, reaches to Strahler segments, and Strahler segments to semantic objects (Argialas, 1986). This "bottom-up" aggregation was necessary because of the method of pattern coding, which took place by digitizing the nodes of the patterns in an interactive graphics system.

Implementation of the hierarchical model required objects to be organized systematically at all levels of the hierarchy (Thomanson and Gonzalez, 1981). Therefore, an attribute list was designed and attached to each node of the object hierarchy to characterize the object of that node. The method of attaching attribute lists to each level of the hierarchy was in the form of relational models, implemented as relational tables. The relational models expressed the relationships among the drainage pattern objects and facilitated representation of those relationships as object attributes (Argialas, 1985).

Attribute Extraction

Attribute extraction involved the computation of the attributes (Table 2) of the drainage pattern models from the information stored in the hierarchical and relational models. Although the computed attributes were numerical-valued, they have been converted to discrete-valued (symbolic) attributes (Table 2) through appropriate thresholds. In this manner, computational descriptions agreed with the photointerpreter's terminology and the qualitative definitions of the patterns. Table 6 shows the steps involved in computing the ranked angle between the MOUTH (mouth of the pattern) and TIP (the node furthest away from the mouth). This angle (ANGLMT) has as a vertex the "center

Table 5. Correspondence of Visual and Computed Attributes for the Semantic Objects of the Trellis Pattern.

Semantic Objects	Visual Attributes	Computed Attributes
TRUNK	(None)	TSHAPE = STRAIGHT BRTYPE = TWOSIDED MAOT = OBTUSE
BRANCHES	Long	BRELON = MEDIUM OR LONG
	Straight	BSHAPE = STRAIGHT, MAOB = OBTUSE
	Parallel to each other	MATB = RIGHT, BSHAPE = STRAIGHT
	Perpendicular to trunk	MATB = RIGHT
	Perpendicular to leaves	MABL = RIGHT
LEAVES	Numerous	RBRBL = MEDIUM OR LARGE
	Short	BRELON = MEDIUM OR LONG
	Perpendicular to branches	MABL = RIGHT, MALL = RIGHT
	Parallel to main stream	BLAZDIF = RIGHT
	Uniform in size	UNLEAF = UNIFORM

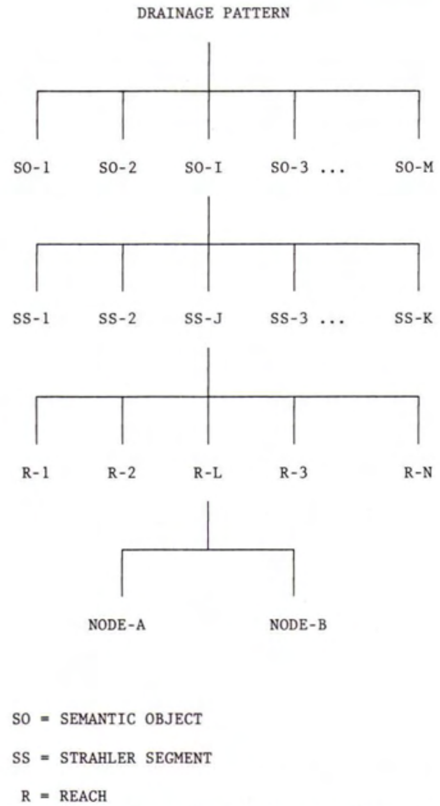

FIG. 2. The hierarchical drainage pattern object organization.

TABLE 6. COMPUTATION OF RANMT-RANKED ANGLE BETWEEN MOUTH AND TIP.

(1)	Compute the coordinates, XBAR, YBAR, of CENTER– the center of gravity of the pattern $XBAR = \frac{\Sigma X_i}{N}, YBAR = \frac{\Sigma Y_i}{N}$ X_i, Y_i are the coordinates of nodes, N is the total number of nodes.
(2)	Define MOUTH as the most downstream node of the TRUNK.
(3)	Search for TIP, the source or exterior node of the pattern that is farthest away from the MOUTH.
(4)	Compute the azimuths of the lines MOUTH-CENTER and CENTER-TIP.
(5)	Based on these azimuths, compute ANGLMT, that is, the angle between the lines MOUTH-CENTER and CENTER-TIP.
(6)	Define RANMT so that RANMT is SMALL if ANGLMT $< 105°$, and RANMT is LARGE if ANGLMT $\geq 105°$.

of gravity" of the pattern (Argialas, 1985). The significance of this attribute was that if its value was lower than approximately 105 degrees, then the pattern was most probably radial or annular. Otherwise it was one of the other patterns.

CLASSIFICATION PROCEDURE

Classification is the process of assigning a pattern to one of many possible classes based on the extracted pattern features. In the structural approach, a series of tests can be designed to evaluate the presence or absence of certain subpatterns, or certain pattern attributes and their values (Fu, 1982). These tests can then be embedded in a decision tree for pattern classification.

TABLE 7. GROUP OF TESTS EMPLOYED FOR THE CLASSIFICATION OF THE TRELLIS PATTERNS.

(1)	If	RANMT	=	LARGE, and
		TSHAPE	=	STRAIGHT
	Then	continue; else go to 7		
(2)	If	RJAW	=	RIGHT
	Then	continue; else go to 7		
(3)	If	RBRBL	=	MEDIUM, and
		BRELON	=	MEDIUM, and
		UNLEAF	=	UNIFORM
	Then	continue; else go to 7		
(4)	The drainage pattern is more likely trellis; continue			
(5)	If	BRELON	=	LONG, and
		RBRBL	=	LARGE, and
		MALL	=	RIGHT, and
		MATB	=	RIGHT, and
		MAOL	=	OBTUSE, and
		BLAZDIF	=	RIGHT, and
		BSHAPE	=	STRAIGHT
	Then continue; else no decision can be made			
(6)	The drainage pattern is definitely trellis			
(7)	Check the next node of the decision tree			

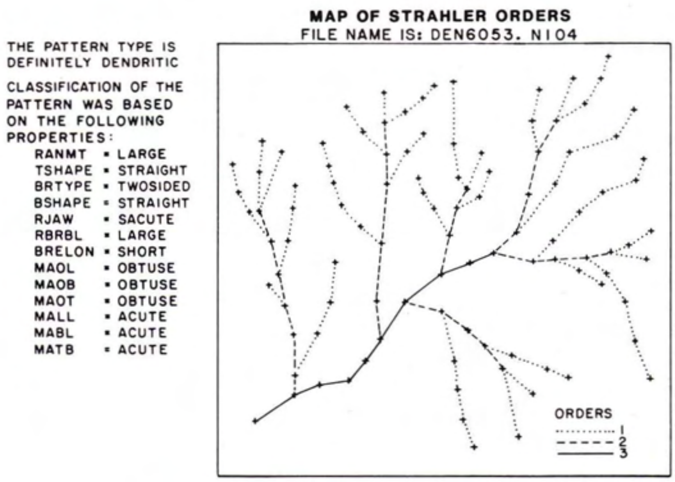

FIG. 3. Classification of a dendritic pattern.

In this effort, a decision tree has been designed for the classification of patterns (Argialas, 1985). The nodes of the decision tree were composed of groups of tests. The tests evaluated the presence or absence of certain combinations of attributes and values. Table 7 shows the group of tests that were employed as a portion of the decision tree and used for the identification of trellis patterns. The tests enabled the establishment or rejection of a decision at each node of the tree. Based on the outcome of these tests, each pattern was assigned to one of the eight predetermined classes.

RESULTS AND DISCUSSION

The methodology was programmed in a software system and named the Drainage Pattern Analysis (DPA) system. As a verification of methodology and system capability, the DPA system has been evaluated with 20 test patterns, both real and artifical, representing the eight major classes. The artifical patterns were drawn by students and colleagues. The real patterns were extracted from books, maps, and aerial photographs. Both artificial and real patterns were evaluated and classified by the authors and other expert photointerpreters to provide for a comparison with the results of the computer classification.

The DPA system classified all the test patterns, and the results

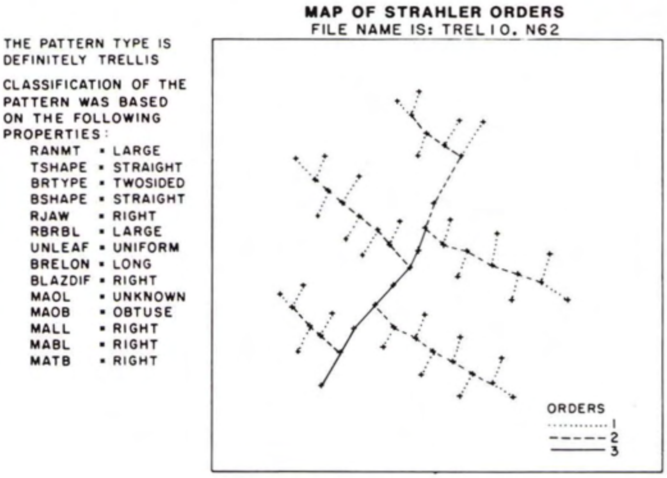

FIG. 4. Classification of a trellis pattern.

were in agreement with those of expert photointerpreters (Argialas, 1985). Examples are shown in Figures 3 and 4. Figure 3 shows the results for the classification of a dendritic pattern and Figure 4 for a trellis pattern.

The present methodology provided a formal framework for describing the intrinsic structure of drainage patterns. It provided both a description and classification of the patterns. Such a computer-assisted classification is potentially valuable in automating the process of engineering terrain analysis. Therefore, it contributed more of a geomorphologic and engineering interpretation than of other approaches. Previously, this problem has only been solved by human interpreters, and computational approaches were limited (Jarvis and Woldenberg, 1984). The present approach offers one step toward a more formal representation of the complex process of human perception of drainage patterns.

CONCLUSIONS

Structural descriptions of drainage patterns were developed to facilitate computer assisted pattern classification. By decomposing drainage patterns into a hierarchical set of objects, and by describing the attributes and relationships among these objects, eight common drainage pattern types were described. The methodology was embedded in a computer program to hierarchically decompose and classify patterns. A verification test of the capability of this approach for classifying patterns resulted in correct classification of 20 examples of drainage patterns.

ACKNOWLEDGMENTS

We would like to thank Professors Charles Moore, Harold Moellering, and B. Chandrasekaran who have provided assistance throughout this research effort. This work was supported in part by the Department of Civil Engineering at Louisiana State University.

REFERENCES

Argialas, D., 1979. *Mapping and Comparison of Landsat Lineaments with Gravity Trends in West Tennessee*. M.S. Thesis, University of Tennessee, Knoxville, Tennessee.

———, 1985. *A Structural Approach Towards Drainage Pattern Recognition*. Ph.D. Dissertation, The Ohio State University, Columbus, Ohio.

———, 1986. Computer-Assisted Recognition of Drainage Patterns. *Proceedings of the Annual Convention of American Society for Photogrammetry and Remote Sensing*, 4:435–444.

Carrol, R., 1983. Automated Gully Delineation using Elevation Data, *Technical paper, 49th Annual Meeting ASP*, pp. 144–151, ACSM-ASP Convention, Washington, D.C., 13–18 March.

Coffman, D., and A. Turner, 1971. Computer Determination of the Geometry and Topology of Stream Networks. *Water Resources Research*, 7:419–423.

Estes, J., E. Hajic, and L. Tinney, 1983. Fundamentals of Image Analysis of Visible and Thermal Infrared Data, *Manual of Remote Sensing* (R. Colwell, editor), American Society of Photogrammetry, Falls Church, Virginia.

Fu, K., 1974. *Syntatic Methods in Pattern Recognition*, Academic Press, New York, New York.

———, 1976. Tree Languages and Syntactic Pattern Recognition, *Pattern Recognition and Artificial Intelligence* (C. Chen, editor) Academic Press, New York, pp. 257–291.

———, 1982. *Applications of Pattern Recognition*, CRC Press, Boca Raton, Florida.

Gelsema, E., and L. Kanal, 1980. *Pattern Recognition in Practice*, North-Holland, Amsterdam, Holland.

Gudilin, I., 1973. Interpretation of Landscape as an Indicator of Geologic Structure, *Landscape Indicators*, (A. Chikishev, editor), Consultants Bureau, New York, New York, pp. 92-105.

Howard, A., 1967. *Drainage Analysis in Geologic Interpretation: A Summation*, American Association of Petroleum Geologists, 51:2246–2259.

Jarvis, R., 1976. Stream Orientation Structures in Drainage Networks, *Journal of Geology*, 84:563–582.

———, 1984. Topology of Tree-like Networks, *Spatial Statistics and Models* (G. Gaile and C. William, editors), D. Reidel Publishing Company, Dordrecht, Holland.

Jarvis, R., and M. Woldenberg, 1984. *River Networks*, Hutchinson Ross Publishing Company, Stroudsburg, Pennsylvania.

Johnston, E., and A. Rosenfeld, 1975. Digitial Detection of Pits, Peaks, Ridges and Ravines, *IEEE Transactions on Systems, Man and Cybernetics*, 5:472–480.

Kanal, L., and A. Rosenfeld, 1981. *Progress in Pattern Recognition*, North-Holland, Amsterdam, Holland.

Mintzer, O., 1983. Engineering Applications, *Manual of Remote Sensing* (R. Colwell, editor), American Society of Photogrammetry, Falls Church, Virginia.

Mintzer, O., and J. Messmore, 1984. *Terrain Analysis Procedural Guide for Surface Configuration*, Technical Report ETL-0352, U.S. Army Corps of Engineers, Engineer Topographic Laboratory, Fort Belvoir, Virginia.

Morisawa, M., 1963. Distribution of Stream-flow Direction in Drainage Patterns, *Journal of Geology*, 71:528–529.

Parvis, M., 1950. Drainage Pattern Significance in Airphoto Identification of Soils and Bedrocks, *Photogrammetric Engineering*, 16:387–409.

Peucker, T., and D. Douglas, 1975. Detection of Surface-Specific Points by Local Parallel Processing of Discrete Terrain Elevation Data, *Computer Graphics and Image Processing*, 4:375–387.

Rayner, J., 1971. *An Introduction to Spectral Analysis*, Pion Limited, London, England.

Scheidegger, A., 1979. The Principles of Antagonism in the Earth's Evolution, *Tectonophysics*, 55:T7-T10.

Salomonson, V., 1983. Water Resources Assessment, *Manual of Remote Sensing* (R. Colwell, editor), American Society of Photogrammetry, Falls Church, Virginia.

Strahler, A., 1964. Quantitative Geomorphology of Drainage Basins and Channel Networks, *Handbook of Applied Hydrology*, (V. Chow, editor), McGraw-Hill, New York.

Thomanson, M., and R. Gonzalez, 1981. Database Representation in Hierarchical Scene Analysis, *Progress in Pattern Recognition* (L. Kanal and A. Rosenfeld), editors, North-Holland, Amsterdam, Holland.

Wang, S., D. Elliott, J. Campell, R. Erich, and R. Haralick, 1983. Spatial Reasoning in Remote Sensing Data, *IEEE Transactions on Geoscience and Remote Sensing*, 21:94–101.

Way, D., 1978. *Terrain Analysis*, McGraw-Hill, New York.

(Received 3 June 1987; revised and accepted 3 December 1987)