# Automatic Line Generalization Using Zero-Crossings

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ABSTRACT: The problem of automating the process of line generalization has been very difficult. It has not been solved yet despite the concerted effort of many private firms as well as government agencies. There does not exist an algorithm which can automatically perform this process when there is a drastic change in scale between the original and generalized maps. In this paper, an algorithm which is successful in automatically generalizing lines from any large scale to any small scale is presented. The algorithm achieves different levels of smoothing the line while preserving the overall shape of the line. The results are compared with those obtained by manual methods. It was found that the results obtained by the algorithm are very close to those obtained by cartographers using manual methods.

## INTRODUCTION

THE SPATIAL INFORMATION in cartography is collected at various scales-usually at large scales-through different means, e.g., by ground survey, by analytical photogrammetric methods, by digitizing (or scanning) existing maps, or from satellite imagery.

Generally, the amount of data collected is much more than what is required to adequately represent a particular feature. Therefore, there is a need for data compaction or data compression.

The spatial information that is displayed in cartography needs to be portrayed at different scales for various purposes. As the scale is reduced, one needs to further compress the data and remove the unwanted and unimportant detail, if one wants the displayed information to be clear, concise, and uncluttered. This can only be achieved through the process of generalization.

Computers have played, and are continuing to play, a major role in the field of cartography. However, the process of map production, especially when a change in scale is required (which is usually the case), has not been automated despite the concerted effort of many government agencies and private firms. The bottleneck for this has been the process of line generalization. There does not exist a satisfactory method which can generalize lines automatically as the scale of the map changes.

This paper addresses the above mentioned problems of cartography. The problem of automatic line generalization from any large scale to any small scale is solved by finding the zero-crossings of the convoluted values of the Gaussian with the signal derived from the digital data.

The evaluation of the generalized lines will be performed by comparing the corresponding lines generalized by zero-crossings algorithm with those performed by cartographers. It was found that the lines generalized by zero-crossings algorithm are almost identical to those produced by cartographers.

## DEFINITION OF LINE GENERALIZATION

The manual process of line generalization is highly subjective, so much so that the same person cannot achieve identical results when generalization of a line is performed at different times. Perhaps it is due to the subjective nature of this process that there is no single definition of line generalization in the literature. Keates (1981) correctly claims that there is no universal agreement on the definition of line generalization. Everyone tends to define line generalization the way they view it. Some cartographers, for example, Robinson and Petchenik (1976), have gone to the extent of calling line generalization as elusive as anything in print.

Probably the definition which explains what is really involved in the practice of line generalization is the one given by Hettner (McMaster, 1983). According to Hettner, "Generalization of a map is first of all a question of restriction and selection of source material. This is achieved partly by simplification of the objects on map, partly by omitting the small or less interesting objects."

### LITERATURE REVIEW

It is known from experience that more than 80 percent of a map consists of lines. Therefore, when one talks about processing maps, one is essentially referring to processing lines. Fortunately, many other disciplines such as image processing, computer graphics, and pattern recognition are also concerned with line processing. They might be interested in recognizing shapes of various objects, industrial parts recognition, feature extraction or electrocardiogram analysis, etc.

Whatever may be the objective of line processing and to whichever field it may be applied, there is one thing in common, *viz*: it is necessary to retain the basic character of the line under consideration.

The problem of line generalization is not very difficult if carried out manually but becomes difficult if one wants to do it by computer. Because of the subjective nature of this problem and because of lack of any criteria for evaluation of line generalization (Lundquist, 1959), it has been very difficult to automate this process. Recently some researchers, for example, Marino (1979) and White (1985), have suggested that one should first find critical points and retain them in the process of line generalization. Note that the critical points in a digital curve are the points of high curvature, points of intersection, and end points.

Others who attempted to make the process of line generalization more objective include Brophy (1973), Dettori and Falcidieno (1982), Douglas and Peucker (1973), Marino (1979), McMaster (1983), Opheim (1982), Reumann and Witkam (1974), Solovitskiy (1974), Srnka (1970), Sukhov (1970), Tobler (1965), Topfer and Pillewiser (1966), and Vanicek and Woolnough (1975).

### PROPOSITION FOR AUTOMATIC LINE GENERALIZATION

The basic assumption behind line generalization is that first the critical points are detected. Different levels of critical points in a curve are detected for different degrees of generalization.

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Once the critical points are detected, a line smoothing technique is used to smooth the line.

The zero-crossings of the second derivative of the Gaussian with the signal derived from the data are rich in information, Especially when the zero-crossings coinciding spatially to at least two channels are taken (Marr and Hildreth, 1980). Note that Marr and Hildreth (1980) discuss this technique with reference to edge detection in image processing. Edges in images are defined as the discontinuities in the first derivative of the gray level. Marr and Hildreth (1980) use gray level as the signal for the convolution with the Laplacian of the Gaussian. They then look for the zero-crossings in convoluted values to detect edges.

This technique may be used to detect critical points in a digitized curve (Asada and Brady, 1986). The level of critical points detection may be changed by simply changing the channel of the Gaussian. The smoothing of the critical points may be achieved by using cubic parametric splines (Spath, 1974), or one can use weighted moving averages.

It was found during the course of investigation that, when line generalization involves a drastic change in scale say from 1:24,000 to 1:250,000, it is not the critical points but the general shape of the curve that one needs to retain. This is contrary to the basic assumption made by cartographers, e.g., White (1985) and at the beginning of this research.

## Line Smoothing and Shape Preservation Using Zero-Crossings

Researchers in the fields of pattern recognition, image processing, and computer vision are interested in detecting edges in images. The fact that there is a change in gray level at the edges of images was long recognized by Rosenfeld and Thurston (1973) and Marr and Hildreth (1980). Consequently, this idea was exploited to detect the edges in images by various researchers.

Marr and Hildreth (1980) suggested that the zero-crossings of the convolution of the gray-level with the Laplacian of the Gaussian across a number of channels are rich in information. This statement is supported by the theoretical work of Logan (1977). In this case zero-crossing across a number of channels means the zero-crossings of the convoluted values of the Laplacian of the Gaussian with the gray-level signal for different values of the scale (sigma) of the Gaussian. In addition, it should be noted that the zero-crossings of the second derivative (or the Laplacian) of the Gaussian are the same as the extremas of the first derivative. The collection of zero-crossings across a number of channels is known as the finger-prints. The plot of these zero-crossings against the different channels is known as the scale space image (Mokhtarian and Mackworth, 1986).

Witkin (1983) demonstrated that, as the scale parameter of the Gaussian decreases, additional zero-crossings may appear but the existing ones, in general, will not disappear. Witkin claimed that the Gaussian is the only function which satisfies this property. In addition, the Gaussian is symmetric and strictly decreasing about the mean; therefore, weighting assigned to the signal values decreases smoothly with distance. Furthermore, the Gaussian is infinitely differentiable.

As stated earlier, different numbers of zero-crossings are obtained for different channels of convolutions, i.e., different values of sigma. In this research, channels and sigma of the Gaussian are interchangeably used. The problem is: how does one combine the zero-crossings from different channels to obtain the optimum information? How many channels are required to accurately detect the edges in images? Marr and Hildreth (1980) speculated that zero-crossings that spatially coincide over several channels are "physically significant." They further suggested that a minimum of two channels, reasonably separated in the frequency domain, are required in order to warrant the existence of an edge. The zero-crossings which are common to both channels indicate the presence of an edge. The edges thus detected are called the primal sketch.

Rosenfeld and Thurston (1973) recognized the fact that the technique that can be used to detect edges may also be used to detect the curvature in a digitized curve. Asada and Brady (1986) proposed the scale space approach to representing the significant changes in curvature. Their approach to represent curvature changes in a curve is termed as curvature primal sketch because of its correspondance with the primal sketch for representing the gray-level changes.

Mokhtarian and Mackworth (1986) used the scale space image and the Gaussian convolution for the description and recognition of planer curves. They compute the zeroes of curvature by convolving the path length with the first and second derivatives of the Gaussian. They used the coastline of Africa as an example.

In this paper, the problem is approached differently. The basic objective is to smooth the line and at the same time retain the basic shape of the digitized curve, whether closed or open.

Unlike Asada and Brady (1986) and Mokhtarian and Mackworth (1986), we convolve the signal only with the second derivative of the Gaussian. We do not use any curvature primitives. The signal (Schenk, Personel communication, 1986) is derived from the freeman chain code instead of the path length. In the next few sections, it is explained exactly how the convolution is performed, what signal is used, and how the post processing is carried out.

## Chain Encoding

Chain coding is a data structure for storing digital curve data in computer memory. Chain codes were devised by Freeman (1974, 1978); hence, it is also called Freeman chain code. Usually the coordinates of the first point are given and the subsequent entries are numbers giving the direction and magnitude between each point in the outline and its neighbor. There are eight possible directions between a pixel and its neighbor. As shown in Figure 1, these numbers are from 0 to 7 counter clockwise (Wilf, 1981). As noted in Thapa (1987), the chain code may not be used as a signal for the convolution because of the presence of discontinuities in it. Therefore, the discontinuities in the chain code must be removed in order to get useful signals for the convolution.

The discontinuities in the chain code may be alleviated by using the following equation (Pavlidis, 1978; Eccles *et al.*, 1977):

$$s(i) = (e(i) - e(i-1) + 11) \bmod 8 - 3. \quad (1)$$

Where $e(i)$ ($i = 1,2, \ldots ,N$) are the Freeman chain codes and $s(i)$ ($i = 1,2, \ldots ,N$) are called curvature elements. Eccles *et al.* (1977) found that $s(i)$ closely approximates the curvature of a digitized line.

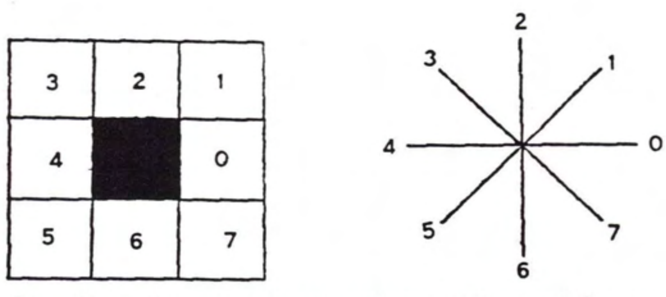


FIG. 1. Simple Freeman chain-code and pixel neighborhood. The dark square at the center shows the pixel in which one is interested. The pixel has 8 neighbors numbered from 0 to 7 Wilf (1987).

### Second Derivative of the Gaussian

The Gaussian function is given by

$$f(t, \sigma) = 1/(\sqrt{2\pi}\,\sigma) * \exp(-t^2/(2\,\sigma^2)) \quad (2)$$

and the second derivative is equal to

$$f''(t, \sigma) = df'(t, \sigma)/dt = C * (\sigma^2 - t^2) * \exp(-t^2/(2 * \sigma^2)) \quad (3)$$

where $C$ is a constant. The value of $C$ is equated to unity because, if one takes the actual value of C, there will be no zero-crossings at higher values of sigma.

### Convolution

Mathematically the convolution may be expressed as:

$$x_n = \Sigma\, s_k * h_{n-k} \quad (4)$$

The above formula applies only to the discrete case. For the continuous case, one has to use the integral form of the convolution. For example, the Gaussian kernel may be convolved with the function $f(x)$ as follows:

$$F(x, \sigma) = f(x) * g(x, \sigma) \quad (5)$$

$$F(x, \sigma) = 1/(\sqrt{2\pi}\sigma) \int f(u) * \exp(-(x-u)^2/(2\,\sigma^2))\, du \quad (6)$$

where '*' is the convolution operator.

### Post Processing Using a Dynamic Mask

The zero-crossings of the convoluted values of the signal (derived from the chain codes of the digitized curve) with the second derivative of the Gaussian include some points which are not critical. One needs to remove these points if one is interested in only the minimum number of critical points which are sufficient to represent the curve. Some researchers, e.g., Marimont (1984), have suggested that dynamic programming should be used to remove these points. But dynamic programming involves many iterations to achieve an optimum solution. Consequently, it is very expensive. Therefore, a dynamic mask is passed through the zero-crossings so that only the critical points are retained from among the zero-crossings.

Note that the dynamic mask used in this section is not the same as the one used in the Gaussian convolution. In this case, the dynamic mask is nothing but a rectangle of width equal to twice the specified tolerance. The length of the mask, as explained in Lozover and Preiss (1983), depends on the complexcity of the line. The concept of passing a dynamic mask in order to throw away the unwanted data was discussed in Lozover and Preiss (1983) and Imai and Iri (1986).

### Step-by-Step Procedures for Critical Points Detection Using Zero-Crossings

The process of finding zero-crossings described above may be outlined as follows:

- If the data are in vector format, transform them to raster form by Bresenham's algorithm (Foley and Van Dam, 1985).
- Compute the chain code and then the curvature from the rasterized data.
- Compute the mask (masks) from the second derivative of the Gaussian for a particular (different values) of sigma using Equation 3.
- Convolve the curvature (the signal) with the mask.
- Look for the zero-crossings in the convoluted values and save them.
- Pass the dynamic mask through the zero-crossings to filter the noise if desired. This step may not be required for larger values of sigma and if the presence of a few extra points is not harmful.

### Results of Zero-Crossings on a Test Figure

The test figure given in Figure 2 was designed to test the applicability of the zero-crossings algorithm to automatic line generalization. Figures 2 and 4 illustrate the fact that as the scale of the Gaussian is increased the number of zero-crossings decreases and the line is further smoothed. In Figure 2 sigma was equal to 6, and it was equal to 12 in Figure 4. Figures 3 and 5 are the straight line connections of the zero-crossings of Figures 2 and 4, respectively. These figures show that smoother results are obtained at higher values of sigma.

### Selection of Lines for Testing and Evaluation

In order to test the usefulness of line generalization using the zero-crossings of the convoluted values of the second derivative

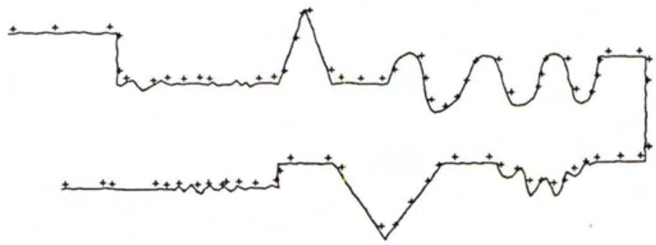

Fig. 2. Zero-crossings of the curvature signal. For sigma = 6 with no thresholding. Note that the zero-crossings indicated by '+' have an offset. (Seventy percent of original size.)

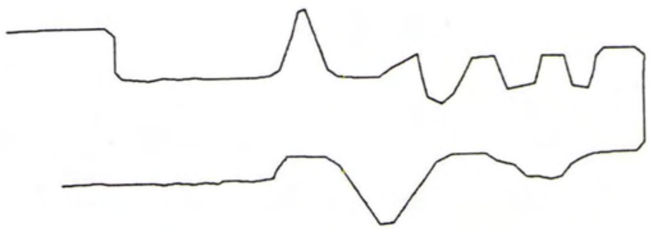

Fig. 4. Zero-crossings for curvature signal with sigma = 12. (Seventy percent of original size.)

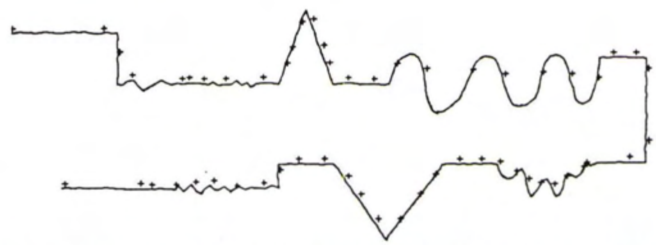

Fig. 3. Connection of the zero-crossings of Figure 2 by straight lines. For sigma = 6 with no thresholding. (Seventy percent of original size.)

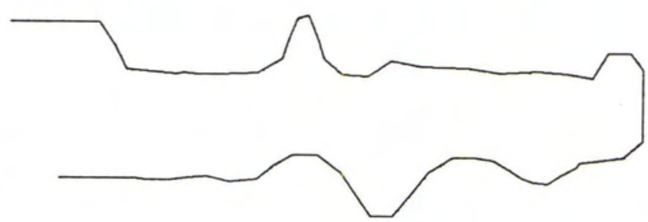

Fig. 5. Connection of the zero-crossings of Figure 4 by straight lines. For sigma = 12 with no thresholding. (Seventy percent of original size.)

of the Gaussian with the curvature signal derived from the chain codes, we need to use test lines from real maps.

For this purpose, USGS maps of Maryland at scales of 1:24,000, 1:100,000, and 1:250,000 were used. Unfortunately, 1:50,000 scale map of the same area was not available. From these maps, Hunting Creek in the river Miles was selected. It is a fairly complex line which has many complex features which can be useful to test the applicability of zero-crossings algorithm for the purpose of line generalization.

The selected feature was very carefully digitized using a Bendix digitizer at all the above scales. There were 620, 253, and 124 digitized points at scales 1:24,000, 1:100,000, and 1:250,000, respectively. Figure 6 shows the selected feature at various scales. These are the lines as manually generalized by cartographers and then digitized for this research.

Another lineal feature, called Greenwood Creek, was also selected from the same map series at the same scales as above. It was also digitized at the various scales. This is a very complicated feature to generalize. In addition, it also includes sections of the line which are very smooth (see Figure 13).

## Inapplicability of Douglas-Peucker Algorithm to Line Generalization

The most popular method of line generalization hitherto used in cartography has been the one described by Douglas and Peucker (1973), or as explained in Duda and Hart (1973), or as explained in Ramer (1972). This method of line generalization is good only for data compression and it can be used for line generalization if the change in the scale between the original line and the generalized line is not very much. For example, from a scale of 1:500 to 1:750. However, if the change in the scale between the original map and the generalized map is drastic, for example, from a scale of 1:24,000 to 1:100,000, the above algorithm may not be used because in such cases it will leave spikes, and the lines look cluttered and aesthetically unpleasing as is clear from Figure 7(A) and Figure 9(A).

The Above fact may be demonstrated using the selected line. Figure 7(A) shows Hunting Creek generalized to scale 1:100,000 from scale 1:24,000 using the Douglas-Peucker algorithm. The tolerance used for this case was 0.127 cm. and there were 88 points selected. The nature of this algorithm dictates that more points be retained if the tolerance is decreased. Figure 7(B) gives the corresponding lines generalized by the cartographers; there are 253 digitized points in this figure. It is clear from the figure that the line generalized by the Douglas-Peucker algorithm looks cluttered and is not anywhere close to the one generalized by the cartographer. This difference, especially around points A and B, is very obvious. At these points, the Douglas-Peucker algorithm has left spikes which are not seen in any maps generalized by the cartographers. Moreover, the lines generalized by the Douglas-Peucker algorithm look clumsy and unaesthetic.

In order to show how the lines generalized by the Douglas-Peucker algorithm look when enlarged, the results of Figure 7(A) were enlarged by 4 times. The enlarged line is shown in Figure 8. In this figure we can see very clearly that the lines generalized by this algorithm do not look acceptable. The existence of various unpleasant spikes has become more pronounced in this figure.

Figure 9(A) shows the results of Hunting Creek generalized

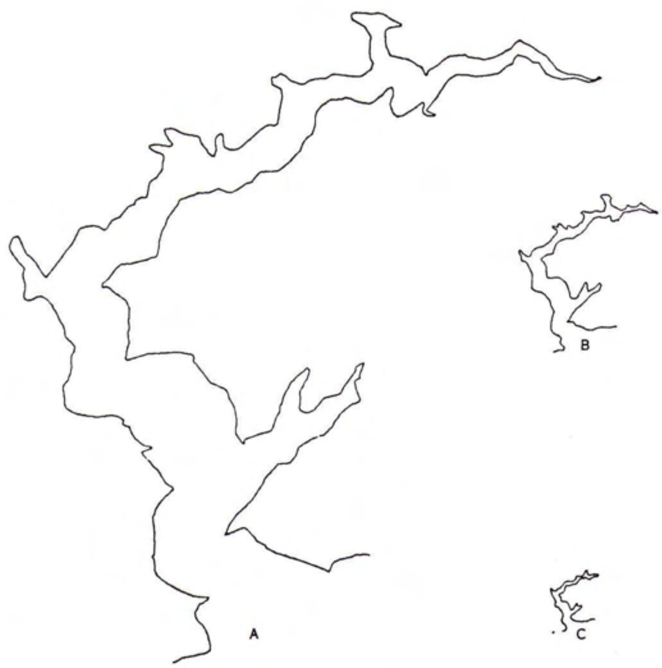


Fig. 6. Hunting Creek along River Miles at various scales. Selected to test the applicability of the zero-crossings algorithm. A, B, and C are at scales 1:24,000, 1:100,000, and 1:250,000, respectively (reduced to two-thirds of original scale; i.e., 1:36,000, 1:150,000, and 1:375,000, respectively).


Fig. 7. Comparison between lines generalized by cartographers and by the Douglas-Peucker algorithm at a scale of 1:100,000. A and B were generalized by the Douglas-Peucker algorithm and by a cartographer, respectively (reduced to two-thirds of original scale).

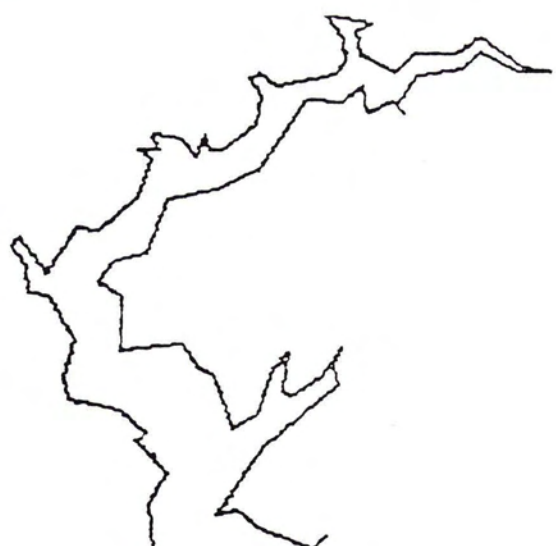

Fig. 8. Result of enlarging Figure 7(A) by four times (reduced to two-thirds of original scale).


FIG. 9. Comparison between lines generalized by cartographers and by the Douglas-Peucker algorithm at a scale of 1:250,000. A and B were generalized by the Douglas-Peucker algorithm and by a cartographer, respectively (reduced to two-thirds of original scale).


FIG. 10. Comparison between lines generalized by cartographers and by the zero-crossings algorithm at a scale of 1:100,000. A and B were generalized by the Zero-Crossings algorithm and by a cartographer, respectively (reduced to two-thirds or original scale).

from scale 1:24,000 to scale 1:250,000 by the Douglas-Peucker algorithm. In this case, the tolerance used is equal to 0.203 cm and there were 66 points selected. Figure 9(B) shows the corresponding lines generalized by the cartographers at the same scale; there are 124 digitized points in this figure. It is again noticed that the lines generalized by the Douglas-Peucker algorithm are not very close to those performed by the cartographers. This example further illustrates the inapplicability of the Douglas-Peucker algorithm to the problem of line generalization. However, as noted earlier, this algorithm may be used for data compaction and for line generalization if it involves only a minor reduction in scale.

### COMPARISON BETWEEN ALGORITHMIC AND MANUAL LINE GENERALIZATION

The zero-crossings of the convoluted values of the second derivative of the Gaussian with the curvature signal derived from the chain codes do not pick up the critical points but preserve the basic shape or the character of the curve. For this reason, this method may be used to generalize lines, especially when there is a drastic reduction in scale, e.g., from 1:24,000 to 1:100,000.

The results of a lines generalized from scale 1:24,000 to 1:100,000 are shown in Figure 10(A). In this figure curvature derived from the chain codes was the signal used and $\sigma = 4$. There were 136 zero-crossings retained. Figure 10(B) shows the corresponding lines generalized by the cartographers; there are 253 points in this figure.

By comparing Figures 10(A) and (B), it is noticed that there is a remarkable similarity between the lines generalized by the cartographer and the zero-crossings algorithm except that in some edges the cartographer has made some exaggerations of detail even when it was not required and in some instances the detail was omitted even though it could have been retained. Moreover, the zero-crossings algorithm used nearly 50 percent fewer points. Despite this fact, the results are pleasingly smooth.

Figure 11 is the enlargement of Figure 10(A) in order to show that the zero-crossings algorithm does not produce a cluttered figure and also leaves no spikes.

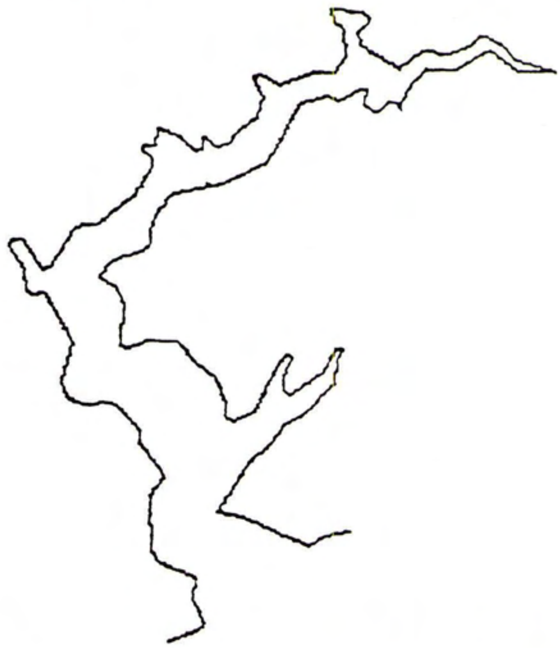

FIG. 11. Result of enlarging Figure 10(A) by four times (reduced to two-thirds of original scale).


FIG. 12. Comparison between lines generalized by cartographers and by the zero-crossings algorithm at a scale of 1:250,000. A and B were generalized by the zero-crossings algorithm and by a cartographer, respectively (reduced to two-thirds of original scale).

Figure 12(A) shows the generalization of Hunting Creek from the scale 1:24,000 to 1:250,000. In this figure curvatures was the signal used and $\sigma = 12$.

There were 79 zero-crossings retained in this figure. Figure 12(B) shows the corresponding lines generalized by a cartographer. There were 124 points digitized in this figure. Note that these numbers are mentioned to give an idea as to how many points the zero-crossings algorithm retains to get results similar those of cartographers. But it is known that the cartographers tend to digitize redundant points.

By comparing these two figures, it can again be claimed that it has been possible to mimic the cartographer even though the reduction in scale is drastic. One also notices that the cartographer was negligent and smoothed the line around point 'a' much more than he/she should have. Maybe the cartographer was not at ease at the time this line was generalized.

In order to prove the flexibility of the zero-crossings method of line generalization, Greenwood Creek was also generalized at the above scales. Figure 13 shows Greenwood Creek at various scales.

Figure 14(A) shows the results of Greenwood Creek generalized from 1:24,000 to 1:100,000. In this case, the curvature was the signal used, $\sigma = 4$, and there were 183 points retained. Figure 14(B) shows the corresponding lines generalized by cartographers in which there are 226 digitized points. If one examines these two figures, one notices especially around points E and F that


FIG. 13. Greenwood Creek along River Miles at various scales. Selected to test the applicability of the zero-crossings algorithm. A, B, and C are at scales 1;24,000, 1:100,000, and 1:250,000, respectively (reduced to two-thirds of original scale).

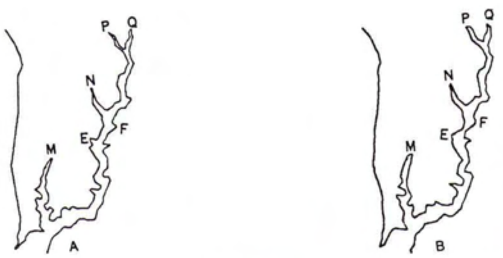


FIG. 14. Comparison between lines generalized by cartographers and by the Zero-Crossings algorithm at a scale of 1:100,000. A and B were generalized by the Zero-Crossings algorithm and by a cartographer, respectively; Greenwood Creek (reduced to two-thirds of original scale).

the cartographer has grossly distorted the character of the line. Detail which could have been easily depicted at this scale was smoothed out. Moreover, the tip ends at points M, N and P, and Q were grossly exaggerated.

Similarly, Figure 15(A) is the result of Greenwood Creek generalization from scale 1:24,000 to 1:250,000. In this case, $\sigma = 10$ and curvature was the signal used. There are 83 points retained in this figure. Figure 15(B) is the corresponding line generalized by a cartographer; there are 135 digitized points in this figure.


FIG. 15. Comparison between lines generalized by cartographers and by the zero-crossings algorithm at a scale of 1:250,000. A and B were generalized by the zero-crossings algorithm and by a cartographer, respectively; Grenwood Creek (reduced to two-thirds of original scale).

Here again the results are similar to those obtained by cartographers except for the tip ends. At Points X, Y, and Z the cartographer has shortened the tip ends and has also exaggerated the width of the tip ends. Other than that, the results obtained by zero-crossings algorithm are close to those obtained by cartographers.

## LINE GENERALIZATION: CRITICAL POINTS DETECTION VS SHAPE PRESERVATION

Many researchers in the field of cartography, such as Marino (1979), McMaster (1983), and White (1985), implied that the critical points are important and must be retained in the process of line generalization. Some of the researchers, e.g., Marino (1979), implied that the detection of critical points is one way of quantifying the subjective process of line generalization.

Now in the light of this research, it has emerged that one cannot preserve all the critical points in the process of line generalization. Some of the critical points which are likely to cause spikes in the generalized lines must be eliminated if the generalized lines are to be smooth, uncluttered, and aesthetically pleasing. On the other hand, as pointed out by many researchers such as Keates (1973), Campbell (1984), and Buttenfield (1985), we need to retain the basic shape of the line or the feature during the process of line generalization. Unfortunately, it was observed that the basic shape or the character of a line is not preserved by preserving all the critical points. This is especially true if the change in the scale is very large. This is clear from many of the examples used from the real map data; e.g., see Figures 10, 13 and 15.

It is not meant that the detection of critical points is useless and therefore should not be pursued. In fact, the detection of critical points is important and very useful for the purpose of data compression in cartography and in many other disciplines (Thapa, 1987). Moreover, critical points are useful in shape recognition if it does not involve a scale change. For this reason, the critical points detection is important in the fields of pattern recognition, image processing, computer vision, computer graphics, etc.

Critical points detection in line generalization is important if one considers it as a two-step process, namely: first, detect the critical points, and second, perform some kind of smoothing of the critical points. In such situations it is perfectly all right to detect the critical points. But it is not done that way in this research because the method of line generalization using the zero-crossings of the convoluted values of the second derivative of the Gaussian with the curvature signal derived from the chain code achieves both smoothing and shape preservation in one step.

It is shown in Thapa (1987) that the zero-crossings algorithm using different signals provides an excellent and reliable means of raster to vector conversion and it is also a very good method for critical points detection in digital curves.

## CONCLUSIONS

- Using the zero-crossings of the convoluted values of the second derivative of the Gaussian with the curvature signal derived from the digitized data, it was possible to automatically generalize lines even when there was a drastic change in scale.
- When the process of line generalization involves a drastic change in scale, e.g., 1:24,000 to 1:100,000 or smaller, what one needs to preserve are the basic shape and general character of a line and not the critical points. The latter conclusion is contrary to the findings and beliefs of some of the previous researchers.
- The superiority of the zero-crossings algorithm for automatic line generalization lies in the fact that this process is faster because it does not require the computation of any square roots nor does it involve any inversion or manipulation of matrices. Moreover, the zero-crossings algorithm for line generalization is better than and different from all the existing algorithms because the zero-crossings algorithm performs line smoothing and data compression at the same time. In addition, unlike the existing methods of line smoothing such as weighting, the line generalized by the zero-crossings algorithm passes through the existing points and does not compute new coordinates. This is exactly the reason why the zero-crossings algorithm gives results which are very close to those obtained by cartographers. In addition, this is an algorithm which can generalize lines in raster form.

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