Considerations in the Implementation of Aerotriangulation with GPS Derived Exposure Station Positions

Raymond J. Hintz and *Mike Z. Zhao*
Department of Surveying Engineering, University of Maine, Orono, ME 04469

ABSTRACT: Kinematic positioning using the Global Positioning Systems (GPS) will be fully possible when the full constellation of GPS satellites is operational. This ability will allow estimation of positions of exposure stations in photogrammetric mapping and densification applications. Simulations by a variety of authors have shown very encouraging trends in aerotriangulation with exposure station positional information derived from GPS estimates, including adjustment of photo blocks without any ground control. This paper will outline strategies for the following practical issues:

- It will be shown how one can obtain reliable results in a single strip of photos, where the linear nature of the measured exposure stations creates problems relating to geometric strength of the photogrammetric network.
- A simple procedure which enables complete use of orthometric heights will be demonstrated, because GPS derived heights are ellipsoidal in nature and thus need to be converted to their orthometric equivalents.
- It will be illustrated that incomplete determination of exposure station positions caused by cycle slips or equipment malfunction will not prevent the aerotriangulation from producing meaningful results.
- An algorithm will be presented which uses the bundle adjustment to iteratively transfer position from the GPS antenna to the exposure station without a *a priori* knowledge of the exposure station's orientation angles.

INTRODUCTION

One of the most important applications of GPS kinematic positioning will be in the determination of exposure station locations in aerial photographic missions. This procedure will result in the capability to perform aerotriangulation without ground control. Various simulations on the potential impacts of the proposed technology have been conducted, both in mapping applications [Schwarz *et al.*, 1984] and in high-accuracy densification of ground control [Lucas, 1987]. A recent study [Mader, 1986] has illustrated that centimetre accuracy may be obtained in kinematic GPS positioning. Other recent simulation studies have covered more aspects of the new procedure in aerotriangulation applications, such as the combination of measured exposure stations and ground control points [Hintz *et al.*, 1988a, 1988b].

Computer simulation has been considered very important before undertaking any new project involving photogrammetric triangulation, as it enables one to predict accuracies from any postulated configuration, and moreover, "to avoid hidden disasters lurking below the surface of things" [Brown, 1980]. The proposed technique involves the use of multiple delicate instruments, requires complicated initial preparation, and heavily depends on weather conditions. These situations compound time constraints due to the limited GPS "window." It is therefore imperative to simulate various situations that are most likely to occur in applications. To serve this purpose, a series of computer programs have been developed at the University of Maine which conduct both simulation and actual computation of aerotriangulation with GPS derived exposure station locations. Detailed descriptions of these programs' features were presented by the authors [Hintz *et al.*, 1988a, 1988b].

At least two tests of the new technology have been recently conducted [Lucas *et al.*, 1988; van der Vegt, 1988]. In both tests conventional aerotriangulation was used to determine exposure positions, which were then related to corresponding GPS antenna positions. Both tests resulted in better than 0.1-m accuracy for GPS positions obtained using differential or combined kinematic GPS positioning techniques. These tests provide valuable stimulis for further research.

This paper will concentrate on some practical issues in the implementation of this new technology, and illustrate possible solutions to problems which can arise.

OBTAINING RELIABLE RESULTS FROM A SINGLE STRIP OF PHOTOS

Single strips of photos may be required due to a linear geometry of the area under study, for example, in coastal areas and in highway construction. In control densification for mapping, the constraint of a linear geometry of photos generally requires a substantial ground control network around the perimeter of the project area. Likewise, the linear nature of measured exposure stations with no ground control would produce poor aerotriangulation results due to geometry constraints. Reliable results can be obtained with the combination of a limited number of ground control points and measured exposure stations, and it may be applied to high precision densification if an appropriate configuration of ground control points is available.

Table 1 illustrates results drawn from simulation of aerotriangulation with a 7 by 7 block using the conventional method, and 7- and 10-photo strips using the proposed technique. Standard error estimates used in the simulation were 0.05 m for each ground control coordinate, 0.1 m for each exposure station coordinate, and 0.003 mm for each photo coordinate. The simulated flying height and focal length were 1520 m (5000 ft) and 152.00 mm (6 in), respectively. Sixty percent endlap and sidelap

TABLE 1. ACCURACY COMPARISON: STRIP VERSUS CONVENTIONAL BLOCK.

Block/ Strip Size	Num. of Ground Ctr. Pts.	Exposure Station Locations	Degrees of Freedom	Average Standard Error of Ground Points (m)		
				X	*Y*	*Z*
7 × 7	5	Unknown	330	0.048	0.053	0.088
1 × 7	5	Measured	45	0.042	0.053	0.078
1 × 10	5	Measured	63	0.046	0.058	0.082

PHOTOGRAMMETRIC ENGINEERING AND REMOTE SENSING,
Vol. 55, No. 12, December 1989, pp. 1731-1735.

0099-1112/89/5512–1731$02.25/0

were used, with the photo x direction assumed to be parallel with the ground X coordinate axis. It can be seen from Table 1 that aerotriangulation with the 7-photo strip using the new method results in somewhat better accuracy than the 7 by 7 block conventional aerotriangulation. The 10-photo strip results are comparable to the 7 by 7 block.

As long as any number of ground control points is available for a single strip of photos, aerotriangulation with the combination of exposure station locations and ground control points will produce meaninful results. It is important to consider the situations where less ground control per strip of photos is available. Simulations of aerotriangulation for strips of 7, 10, 20, and 30 photos, combined with three three-dimensional ground control points, is illustrated in Figure 1.

It can been seen from Figure 1 that reliable results can be obtained for a strip of photos if a minimal ground control network exists. Figure 1 also shows that as the number of photos grows the accuracy in the Y direction eventually becomes worse than that in the Z direction. The accuracies in the Y and Z directions are similar for strips of 30 photos.

Aerotriangulation using this combination technique could result in significant advantages. The technique could save field work time, decrease ground control identification problems in aerotriangulation data collection, and increase densification accuracy. A comparison of simulation results for varying control configurations for a strip of 30 photos is presented in Table 2.

Simulations of a strip of seven photos with various configurations of ground control and measured exposure station locations are presented in Figure 2. This figure shows that the reduction of planimetric ground control does not increase the standard errors in the X and Z dimensions significantly. In the Y dimension, however, the standard error increases dramatically as planimetric ground control is reduced.

To clarify the influence of planimetric ground control on a single stirp of photos, results from simulations utilizing different numbers of planimetric ground control points are presented in Figure 3. The figure illustrates that, as the number of ground control points is reduced, the accuracies in the X and Z dimensions decrease in nearly a linear fashion. The decrease of accuracy in the Y dimension has a very different pattern. There is a notable increase in the standard error in the Y dimension (S_y) when the number of ground control points is reduced by one from an odd number to an even number. There is little change in S_y when an even number of ground control points is reduced by one. This phenomenon is attributable to control point geometry. It indicates that increasing an odd number of ground control by one results in increased control point redundancy without any significant increase in accuracy.

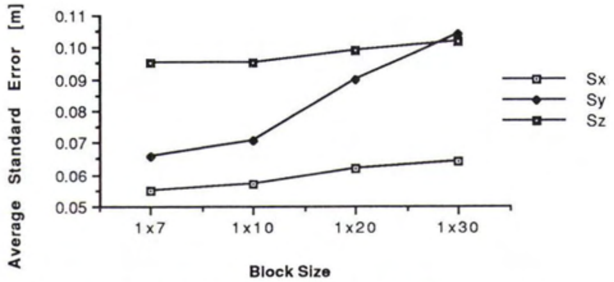

FIG. 1. Stripe size versus accuracy with three-dimensional ground control points and measured exposure station locations.

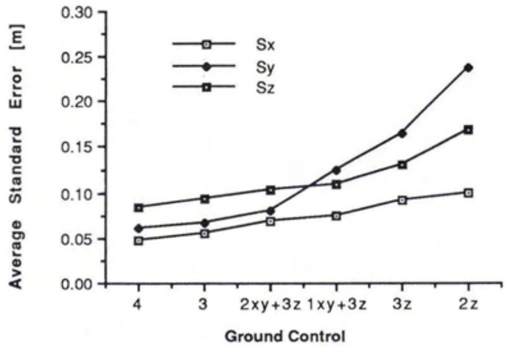

FIG. 2. Seven-photo strip with measured exposure stations and differing ground control parameters.

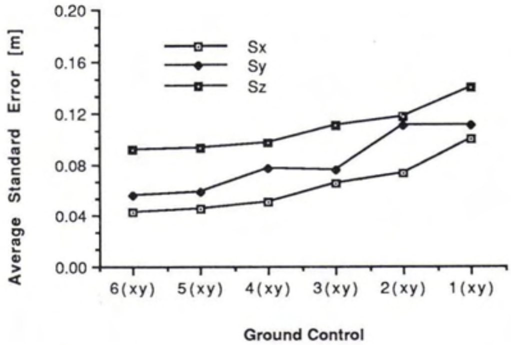

FIG. 3. Seven-photo strip with measured exposure stations and horizontal ground control parameters.

TABLE 2. ACCURACY OF AEROTRIANGULATION VERSUS CONTROL CONFIGURATION FOR A STRIP OF 30 PHOTOS.

Num. of Ground Ctr. Pts.	Exposure Station Locations	Degrees of Freedom	Average Standard Error of Ground Points (m)		
			X	Y	Z
3	Measured	177	0.064	0.104	0.102
4	Unknown	90	0.381	0.352	1.362
6	Unknown	96	0.150	0.154	0.432
8	Unknown	102	0.122	0.127	0.254
16	Unknown	126	0.065	0.078	0.152

From Table 1 and Figures 1, 2, and 3 the following conclusions can be drawn regarding aerotriangulation of a single strip of photos:

- If more than two horizontal ground control points are available, their inclusion with measured exposure locations will generate results with high accuracy in all three coordinates.
- With a configuration of less than two horizontal ground control stations, a significant decrease in accuracy occurs in the direction perpendicular to the flight line. Increasing the number of vertical control points does not resolve this problem.
- Additional horizontal ground control points efficiently improve the densification accuracies in all three coordinates, particularly in the dimension perpendicular to the flight line.
- In general, aerotriangulation of single strips involving both GPS measured exposure station locations and a suitable ground control network will produce accuracies similar to those resulting from blocks of photos.

ELEVATION ISSUE

The elevation information generated by GPS is an ellipsoidal height, and it must be corrected by the geoidal height to obtain orthometric height [Hoar, 1982]. Orthometric height is the elevation reference generally used in mapping applications. Because the knowledge of geoidal height may not be precise or available due to the limited knowledge of the gravity field in a region, an alternative method of determining orthometric heights must be implemented in situations where reliable geoidal height is not available.

In most regions in the United States there are generally more "benchmarks" available than planimetric control points. Shorelines and certain other natural features often have an elevation associated with them, and thus serve as a possible vertical control reference. It is usually easier to establish a vertical control network (differential leveling) as opposed to a horizontal control network. Users of the new technology will have the alternative of using elevation control points on the ground, combined with GPS derived planimetric positions "in the air."

Table 3 lists simulations which compare this approach to that of conventional aerotriangulation. From the table it is seen that the alternative method results in comparable accuracy to the conventional method.

Results of this simulation are very meaningful because they present a practical solution to the problem of converting GPS derived elevations to their orthometric equivalents.

INCOMPLETE DETERMINATION OF EXPOSURE STATION LOCATIONS

It is possible in typical operations thay only some of the exposure station locations could be measured. This problem could arise because GPS "cycle slips" are a bigger problem in kinematic GPS than in static positioning. It is fortunate that the problem should not occur as often because the receiver on the airplane should allow unobstructed satellite visibility. An exception is when the aircraft banks, and a portion of the aircraft disrupts the satellite visibility.

A cycle slip happens when a satellite signal is obstructed in any way. When signal lock is reacquired, the fractional part of the measured phase would still be the same, as if signal tracking has been maintained, but there actually is a discontinuity (cycle slip) in the measured phase. This is because a new initial integer must be used to count phase rotations. A technique has to be developed to precisely relate the phase data, after the loss of lock, to that before the cycle slip. This could be accomplished by adding an appropriate integer to all the measured phases after the cycle slip.

The accuracy of GPS positioning is heavily based on the editing of cycle slips. Various techniques on editing cycle slips have been developed [Mader, 1986; Wells, 1987; Bastos and Landau, 1988]. A linear extrapolation of the phase behavior before the cycle slip's occurrence can be used after the recovery of a cycle slip. An accuracy of a few cycles (1 cycle = 20 cm) in determining the integer ambiguity can be obtained using this method. When dual frequency phase observations are available, a parameter called the ionospheric residual can be used in detecting cycle slips. This parameter is more sensitive to cycle slips and may be computed at a noise level of a small fraction of a cycle [Mader, 1986]. This technique enables more accurate estimates of the integer ambiguities for both phases.

Table 3. Aerotriangulation with Combined GPS Derived Planimetric Exposure Station Location and Ground Elevation Control versus Conventional Procedures (7 by 7 Block).

Ground Control	Exposure Station Locations	Average Standard Error of Ground Points (m)		
		X	Y	Z
Five 3-D Control Points	Unknown Station Locations	0.048	0.053	0.088
5 Vertical Control Points	Measured Planimetric Locations	0.053	0.051	0.079

A major concern is not only the decrease of positioning accuracy caused by cycle slips, but also the problem of missing exposure station locations when a cycle slip, or another reason for loss of signal lock, occurs during a photographic mission. In kinematic positioning, discrete exposure station positions that fall in the time period after the cycle slip happens, and before it recovers, cannot be measured. Other situations, such as a limited satellite window period and recording instrument malfunctions, can arise. Then prevents GPS positioning for all exposure stations. It is therefore necessary to simulate situations where the exposure station locations in a block are only partially determined. In this paper the following three special situations have been simulated:

- all exposure station locations are measured except those on a strip of photos in the middle of a block,
- exposure station locations are known only around the perimeter of a block, and
- exposure station locaitons are known only in the first and last strips.

All of the situations could be realistic if alternatives to existing flight patterns for photographic missions are considered. End strips of photos in a block could be photographed first, instead of the conventional sequential approach.

The average precisions of densified ground points in these situations are listed in Table 4. These data illustrate only a slight decrease in the average precisions of the ground points. The worst accuracy, as would be expected, results when exposure locations are known only on the first and last strips. However, the difference in accuracy between this situation, and that of knowing all exposure station locations (first line, Table 4), is less than 3.4 cm. This indicates that, if some exposure station locations are not determined, a decrease in accuracy is often not significant, and the aerotriangulation can still be accomplished.

In the test presented by Lucas and Mader (1988), some GPS data were lost due to malfunctioning of the recording mechanism. The lost antenna positions were interpolated from other GPS data points. Because the missing antenna positions fell within the area where the GPS determined camera positions were judged questionable, it is plausible that the discontinuity of the camera positions could be attributable to deficiencies in the data. However, other missing observations in the same test do not appear to cause any problem. The simulations illustrated in this paper support these results.

TRANSFORMATION BETWEEN ANTENNA LOCATIONS AND EXPOSURE STATIONS

The results of kinematic GPS are a series of instantaneous antenna positions, while the presented aerotriangulation simulations were based on measured exposure station locations. A methodology must be developed to relate the two positions.

Table 4. Aerotriangulation with Incomplete Determination of Exposure Station Locations (7 by 7 Block).

Exposure Station Locations (measured)	Average Standard Error (m)		
	X	Y	Z
All	0.045	0.049	0.062
All except one strip	0.047	0.051	0.064
Perimeter	0.056	0.061	0.080
The first & last strip	0.066	0.080	0.096

Several authors have mentioned this issue without resolving the situation completely [Lucas, 1987; van der Vegt, 1988]. In the investigation conducted by Lucas (1987), the accuracy comparison was based on the difference of adjustment results of conventional aerotriangulation using ground control points, and aerotriangulation with simulated antenna positions computed from Equation 1, respectively:

$$\begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} = \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix} + \mathbf{M}^{\mathrm{T}} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \tag{1}$$

where Y_C, X_C, and Z_C are exposure station coordinates; X_A, Y_A, and Z_A are GPS antenna coordinates; and ΔX, ΔY, and ΔZ are the antenna-camera offsets in three-dimensional coordinate shifts. **M** is the transformation orientation matrix derived from the three angular exterior orientation components of an exposure station. It was indicated that the offsets can be estimated from preflight measurements, but that the three orientation angles are difficult to resolve prior to aerotriangulation. Lucas (1987) also indicated that pre-bundle adjustment estimates of the orientation angles within 1.5 degrees of their actual values is sufficient if no more than 2 metres separates the camera and antenna. This suggests that reliable estimates of the three angles can generally satisfy the tranformation.

The independent model adjustment was used in van der Vegt's (1988) investigation. The three angular elements for each exposure were obtained as a result of this process. It was indicated that variations in these orientation elements proved to be very small within a given strip. For this reason, the angular orientation components were computed for the first photo in each strip, and used in the triangulation for all other photos within that strip [van der Vegt, 1988].

An algorithm has been developed that requires none of the forementioned assumptions in performing the transformation from antenna to exposure station for a bundle block adjustment. A computer program which successfully implements the algorithm within the bundle adjustment has also been developed and tested statistically [Hintz *et al.*, 1988b]

Rearranging Equation 1, which performs the transformation from exposure station to GPS antenna, the transformation from GPS antenna to exposure station can be written as:

$$\begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} - \mathbf{M}^{\mathrm{T}} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \tag{2}$$

The algorithm, illustrated in Figure 4, for the transformation from antenna to camera is based on two properties of aerotriangulation geometry. The first property is that initial estimates of zero for omega and phi are valid in conventional aerotriangulation, and kappa can be readily approximated by knowing the direction of the flight line relative to the axes of the ground coordinate system. The flight line direction is easily computable from GPS antenna positions. The second requirement of the algorithm is that the receiver antenna can be set up at a consistent location relative to the camera each time the photographic collection mission is being performed.

The transformation to the exposure station from the antenna is then performed using Equation 2. The bundle adjustment is then performed, specifically for determination of new estimates of the angular orientation parameters of the exposure stations.

These angles are used in the standard rotation matrix to determine new estimates of the X, Y, Z components of each antenna-camera offset. New estimates of the positions of the camera exposure stations can thus be determined. If these do not differ appreciably from the previous initial estimates (a fractional per-

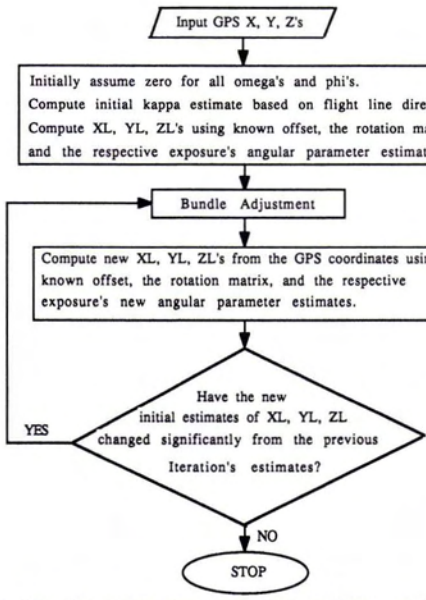

FIG. 4. Algorithm for conversion from GPS antenna positions to the exposure station positions.

cent of the standard error estimate could be used as a maximum), the process would be terminated. If not, a new bundle adjustment would occur, with the number of iterations minimized by using updated approximations from the previous solution. This process is successful in conventional aerotriangulation situations because a small error in a camera position will not create a large error in the exterior orientation angles.

A statistical test was carried out to evaluate the transformation using the algorithm. The test procedure steps were

(1) Simulated photo and ground data were created using a computer program named SIMUL [Hintz *et al.*, 1988a].
(2) The data were perturbed using a random number generator. The generator was based on a normal distribution and the error estimates for all measured quantities. The exposure station locations, not the GPS receiver locations, were perturbed.
(3) A bundle adjustment was performed using the exposure location estimates that were perturbed by the random number generator.
(4) Fictitious GPS antenna positions were generated using Equation 1, based on the exposure station positions and angular orientation solutions determined in Step (3). A receiver-to-exposure station offset of two metres was used in all tests.
(5) Using these antenna positions, the recursive bundle adjustmet procedure described in Figure 4 was performed, and the resulting ground positions were compared to those derived from the bundle adjustment in Step (3). Note initial estimates of zero for all omega and phi were used in all adjustments.
(6) The RMS ground coordinate differences derived from the results of the two adjustments were subjected to a T-test. The test is based on the premise that the coordinates were drawn from two normal distributions with identical variances, which was evaluated using the F-test [Ott, 1984].

A series of tests on 4 by 4, 7 by 7, and 10 by 10 blocks were used to evaluate the two adjustments, and they were found to be equivalent at the 95 percent confidence level in all three coordinates. The algorithm for conversion from GPS antenna to exposure station location presented in Figure 4 could prove extremely useful in implementation of the new technology. The iteration of the exposure station estimates was performed outside of the actual adjustment, which indicated existing bundle adjustment programs would need very minor modifications to adapt the algorithm.

CONCLUSION

Kinematic GPS positioning will potentially allow new techniques to be utilized in control densification by aerotriangulation and/or photogrammetric mapping. A series of issues regarding the implementation of this technology have been addressed. It is hoped that the provided strategies for solving these issues will provide assistance in evaluating and advancing the technology, and facilitate its implementation in traditional practice.

REFERENCES

Bastos, L., and H. Landau, 1988. Fixing Cycle Slips in Dual-Frequency Kinematic GPS-Application Using Kalman Filtering, *Manuscripta Geodetica*, Vol. 13, pp. 249–256.

Brown, D. C., 1980. *Application of Close-Range Photogrammetry to Measurements of Structures in Orbit*, Vol. 1, Final Report, Geodetic Services, Inc., 131p.

Hintz, R. J., M. Z. Zhao, and J. M. Kang, 1988a. *The Simulation of Aerotriangulation with GPS Derived Estimates for Exposure Station Parameters*, Technical Report of the Surveying Engineering Program, University of Maine, Orono, Maine, 18p.

———, 1988b. *Practical Issues in Aerotriangulation with GPS Derived Exposure Station Locations*, Technical Report of the Surveying Engineering Program, University of Maine, Orono, Maine, 17p.

Hoar, G. J., 1982. *Satellite Surveying*, Magnavox Advanced Products and Systems Co., Torrance, California, 146p.

Lucas, J. R., 1987. Aerotriangulation without Ground Control, *Photogrammetric Engineering and Remote Sensing*, Vol. 53, No. 3, pp. 311–314.

Lucas, J. R., and G. L. Mader, 1989. Recent Advances in Kinematic GPS Photogrammetry, *Journal of Surveying Engineering*, Vol. 115, No. 1, pp. 78–92.

Mader, G. L., 1986. Dynamic Positioning Using GPS Carrier Phase Measurements, *Manuscripta Geodetica*, Vol. II, pp. 272–277.

Ott, Lyman, 1984. An *Introduction to Statistical Methods and Data Analysis*, PWS Publishers, Boston, Massachusetts, 775p.

Schwarz, K. P., C. S. Fraser, and P. C. Gustafson, 1984. Aerotriangulation without Ground Control, *International Archives of Photogrammetry and Remote Sensing*, Vol. 25, Part A1, Rio de Janeiro, Brazil, pp 237–250.

van der Vegt, J. W., 1988. Differential GPS: An Efficient Tool in Photogrammetry, Unpublished paper, Presented at the May 11-14, 1988, ASCE Specialty Conference "GPS 88 - Engineering Applications of GPS Satellite Surveying Technology", Nashville, Tennessee, 14p.

Wells, D., 1987. *Guide to GPS Positioning*, Canadian GPS Assoc., Fredericton, New Brunswick, Canada, 500p.

(Received 3 February 1989; accepted 10 March 1989; revised 11 April 1989)