Periodic Inspection of Industrial Tooling by Photogrammetry*

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ABSTRACT: Industrial photogrammetry is now an accepted method for periodic inspection of assembly tooling in aerospace manufacturing. Periodic inspection surveys for quality assurance constitute a deformation monitoring task which requires very high accuracies (1 part in 100,000 and better), and photogrammetry affords advantages in this area as well as in practicability and economy. This paper briefly overviews the application of photogrammetry to periodic inspection of assembly tooling in aircraft manufacturing and then reviews a deformation analysis strategy which emphasizes the localization of single point movements. Specific covariance matrix characteristics are exploited to arrive at an analysis scheme for tooling inspection which can serve as a simple alternative to conventional multi-epoch deformation analysis approaches.

INTRODUCTION

AT THE PRESENT TIME a dozen or so of the leading aerospace manufacturing companies in North America are employing photogrammetry as a measurement tool at various stages in the design and production cycle of aircraft and space vehicles. Recently reported applications include wind tunnel testing, full-scale aircraft inspection, structural calibration, tool and component measurement, and tooling inspection (Kilburg and Rathburn, 1984; Powell, 1984; Fraser and Brown, 1986; Mallison, 1987). It is the last of these areas which forms the topic of this paper.

Accompanying the stringent demands for quality and structural integrity imposed in the production stage of aerospace vehicle manufacture is the requirement to exhaustively inspect and verify manufacturing tooling, be it large component assembly jigs or smaller tool configurations. Quality assurance inspection of tooling fixtures entails the measurement of position and possibly orientation of devices called locators which are used to position the components being assembled in the tool. To better illustrate the concept of a jig, three are shown in Figures 1, 2 and 3. The first, Figure 1, is a 4.3 by 2.6-m tool which is used for wing assembly on the F16 fighter aircraft. The second, Figure 2, shows a "barrel" tool for the assembly of the nose cone on a Harrier II, whereas the third, Figure 3, is for the assembly of outer wing tip sections on an F/A 18 Hornet fighter. Figure 4 provides a good illustration of the concept of locator points and reference surfaces. Note in this figure and in Figure 3 the photogrammetric targets which serve as measurement points on the locators.

The intent of utilizing photogrammetry for periodic inspection of tooling is both to improve production by minimizing or eliminating tool downtime, and to greatly enhance cost effectiveness by reducing both the time the tool is out of production and the substantial costs associated with inspection by gauging. Gauging involves the use of a "perfect" part or master gauge which at the time a tool is inspected is brought in and positioned in the jig, thus allowing the dimensional stability of locators to be checked. With this contact measurement technique the periodic "recycling" of a large tool can take days and involve a number of people, even if the tool has no discrepancies. A further shortcoming of gauging is that all positional reference is with respect to the master gauge itself and not explicitly to the computer aided design (CAD) database which references locator positions in terms of the vehicle coordinate system. When coupled with the complementary measurement technique of multiple-head digital theodolite systems for locator realignment, photogrammetry offers a non-contact measurement tool

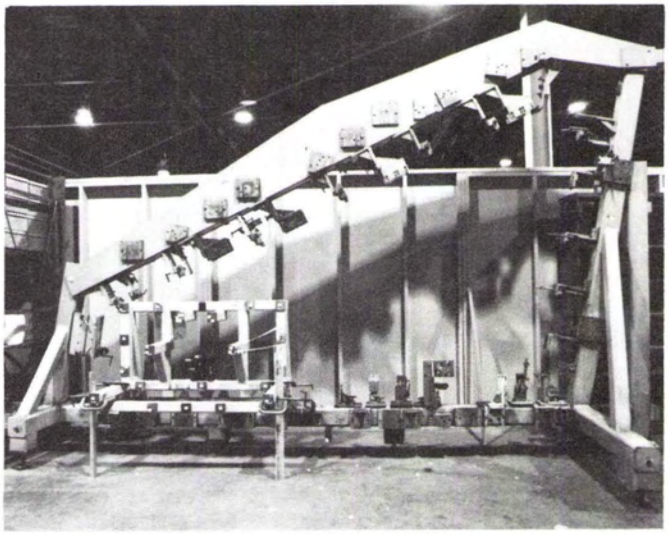

FIG. 1. Wing assembly jig for F16 aircraft (photo courtesy of General Dynamics Corporation).

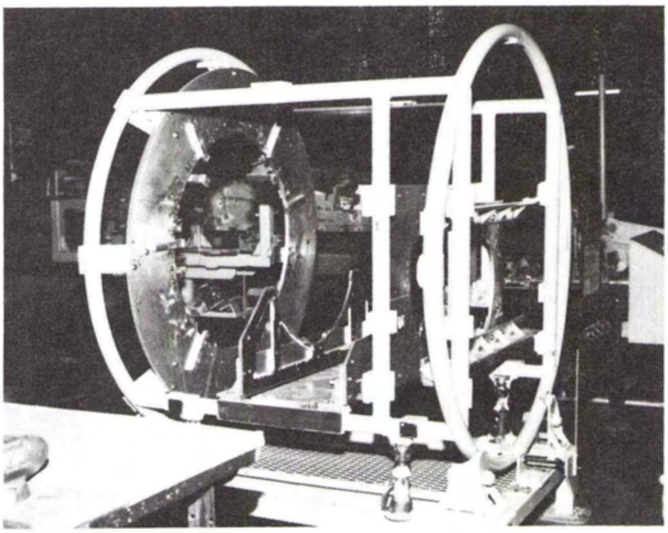

FIG. 2. Nose cone assembly tool for Harrier II aircraft (photo courtesy of McDonnell Douglas Corporation).

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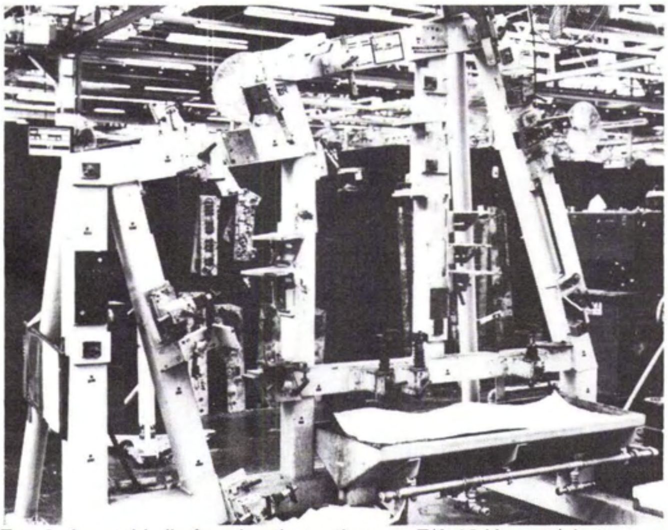

FIG. 3. Assembly jig for wing tip sections on F/A 18 Hornet (photo courtesy of McDonnell Douglas Corporation).

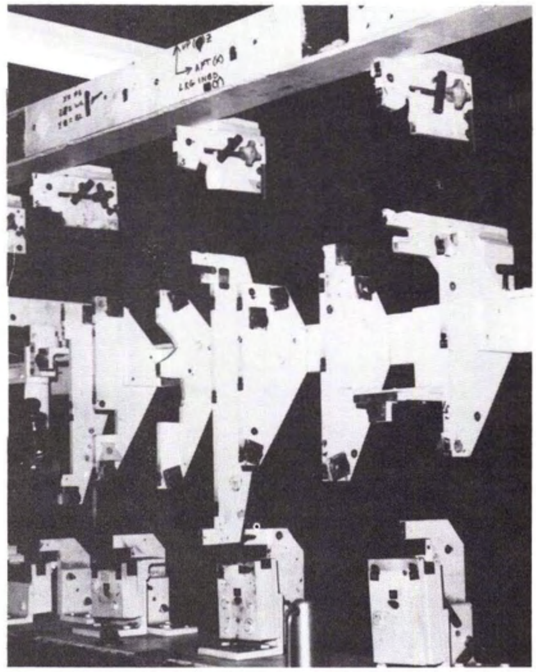

FIG. 4. Example of locator points and reference surfaces on a tool (photo courtesy of McDonnell Douglas Corporation).

which makes feasible gaugeless tooling, for both the production and periodic inspection of tooling (e.g., Kilburg and Rathburn, 1984).

Placed into an engineering surveying context, it can be seen that the periodic inspection of assembly tooling, be it in the aerospace, automobile, or other manufacturing industry, constitutes a multi-epoch, three-dimensional deformation monitoring survey. Hence, the question of deformation analysis must be addressed. In the aerospace industry proper analysis of locator stability is of considerable importance because of the stringent accuracy tolerances imposed. Engineering tolerances for locator position of 0.1mm or less are common for moderately large tools, and, consequently, measurement accuracies exceeding 1 part in 100,000 of the jig's largest dimension are routinely called for. When one or more locators are found to have moved in excess of their positional tolerances, a tool "realignment" is required, thus necessitating removal of the jig from the production line. It is therefore very important to ensure that measured locator displacements do in fact represent real tool deformation and not the results of random or systematic measurement errors. The cost implications of a wrong decision in this area are significant, especially in cases where a master guage is brought in for the realignment. Consequently, an appropriate deformation analysis strategy is warranted in most circumstances where the standard error of measurement is greater in magnitude than say one tenth the engineering tolerances imposed.

In the following section a commonly used strategy for the localization of point displacements in a multi-epoch deformation monitoring survey is reviewed, and features pertinent to tooling inspection by photogrammetry are highlighted. One significant feature of well designed multi-station close-range photogrammetric networks is that they yield near orthogonal covariance matrices for the object point coordinates when adjusted by means of a free-network approach. This characteristic impacts on deformation measurement networks as it offers the possibility of simplifying the analysis procedure, as discussed later. The effectiveness of such a simplification for application to tooling inspection is then investigated for the case of the wing assembly jig shown in Figure 1.

DEFORMATION ANALYSIS

ANALYSIS APPROACH

Deformation monitoring networks are characterized as being either absolute or relative in nature. In an absolute or reference network, concern centers on the stability of a given set of reference points and the localization of point movements with respect to these reference points. In a relative network all points are assumed to lie on a deformable body and are all thus subject to movement when the object changes in shape or scale. In periodic inspection applications, the monitoring networks are conventionally of absolute type, though relative networks are often encountered, for example, when monitoring bond molds. Irrespective of type, the aim of photogrammetric monitoring networks for tooling inspection is a localization in space and time of locator movements which exceed engineering tolerances. With this in mind, the following general deformation analysis procedure can be adopted for any two measuring epochs:

- Determine which points form a stable reference point array,
- Transform the coordinate system of each of the networks being compared into a common datum defined by the stable points,
- Determine individual displacements of tooling locators within the reference system and identify those exceeding applicable tolerances, and
- Verify that the point movements constitute a statistically significant deformation.

The procedure is followed in essentially the order given in the case of absolute networks. Here, the stability of the reference array is tested, following which the analysis simply involves significance testing of normalized point movements; i.e., one tests the null hypothesis that the locators did not move.

Although the analysis of relative networks implicitly includes the four phases listed above, the adopted procedure does not explicitly proceed in the order given, but rather comprises a repeated procedure of congruency testing and localization of single-point displacements. This approach is more general in nature and is also applicable to reference networks. Thus, it alone will be described in the following sections.

THE GLOBAL CONGRUENCY TEST

The analysis procedure commences with the global congruency test, which examines the null hypothesis that the network of object points is stable over all measuring epochs. For any two epochs i and i+1 the hypothesis can be formulated as:

$$H_0 : \mathrm{E}\,\{\mathbf{x}_{i+1} - \mathbf{x}_i\} = 0 \text{ against } H_1 : \mathrm{E}\,\{\mathbf{x}_{i+1} - \mathbf{x}_i\} \neq 0 \quad (1)$$

where $\mathbf{x}_i$ is the vector of XYZ point coordinates at epoch i. A standard F-test, in which the quadratic form of the deformations is compared to the *a posteriori* variance factor, determines whether the null hypothesis, H_o, will be rejected in favor of the alternative, H_1, (e.g. Pelzer, 1971; Niemeier, 1981).

Implementation of the congruency test requires that all coordinates, x_i, and their corresponding covariance matrices, $\mathbf{Q}_x^{(i)}$, must have the same datum. It is, therefore, possible that a datum transformation may be necessary at the outset to bring the two networks into the same computational base, which is formed by a suitable subset of points common to each epoch. To transform $\mathbf{x}$ and $\mathbf{Q}_x$ to the required datum, Baarda's S-transformations can be employed: i.e.,

$$\mathbf{x}_s = \mathbf{S}\ \mathbf{x} \quad (2)$$

and

$$\mathbf{Q}_{xs} = \mathbf{S}\ \mathbf{Q}_x\ \mathbf{S}^{\mathrm{T}}. \quad (3)$$

For a comprehensive account of S-transformations and their role in deformation analysis, the reader is referred to van Mierlo (1981).

For the congruency test the quadratic form of the point displacements is determined and the value of a test statistic ω computed:

$$\omega = \frac{\Omega}{h\ \sigma_0^2} = \frac{\mathbf{d}^{\mathrm{T}}\ \mathbf{Q}_d^+\ \mathbf{d}}{h\ \sigma_0^2} \quad (4)$$

and

where σ_0^2 is the common variance factor, $\mathbf{d} = (\mathbf{x}_{i+1} - \mathbf{x}_i)$, $\mathbf{Q}_d = (\mathbf{Q}_x^{(i+1)} + \mathbf{Q}_x^{(i)})$, and h is the rank of $\mathbf{Q}_d$. With r the degrees of freedom, the test of ω is against $F_{h,r,1-\alpha}$, generally at a significance level of $\alpha = 0.05$. If the test passes, the null hypothesis that the network has not undergone a change in shape is accepted. If, on the other hand, the test fails, it is due to the presence of statistically significant point displacements. The next step is to locate these individual movements of locators on the tool.

Localization of Deformations

The quadratic form Ω provides a measure of the non-congruence of the two networks. Thus, it is possible to examine the contribution Ω_j of each point displacement $\mathbf{d}_j = (dX, dY, dZ)_j^{\mathrm{T}}$ to this value. The point with the maximum value is deemed to be a significant deformation. Individual Ω_j values are computed using the following conformal partitioning procedure (Niemeier, 1979; van Mierlo, 1981):

$$\Omega_j = \bar{\mathbf{d}}_j^{\mathrm{T}}\ \mathbf{P}_j\ \bar{\mathbf{d}}_j \quad (5)$$

$$\text{where } \bar{\mathbf{d}}_j = \mathbf{P}_j^{-1}\ \mathbf{P}_{rj}\ \mathbf{d}_r + \mathbf{d}_j \quad (6)$$

with $\mathbf{d}$ and $\mathbf{Q}_d^+$ partitioned as

$$\mathbf{d} = \begin{pmatrix} \mathbf{d}_r \\ \mathbf{d}_j \end{pmatrix} \text{ and } \mathbf{Q}_d^+ = \begin{pmatrix} \mathbf{P}_r & \mathbf{P}_{rj} \\ \mathbf{P}_{rj}^{\mathrm{T}} & \mathbf{P}_j \end{pmatrix} \quad (7)$$

Following the location of the point displacement which contributes most significantly to the non-congruence of the networks, it must be determined if that point is the only one to have moved. To answer this question, the congruency test must be repeated, but this time the influence of the first-located movement, $\mathbf{d}_j$, is eliminated from the common network datum. Hence, the congruency test is no longer global but rather restricted to a part of the network. This requires a partitioning of both the vector $\mathbf{d}$ and the covariance matrix $\mathbf{Q}_d$ as follows:

$$\mathbf{d} = \begin{pmatrix} \mathbf{d}_r \\ \mathbf{d}_j \end{pmatrix} \text{ and } \mathbf{Q}_d = \begin{pmatrix} \mathbf{Q}_r & \mathbf{Q}_{re} \\ \mathbf{Q}_{re}^{\mathrm{T}} & \mathbf{Q}_e \end{pmatrix} \quad (8)$$

where $\mathbf{d}_e$ is the vector of "eliminated" points formed by the individual deformations, $\mathbf{d}_j$, located in Equation 5. To remove $\mathbf{d}_e$ from the computational base, an S-transformation can be used. Alternatively, a computationally more straightforward method involving a sequence of matrix inversion operations (Gruendig *et al.*, 1985a) can be applied, or the networks could conceivably be re-adjusted using a free-network approach where inner constraints are imposed only on the retained points forming $\mathbf{d}_r$, but this approach is generally not practical.

To apply the S-transformation, the S-matrix is formed as

$$\mathbf{S} = \begin{pmatrix} \mathbf{I} - \mathbf{G}_r\ \mathbf{R} & 0 \\ -\ \mathbf{G}_e\ \mathbf{R} & \mathbf{I} \end{pmatrix} \quad (9)$$

where $\mathbf{G}_e$ and $\mathbf{G}_r$ are similarity transformation matrices corresponding to $\mathbf{x}_e$ and $\mathbf{x}_r$, respectively, and $\mathbf{R} = (\mathbf{G}_r^{\mathrm{T}}\ \mathbf{G}_r)^{-1}\ \mathbf{G}_r^{\mathrm{T}}$. The G-matrices are those which are routinely applied to impose datum constraints in free-network adjustment by means of inner constraints (e.g., van Mierlo, 1981; Fraser and Gruendig, 1985). The transformed deformation vector (now with a datum free from the influence of the eliminated points forming $\mathbf{d}_e$) and the corresponding covariance matrix follow from

$$\begin{pmatrix} \mathbf{d}_{sr} \\ \mathbf{d}_{se} \end{pmatrix} = \mathbf{S} \begin{pmatrix} \mathbf{d}_r \\ \mathbf{d}_e \end{pmatrix} \quad (10)$$

and

$$\mathbf{Q}_{ds} = \mathbf{S}\ \mathbf{Q}_d\ \mathbf{S}^{\mathrm{T}} \quad (11)$$

At this point the congruency test is again performed to examine whether the vector $\mathbf{d}_{sr}$ contains any further significant deformations.

If there are k eliminated points in $\mathbf{d}_{se}$, the congruency test for the partial network formed by the retained object points is performed as

$$\frac{\mathbf{d}_{sr}^{\mathrm{T}}\ \mathbf{P}_{sr}\ \mathbf{d}_{sr}}{(h - 3k)\ \sigma_0^2} < F_{h-3k,r,1-\alpha} \quad (12)$$

where $\mathbf{P}_{sr}$ is the sub-matrix of the pseudo inverse $\mathbf{Q}_{ds}^+$ corresponding to $\mathbf{d}_{sr}$. The null hypothesis that the partial network has not changed in shape is accepted if the test, Equation 12, passes. Thus, the elements of $\mathbf{d}_{se}$ indicate the significant deformations with respect to the stable target point network described by $\mathbf{d}_{sr}$. If the congruency test fails, the point with the largest Ω_j component is again determined by means of Equation 5. The three corresponding coordinate differences dX, dY, and dZ are then added to the $\mathbf{d}_e$ vector, k is incremented by one, and both the S-transformation and the partial network test are repeated. At the completion of the deformation localization phase, $\mathbf{d}_{se}$ represents the vector of detectable deformations.

Upon completion of this localization procedure, it is possible to apply a further test which is aimed at confirming the findings of the congruency testing. Such a confirmation is valuable because during the localization the existence of a movement of a group of points may affect the sequence of elimination of single points. In the presence of a group movement, it is conceivable that a stable point may be flagged as having undergone significant movement, because the hypothesis tested deals only with single-point displacements. Considering the single-point case, the formulated null hypothesis to be tested is that $\mathbf{d}_j = 0$. This hypothesis is then examined by the test

$$T_j = \frac{\mathbf{d}_j^{\mathrm{T}}\ \mathbf{Q}_{dj}^{-1}\ \mathbf{d}_j}{\sigma_0^2} < 3F_{3,r,1-\alpha} \quad (13)$$

where $\mathbf{Q}_{dj}$ is the 3 by 3 cofactor matrix of the displacement vector $\mathbf{d}_j$. Because this single-point t-test does not account for point-to-point correlation information, it is conceivable that deformations located in the congruency testing phase will pass the t-test as non-significant point movements. Experience suggests that for well-designed networks this generally occurs only when the deformation is marginally significant, i.e., when it just passes congruency testing and just fails t-testing. It is recommended that the t-test always be applied because it is more sensitive to single-point movements than the congruency test.

The localization process outlined is embodied in some commercially available software packages for deformation analysis. One such program is LOKAL which was written by Gruendig and colleagues at the University of Stuttgart (Gruendig et al., 1985b). This program is mentioned because it has been utilized for the deformation analysis example discussed later.

As presented above, the deformation localization procedure comprises an examination of two epochs of data. Thus, the location in time problem is trivial; deformations are deemed to have occurred at the latest epoch examined. If the number of epochs of data exceeds two, a more comprehensive method can be applied to isolate the time at which particular deformations took place (Niemeier, 1981). The two-epoch approach to localization in space can also be expanded by means of the combined adjustment of networks when multiple epochs of data are available. For the majority of tooling inspection applications, however, the two-epoch approach presented is adequate. The aim at any inspection period is to determine whether the locator movements with respect to the base epoch exceed engineering tolerances. The deformation patterns at successive epochs do give valuable information on the positional behavior of locators, but they are of no consequence in the production cycle until movement of a locator from its base position is such that realignment is warranted.

CHARACTERISTICS OF PHOTOGRAMMETRIC NETWORKS

Apart from practical advantages in the area of accuracy and economy, photogrammetry also exhibits network characteristics which greatly assist in the localization of single-point movements in a deformation analysis. Foremost among these features is the fact that "strong" photogrammetric networks (those exhibiting high internal reliability) generally exhibit limited point-to-point correlation and reasonably homogeneous point-to-point precision when subjected to free-network adjustment. The presence of a near-orthogonal covariance structure enhances the deformation testing procedure, especially in regard to the sensitivity of the local t-testing phase. Moreover, it offers the possibility of simplifying the localization procedure reviewed in the previous section. This possibility will shortly be examined.

In a photogrammetric bundle adjustment using the method of inner constraints, the covariance matrix of parameters is obtained from the normal equations as

$$Q_x = \begin{pmatrix} Q_1 & Q_{12} & \cdot \\ Q_{21} & Q_2 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} N_1 & \bar{N} & 0 \\ \bar{N}^T & N_2 & G \\ 0 & G^T & 0 \end{pmatrix}^{-1} \quad (14)$$

where $\mathbf{N}_1$ corresponds to the exterior orientation and self-calibration parameters and $\mathbf{N}_2$ to the object point coordinates. The datum constraint matrix $\mathbf{G}$ is the same as that used to construct the S-transformation (Equation 9). Of the covariance matrix $\mathbf{Q}_x$, only the portion $\mathbf{Q}_2$ corresponding to the object point coordinates is of importance for deformation analysis.

As has been shown in Fraser (1987), $\mathbf{Q}_2$ invariably displays a near-orthogonal covariance structure. Thus, this matrix, the computation of which is not practical for large networks, can often be represented as a block-diagonal matrix comprising 3 by 3 covariance matrices $\mathbf{Q}_{2j}$ for the X, Y, and Z coordinates of each point j: i.e.,

$$\mathbf{Q}_{2j} = \mathbf{N}_{2j}^{-1} + \mathbf{N}_{2j}^{-1} (\bar{\mathbf{N}}^T \mathbf{G})_j \, \mathring{\mathbf{Q}}_1 \begin{pmatrix} \bar{\mathbf{N}} \\ \mathbf{G}^T \end{pmatrix}_j \mathbf{N}_{2j}^{-1} \quad (15)$$

where $\mathring{\mathbf{Q}}_1$ includes seven extra rows and columns corresponding to the Lagrangian multipliers of the inner constraint solution. The simplification process for $\mathbf{Q}_2$ can be continued by noting that for most well-designed networks $\mathbf{Q}_{2j} \approx \mathbf{N}_{2j}^{-1}$, as shown in Fraser (1987). For this discussion, however, only $\mathbf{Q}_2$ from Equation 15 will be considered.

A SIMPLIFIED LOCALIZATION APPROACH

In the presence of an orthogonal point-to-point covariance structure in the photogrammetric networks for the two epochs being examined, the localization procedure is greatly simplified. Once the two networks are brought into the same datum (nominally using the inner constraint adjustment for each epoch), the localization proceeds first with computation of the global congruency test statistic, which reduces to

$$\omega = \frac{\Omega}{h\sigma_0^2} = \frac{1}{h\sigma_0^2} \sum_j \mathbf{d}_j^T \mathbf{Q}_{dj}^{-1} \mathbf{d}_j = \frac{1}{h} \sum_j T_j. \quad (16)$$

If the congruency test fails, the largest Ω component is sought. But, because $\mathbf{Q}_d$ is block-diagonal the largest Ω component is simply the largest T_j value. This point can then be eliminated and the datum transformed accordingly. Here, too, the computational procedure is more straightforward: i.e.,

$$\begin{pmatrix} \mathbf{d}_{sr} \\ \mathbf{d}_{se} \end{pmatrix} = \mathbf{S} \begin{pmatrix} \mathbf{d}_r \\ \mathbf{d}_e \end{pmatrix} \quad (17)$$

and

$$\mathbf{Q}_{ds} \approx \mathbf{Q}_d. \quad (18)$$

The transformation, Equation 17, need not explicitly be carried out by means of an S-transformation. For example, a similarity transformation could be used to transform the coordinates $\mathbf{x}_{i+1}$ into the $\mathbf{x}_i$ datum, using only those points in $\mathbf{d}_r$ as common to both networks. The vectors, $\mathbf{d}_{sr}$ and $\mathbf{d}_{se}$, are then given as the differences between the $\mathbf{x}_i$ and transformed $\mathbf{x}_{i+1}$ coordinates. This approach is computationally very straightforward. For a strong photogrammetric network comprising at least many tens of points, it is the author's experience that $\mathbf{Q}_2$ is largely invariant with small changes in the datum equations in an inner constraint adjustment (recall also that $\mathbf{Q}_2 \approx \mathbf{N}_2^{-1}$). Thus, as displaced points are eliminated in the localization procedure, the $\mathbf{Q}_{ds}$ values of remaining points remain essentially unchanged, at least for cases where there are more stable than displaced points.

In the simplified localization approach the confirming t-test for single point movements is carried out implicitly with the congruency test because the final Ω_j values, when divided by the variance factor, yield the final T_j values.

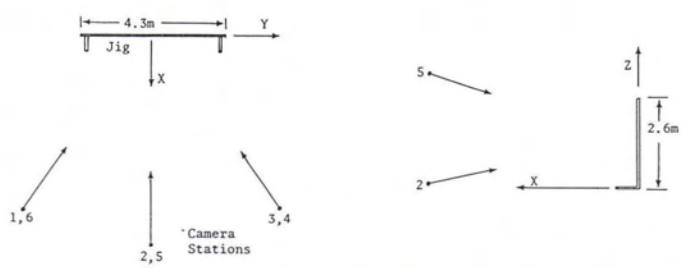

FIG. 5. Geometry of 6-photo network for the wing jig (Figure 1).

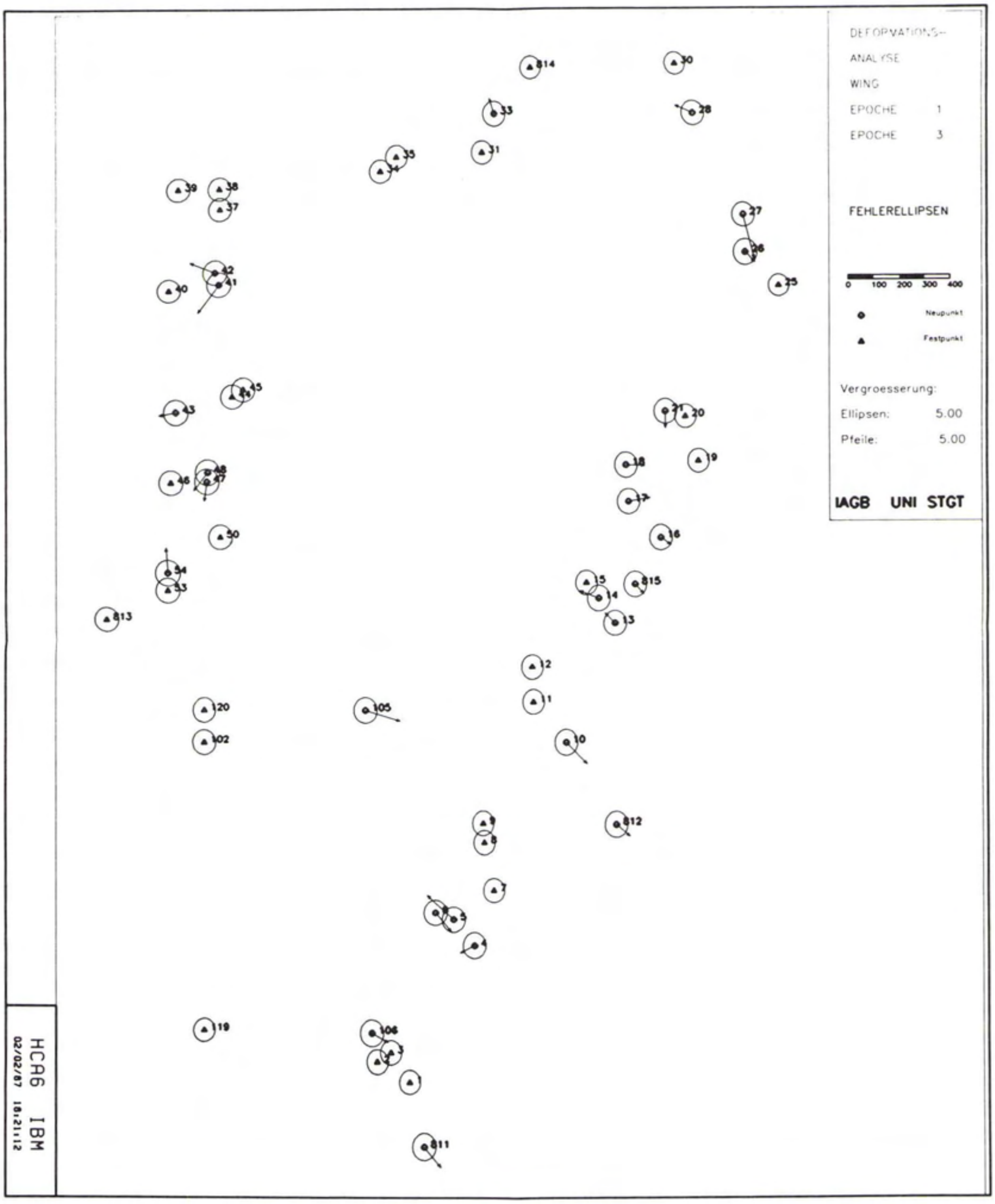

FIG. 6. Results of deformation analysis of Epoch 3 versus Epoch 1. Displacement vectors indicate significant point movements in *XY*. Stable points are indicated by triangles. Note near circularity of point error ellipses.

Apart from the fact that it can yield some insight into the mechanics of the localization procedure, i.e., the analysis comprises the successive testing of normalized point movements (displacement divided by its standard error) until no shape change is detectable in the network, the simplified approach would be largely an academic exercise were it not for the structure of $\mathbf{Q}_2$ in photogrammetric networks. For periodic inspection measurements of tooling, the simplified localization approach thus has the potential of offering a practical means for the determination of locator movements.

A PRACTICAL APPLICATION

As an example of a periodic inspection measurement undertaken by photogrammetry, the jig shown in Figure 1 is considered. Geodetic Services, Inc. had previously carried out a two-epoch deformation measurement of this tool as part of a "benchmark" test in which locator movements were measured to an accuracy of better than 0.03 mm or about 1 part in 180,000 (Fraser and Brown, 1986). In that test, however, all locator displacements were relatively large (0.4 mm or greater) and therefore easily locatable. To facilitate a more demanding test for both the rigorous and simplified localization procedures, three epochs of photogrammetric data were simulated, using the same basic geometry as in the previously conducted measurements.

At each epoch a six-photo convergent imaging geometry was adopted for the network design, as shown in Figure 5, and on the tool some 55 locators were positioned. For the second epoch some 20 locators were displaced in various directions, the magnitudes of the movement being from 2 $\bar{\sigma}_c$ to 10 $\bar{\sigma}_c$, where $\bar{\sigma}_c$ is the mean positional standard error of the point. The majority were in the range of 3 $\bar{\sigma}_c$ to 5 $\bar{\sigma}_c$. At the third epoch 26 locators were moved, the movements being of similar magnitude to those for Epoch 2. Included in this second set of simulated deformations were 16 points which had undergone a movement at the second epoch. Superimposed on the introduced locator movements was the random error component which resulted from the photogrammetric bundle adjustment at each epoch. Photographic image coordinates were randomly perturbed according to a standard error of 2 micrometres.

The aim of the test was to examine the effectiveness of the simplified deformation analysis approach as compared to the more rigorous single-point localization procedure. Because introduced point movements were in places as small as 2 $\bar{\sigma}_c$, it was not anticipated that all simulated deformations would be located. From the data, however, it was hoped that some insight could nevertheless be gained into the sensitivity of the two deformation analysis procedures. As can be seen from Equation 13, maximum t-test sensitivity could be expected for networks with a homogeneous and isotropic covariance matrix Q_2. In such cases, $\sigma_X = \sigma_Y = \sigma_Z = \bar{\sigma}_c$, and Q_2 is diagonal. Thus, displacement vectors $\mathbf{d}_j$ with magnitudes exceeding 2.8 $\bar{\sigma}_c$ would constitute detectable deformations.

In the photogrammetric networks at each epoch, a near homogeneous and isotropic distribution of precision was achieved in the *XY* plane, as shown, for example, by the error ellipses in Figure 6. Standard errors σ_Z, while varying little from point-to-point, were about 40 percent higher than those in *XY* due to the moderate convergence angles employed in the six-photo imaging geometry. Mean square error estimates for the *XY* plane and Z coordinate were close to 0.024 mm and 0.034 mm, respectively, thus leading to a mean positional standard error $\bar{\sigma}_c$ of 0.028 mm for the 55 locators. Overall, point-to-point and coordinate-to-coordinate correlation was very low with there being less than 0.2 percent of correlation coefficient values in Q_2 exceeding 0.15; none exceeded 0.35. Such a covariance structure augurs well for the success of the simplified deformation analysis approach.

For the three epochs of data, deformation analyses were carried out for Epoch 2 versus 1, 3 versus 1, and 3 versus 2. The program LOKAL was used for the rigorous localization procedure, the computations being performed at the Institute of Applied Geodesy, University of Stuttgart. The results of the LOKAL runs were then compared to the findings of the simplified approach. In the analysis of Epoch 2 versus 1, LOKAL yielded 20 detectable locator displacements whereas the simplified method yielded the same 20 significant movements plus one extra, it being a marginally significant displacement of magnitude 3.6 $\bar{\sigma}_c$. The next largest and undetectable locator displacement was found to correspond to 3.3 σ_c. For the case of Epoch 3 versus 1, LOKAL yielded 25 significant point movements while 22 were found with the simplified approach. Once again, the three displacements identified in the one method but not the other were only marginally detectable, their magnitudes ranging from 3.5 $\bar{\sigma}_c$ to 3.8 $\bar{\sigma}_c$. For Epoch 3 versus 2, similar results were found. LOKAL yielded 15 significant locator displacements while the alternative method produced 14 of these. The movement which was not flagged as significant in the simplified approach had a magnitude of 3.6 $\bar{\sigma}_c$.

In summarizing the results of the two deformation analysis approaches, it can be stated that the simplified method by and large yielded the same results as LOKAL. All clearly significant locator movements were identified by each method. As regards the marginally detectable deformations, it is less than clear which conclusion should be accepted. The rigor of the congruency testing procedure in LOKAL has to be weighed against the sensitivity of the final t-tests in the approximate method. In a well designed deformation measurement network for tool inspection, such questions can be avoided by ensuring that the minimum detectable locator displacement is well below the threshold magnitude that would indicate the necessity for locator realignment. In the application discussed, all locator movements of greater than 4 σ_c were detectable by both methods at the 95 percent confidence level. For any network, the distribution of precision impacts on the sensitivity of the deformation analysis and must be accounted for in the network design.

CONCLUDING REMARKS

As photogrammetry becomes more widely employed for the periodic inspection of tooling fixtures, so more interest will be generated in the problem of how to adequately locate and quantify locator displacements which exceed engineering tolerances. Due to concerns regarding potential interruptions in the production cycle, decisions as to whether or not realignment is warranted need to be based on a proper analysis of the measured locator movements. The deformation analysis procedure outlined earlier is well suited to tooling inspection as it emphasizes the localization of single-point movements. Moreover, with the success of the simplified localization strategy in the wing jig example and other applications, it is appealing to propose this method for wider usage. Such a recommendation can, however, only be made with the proviso that the networks of object points must have covariance matrices of *XYZ* coordinates which exhibit a near-orthogonal structure. Fortunately, such a property is generally found in well-designed photogrammetric networks which are subjected to free-network adjustment.

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