Photogrammetric Work Without Blunders

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ABSTRACT: The method of least squares is unsuitable for dealing with blunders in measurements. Other so called robust computation methods are explained and applied to photogrammetric problems (relative and absolute orientation) enabling the automatic detection and elimination of blunders. It is shown that these robust methods considerably improve the photogrammetric mapping process.

REVIEW OF ROBUST ESTIMATION METHODS

ALTERNATIVE METHODS to least-squares adjustment have been proposed in recent years (see, for example, Fredriksen *et al.* (1985) and Kubik *et al.* (1986)) which reduce the influence of blunders in the computation process. These are the so called robust estimation methods. Robust estimators are estimators which are relatively insensitive to limited variations in the frequency distribution function of the measurements and, thus, to the presence of outlyers and blunders. All these methods have in common: they do not minimize the (weighted) square sum of residuals, i.e.,

$$\sum_{i=1}^{n} r_i^2 \rightarrow \min; \; r_i \text{ residuals} \tag{1}$$

but, rather, a properly chosen function Φ of r_i which, for large values of r_i, increases less rapidly than r_i^2, i.e.,

$$\sum_{i=1}^{n} \Phi\,(r_i) \rightarrow \min. \tag{2}$$

Examples for the choice of ϕ are

$$\Phi(r_i) = |r_i| \text{ and}$$

$$\Phi(r_i) = r_i^2 \exp\,(-r_i^2/a^2).$$

The numerical solution of Equation 2 can be obtained by using the method of "reweighted least squares." Equation 2 can be written as

$$\Sigma p(r_i) \times r_i^2 \rightarrow \min$$

where the weight function $p(r_i)$ is given by

$$p(r_i) = \frac{\Phi(r_i)}{r_i^2} \tag{3}$$

The procedure is iterative, starting with the least-squares solution ($p = 1$).

In the second iteration the individual weights are calculated from Equation 3 using the residuals from the least-squares solution. In the third and following iterations, the residuals from the previous iteration are used for calculating new weights. This procedure is continued until convergence is obtained.

ROBUST ESTIMATION OF A MEAN VALUE

This section presents results from the application of three robust alternatives to the least-square method for estimating the mean value of a set of measurements. These examples serve to illustrate the flow of computation.

Let us assume the following sample of five values given: Z = (10, 11, 11, 12, 100) and σ = 5 units. The least-squares estimate for the mean is equal to m = 29 units.

LEAST-SUM ESTIMATION

In least-sum estimation, the weight function is chosen equal to $p_i = \dfrac{1}{|r_i|}$, thus minimizing $\Sigma|r_i| \rightarrow \min.$*). In Table 1, the iterative computation is shown. The second column of the table lists the weights P_i for the five measurements used for the weighted average computations. The last column lists the five residuals $|r_i|$ = abs (Z_i-m) for every iteration.

The proper solution m = 11 is obtained after ten iterations. This example illustrates the necessity to limit the absolute magnitude of P_i; otherwise, some measurements receive very large weights and the mean estimate becomes equal to these measurements.

HUBERS ESTIMATE

In Hubers estimate (Kubik *et al.*, 1986), the weight is limited in magnitude according to the following rule:

$$P = 1 \text{ for } |r| < a$$

$$P = \frac{a}{|r|} \text{ for } |r| \geq a$$

We use an iterative algorithm similar to the previous one and assume $a = 2.\sigma = 10$ units. Table 2 summarizes the results.

The computation converges after three iterations to the mean estimate, 13.5. This estimate is different from before, as we are minimizing a different function Equation 2.

TABLE 1. LEAST SUM ESTIMATION OF MEAN VALUE

Sample Values: 10 11 11 12 100

Iteration No.	Weights p for Samples 1	2	3	4	5	Weighted Mean M	Residuals for Sample 1	2	3	4	5
1	1.00	1.00	1.00	1.00	1.00	28.8	18.8	17.8	17.8	16.8	71.2
2	0.05	0.06	0.06	0.06	0.01	16.3	6.3	5.3	5.3	4.3	83.7
3	0.16	0.19	0.19	0.24	0.01	12.4	2.4	1.4	1.4	0.4	87.6
4	0.41	0.69	0.69	2.25	0.01	11.7	1.7	0.7	0.7	0.3	88.3
5	0.59	1.42	1.42	3.39	0.01	11.6	1.6	0.6	0.6	0.4	88.4
6	0.64	1.79	1.79	2.26	0.01	11.4	1.4	0.4	0.4	0.6	88.6
7	0.71	2.47	2.47	1.68	0.01	11.3	1.3	0.3	0.3	0.7	88.7
8	0.79	3.73	3.73	1.37	0.01	11.2	1.2	0.2	0.2	0.8	88.8
9	0.86	0.09	6.09	1.20	0.01	11.1	1.1	0.1	0.1	0.9	88.9
10	0.91	10.64	10.64	1.10	0.01	11.1	1.0	0.0	0.0	1.0	89.0
11	0.95	19.57	10.57	1.05	0.01	11.0	1.0	0.0	0.0	1.0	89.0
12	0.97	37.31	37.31	1.03	0.01	11.0	1.0	0.0	0.0	1.0	89.0

* This estimator is equal to the median, being the value corresponding to the 50 percent percentile in the cummulative frequency distribution. For our example, the median is equal to 11 units.

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TABLE 2. HUBER'S ESTIMATE FOR MEAN VALUE

Sample Values: 10 11 11 12 100											
Iteration	Weights p for Samples					Weighted Mean	Residuals for Sample				
No.	1	2	3	4	5	M	1	2	3	4	5
1	1.00	1.00	1.00	1.00	1.00	28.8	18.8	17.8	17.8	16.8	71.2
2	0.53	0.56	0.56	0.60	0.14	16.3	6.3	5.3	5.3	4.3	83.7
3	1.00	1.00	1.00	1.00	0.12	13.6	3.6	2.6	2.6	1.6	86.4
4	1.00	1.00	1.00	1.00	0.12	13.5	3.5	2.5	2.5	1.5	86.5

TABLE 3. THE DANISH METHOD

Sample Values: 10 11 11 12 100											
Iteration	Weights p for Samples					Weighted Mean	Residuals for Sample				
No.	1	2	3	4	5	M	1	2	3	4	5
1	1.00	1.00	1.00	1.00	1.00	28.8	18.8	17.8	17.8	16.8	71.2
2	0.03	0.04	0.04	0.06	0.00	11.2	1.2	0.2	0.2	0.8	88.8
3	1.00	1.00	1.00	1.00	0.00	11.0	1.0	0.0	0.0	1.0	89.0
4	1.00	1.00	1.00	1.00	0.00	11.0	1.0	0.0	0.0	1.0	89.0

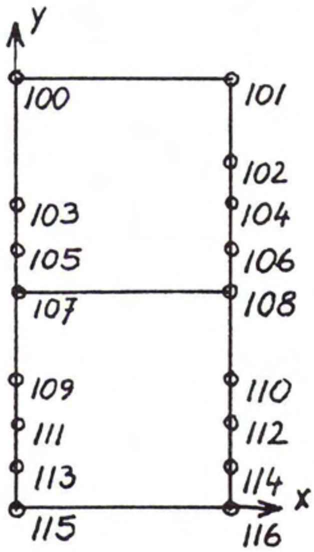

FIG. 1. Point Distribution for Relative Orientation, Photo No. 2

THE DANISH METHOD

In the Danish Method, (Kubik *et al.*, 1986) the weight of outlying measurements is reduced as the value of their residuals increases. The following weighting rule is used:

$$P = 1 \text{ for } |r| < a$$

$$P = \exp\left(-\frac{|r|^2}{a^2}\right) \text{ for } |r| \geq a$$

The results of the iterations are summarized in Table 3 (using $a = 2.\sigma$).

Because the Danish Method uses exponential weights, measurements outside a chosen confidence interval are completely rejected, thus resulting in a truncated sample and mean estimator. Convergence of the iterations is extremely fast, and that is why we favor the exponential weight method over other methods.

NUMERICAL RELATIVE ORIENTATION

Let us now consider the application of these robust methods to numerical relative orientation, based on the well known collinearity conditions. A simulated model with 17 points for relative orientation, as shown in Figure 1, is used for this purpose. At first, random errors of $\sigma = 3$ μm were superimposed on the ideal image coordinates and a least-squares adjustment was performed. The results are shown in Table 4, and the residual y parallaxes are consistent with the initially introduced perturbance. Next, the least-squares relative orientation is repeated, but now a blunder of 40 μm is introduced in point 100. The results of the computations are shown in Table 5; the largest real parallax appears at point 103, thus making it a non-trivial task to properly locate the blunder, even with so many relative orientation points. The computation is repeated but now using robust estimation according to the least sum principle, and results are presented in Table 6. The blunder can now be clearly recognized in point 100 (as 2×13.9 μm $\sim$ 28 μm).

Finally, the Danish Method is applied to this example. The results of Table 7 show that the total amount of the blunder is correctly found and eliminated. The magnitude of the estimated error is 2×20.5 μm $= 41$ μm, corresponding to the 40 μm originally introduced, demonstrating the superiority of this method. The measurements at point 100 were iteratively weight reduced and finally, after 2 iterations, were assigned weight θ.

TABLE 2. HUBER'S ESTIMATE FOR MEAN VALUE
TABLE 4 LEAST-SQUARES RELATIVE ORIENTATION — UNPERTURBED CASE

Point No.	Photo No.	X' (mm)	Y' (mm)	VX' (μm)	VY' (μm)
100	1	−100.0000	100.0000	0.0	0.2
	2	0.000	100.0000	−0.0	−0.2
101	1	0.0000	100.0000	0.0	0.6
	2	100.0000	99.9960	−0.0	−0.6
102	1	0.0000	60.0000	0.0	−1.1
	2	100.0000	60.0000	0.0	1.1
103	1	−100.0000	40.0000	0.0	−2.1
	2	0.0000	40.004	−0.0	2.1
104	1	0.0000	40.0000	0.0	1.1
	2	100.0000	39.9960	−0.0	−1.1
105	1	−100.0000	20.0000	0.0	1.3
	2	0.0000	19.9970	−0.0	−1.3
106	1	0.0000	20.0000	0.0	0.3
	2	100.0000	19.9980	−0.0	−0.3
107	1	−100.0000	0.0000	−0.0	−0.3
	2	0.0000	0.0000	0.0	0.3
108	1	0.0000	0.0001	0.0	−1.0
	2	100.0000	0.0010	−0.0	1.0
109	1	−100.0000	−40.0000	0.0	2.1
	2	0.0000	−40.0000	−0.0	−2.1
110	1	0.0000	−40.0000	0.0	0.9
	2	100.0000	−40.0000	−0.0	−0.9
111	1	−100.0000	−60.0000	0.0	1.0
	2	0.0000	−60.0000	−0.0	−1.0
112	1	0.0000	−60.0000	0.0	−2.4
	2	100.0000	−59.9950	−0.0	2.4
113	1	−100.0000	−80.0000	0.0	−1.6
	2	0.0000	−79.0000	−0.0	1.6
114	1	0.0000	−80.0000	0.0	0.7
	2	100.0000	−80.0000	−0.0	−0.7
115	1	−100.0000	−100.0000	−0.0	−0.6
	2	0.0000	−100.0000	−0.0	0.6
116	1	0.0000	−100.0000	0.0	0.9
	2	100.0000	−100.0000	−0.0	−0.9

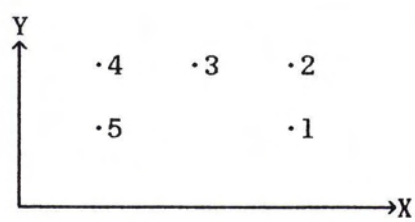

FIG. 2. Control Points Used for Numerical Absolute Orientation

TABLE 5 LEAST-SQUARES RELATIVE ORIENTATION — BLUNDERS IN POINT 100

Point No.	Photo No.	X (mm)	Y (mm)	VX (μm)	VY (μm)
100	2	−100.0000	99.9600	0.0	−5.6
	1	100.000	100.0000	−0.0	−5.6
101	2	100.0000	99.9960	−0.0	3.2
	1	0.0000	100.0000	0.0	−3.2
102	2	100.0000	60.0000	−0.0	1.0
	1	0.0000	60.0000	0.0	−1.0
103	2	0.0000	40.0000	−0.0	7.3
	1	−100.0000	40.0000	−0.0	−7.3
104	2	100.0000	39.9960	0.0	−2.4
	1	0.0000	40.0000	−0.0	2.4
105	2	− 0.0000	19.9970	−0.0	1.7
	1	−100.0000	20.0000	0.0	−1.7
106	2	100.0000	19.9980	0.0	−2.3
	1	0.0000	20.0000	−0.0	2.3
107	2	0.0000	0.0000	−0.0	1.5
	1	−100.0000	0.0000	0.0	1.5
108	2	100.0000	.0010	0.0	−1.3
	1	0.0000	0.0000	−0.0	1.3
109	2	0.0000	−40.0050	0.0	−3.0
	1	−100.0000	−40.0000	−0.0	3.0
110	2	100.0000	−40.0020	0.0	−2.4
	1	0.0000	−40.0000	−0.0	2.4
111	2	0.0000	−60.0000	0.0	−2.3
	1	−100.0000	−60.0000	−0.0	2.3
112	2	100.0000	−59.9950	−0.0	1.9
	1	0.0000	−60.0000	0.0	1.9
113	2	0.0000	−79.9980	0.0	0.4
	1	−100.0000	−80.0000	0.0	−0.4
114	2	100.0000	−80.0010	−0.0	0.2
	1	0.0000	−80.0000	0.0	−0.2
115	2	0.0000	−100.0000	0.0	−0.1
	1	−100.0000	−100.0000	−0.0	0.1
116	2	100.0000	−100.0000	0.0	2.0
	1	0.0000	−100.0000	0.0	−2.0

TABLE 6 RELATIVE ORIENTATION — LEAST SUM PRINCIPLE

Point No.	Photo No.	X˙ (mm)	Y˙ (mm)	VX˙ (μm)	VY˙ (μm)
100	1	−100.0000	100.0000	0.0	13.9
	2	0.0000	99.9600	−0.0	−13.9
101	1	0.0000	100.0000	0.0	−0.1
	2	100.0000	99.9960	−0.0	0.1
102	1	0.0000	60.0000	0.0	−0.6
	2	100.0000	60.0000	−0.0	0.6
103	1	−100.0000	40.0000	0.0	−4.5
	2	0.0000	40.0000	−0.0	4.5
104	1	0.0000	40.0000	−0.0	1.9
	2	100.0000	39.9960	0.0	−1.9
105	1	−100.0000	20.0000	0.0	−0.2
	2	0.0000	19.9970	−0.0	0.2
106	1	0.0000	20.0000	−0.0	1.2
	2	100.0000	19.9980	0.0	−1.3
107	1	−100.0000	0.0000	0.0	−1.0
	2	0.0000	0.0001	−0.0	1.0
108	1	0.0000	0.0001	0.0	−0.0
	2	100.0000	0.0010	−0.0	0.0
109	1	−100.0000	−40.0000	−0.0	2.4
	2	0.0000	−40.0000	0.0	−2.4
110	1	0.0000	−40.0000	−0.0	1.3
	2	100.0000	−40.0000	0.0	−1.3
111	1	−100.0000	−60.0000	−0.0	1.6
	2	0.0000	−60.0000	0.0	−1.6
112	1	0.0000	−60.0000	0.0	−2.5
	2	100.0000	−59.9950	−0.0	2.5
113	1	−100.0000	−80.0000	0.0	−0.9
	2	0.0000	−79.9980	−0.0	0.9
114	1	0.0000	−80.0000	0.0	0.0
	2	100.0000	−80.0000	0.0	−0.0
115	1	−100.0000	−100.0000	0.0	0.0
	2	0.0000	−100.0000	0.0	−0.0
116	1	0.0000	−100.0000	0.0	−0.6
	2	100.0000	−100.0000	−0.0	0.6

NUMERICAL ABSOLUTE ORIENTATION

As a final example, we illustrate the application of robust estimation to absolute orientation from an application in industrial photogrammetry. Consider five points to be given in the model and ground coordinate system as listed in Table 8 and shown in Figure 2. All magnitudes are transferred to mm at image scale.

In the X and Y coordinates of the last two points 9 and 11, blunders of 10 μm have been added to the model coordinates. Tables 9 and 10 summarize the results of absolute orientation according to the least-squares method and the Danish Method, respectively.

After least squares, the four blunders cannot be located in the residuals. There are large residuals at all points. The largest residual appears in point 1, and even advanced (Baarda) test theory does not allow a proper identification of the blunders. On the other hand, the Danish method clearly locates and eliminates the four blunders of 10 μm, thus preserving the high accuracy of the other measurements of this example.

TABLE 7 RELATIVE ORIENTATION — DANISH METHOD

Point No.	Photo No.	X˙ (mm)	Y˙ (mm)	VX˙ (μm)	VY˙ (μm)	WT.
100	2	0.0000	99.9600	−0.0	−20.5	0.0
	1	−100.0000	100.0000	0.0	20.5	0.0
101	2	100.0000	99.9960	−0.0	−0.7	
	1	0.0000	100.0000	0.0	−0.7	
102	2	100.0000	60.0000	0.0	1.1	
	1	0.0000	60.0000	0.0	−1.1	
103	2	0.0000	40.0000	−0.0	2.0	
	1	−100.0000	40.0000	0.0	−2.0	
104	2	100.0000	39.9960	−0.0	−1.1	
	1	0.0000	40.0000	0.0	1.1	
105	2	0.0000	19.9970	−0.0	−1.4	
	1	−100.0000	20.0000	0.0	1.4	
106	2	100.0000	19.9980	−0.0	−0.2	
	1	0.0000	20.0000	0.0	0.2	
107	2	0.0000	0.0000	0.0	0.2	
	1	−100.0000	0.0000	−0.0	−0.2	
108	2	100.0000	0.0010	−0.0	1.1	
	1	0.0000	0.0000	0.0	−1.1	
109	2	0.0000	−40.0050	−0.0	−2.1	
	1	−100.0000	−40.0000	0.0	2.1	
110	2	100.0000	−40.0020	−0.0	−0.8	
	1	0.0000	−40.0000	0.0	0.8	
111	2	0.0000	−60.0000	−0.0	−1.0	
	1	−100.0000	−60.0000	0.0	1.0	
112	2	100.0000	−39.9950	−0.0	2.5	
	1	0.0000	−60.0000	0.0	−2.5	
113	2	0.0000	−79.9980	−0.0	1.6	
	1	−100.0000	−80.0000	0.0	−1.6	
114	2	100.0000	−80.0010	−0.0	−0.8	
	1	0.0000	−80.0000	0.0	0.8	
115	2	0.0000	−100.0000	−0.0	0.6	
	1	−100.0000	−100.0000	−0.0	−0.6	
116	2	100.0000	−100.0000	−0.0	−1.0	
	1	0.0000	−100.0000	0.0	1.0	

TABLE 8. INPUT DATA FOR ABSOLUTE ORIENTATION (IN UNITS OF MM AT PHOTOSCALE)

	Model Coordinates			Ground Coordinate		
Point	X	Y	Z	X	Y	Z
1	30.523	280.731	27.717	30.519	280.731	27.715
2	46.89	280.307	133.076	46.892	280.307	133.078
3	49.144	148.555	133.134	49.143	148.55	133.13
4	46.91	52.798	133.44	46.919	52.8	133.437
5	89.98	52.175	57.744	89.689	52.176	57.745

TABLE 9. RESIDUALS AFTER LEAST-SQUARES ABSOLUTE ORIENTATION

Point	dx	dy	dz
1	0.007	−0.000	0.001
2	0.001	−0.000	−0.003
3	0.004	0.005	0.003
4	−0.006	−0.002	0.002
5	−0.006	−0.001	−0.002

TABLE 10. RESIDUALS AFTER THE DANISH METHOD

Point	dx	dy	dz
1	0.002	−0.000	0.001
2	−0.003	−0.000	−0.002
3	0.000	0.001	0.004
4	−0.010	0.009	0.003
5	−0.009	0.008	−0.003

CONCLUSIONS

These few examples demonstrate how important it is in practical photogrammetry to use robust adjustment methods in simple operations like relative and absolute orientation, as well as in block adjustment. The reader should be aware that today many blunders go unnoticed, because the least-squares methods distribute them evenly over all measurements. Therefore, we strongly recommend that robust adjustment methods be used as a replacement for or error check of the conventional least-squares method. In particular, the Danish Method, developed by Krarup (personal communication) and further refined by the authors, proves to be a very effective tool for modern photogrammetric computations.

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