Aerial Photo System Calibration Over Flat Terrain

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ABSTRACT: A procedure to calibrate metric quality aerial cameras from photography taken over level terrain was developed. The method uses aerial photography collected under circumstances closely conforming to those found operationally. The development of the procedure is described, and results from both synthetic data simulations and real data are presented. By merging in a common adjustment high oblique photography with vertical photography exposed according to tightly prescribed orientations, it is possible to separate the parameters that describe interior from exterior orientation. Results from real data indicated an RMS error of photo coordinate residuals of 3.0 micrometres with over 600 degrees of freedom. The new oblique aerial method provides a means for replacing laboratory methods of calibration where recovery of the complete spatial geometry is essential, such as in the establishment of a measurement system for analytical photogrammetry in accord with the principles of Eisenhart.

INTRODUCTION

THE CONCEPT of instrument and measurement system calibration is thoroughly discussed and documented by Churchill Eisenhart (1963). Based on principles and methods used in production quality control, Eisenhart provides a logical approach to define and evaluate the metric performance of a measurement system.

In summary, Eisenhart states that, to achieve a measurement system calibration, it is first necessary to define the total measurement procedure in terms of a specification. This specification is defined as the "method of measurement." Given the "method of measurement," it remains to establish a "state of statistical control" by repeated comparisons of the final measurement product to a standard of the same measurements produced at a higher accuracy. As long as the measurement system performs as required in terms of accuracy and precision, it is said to be in a "state of statistical control" and is termed a "measurement process." This requires that the measurement procedure be repeated to assure that it is producing the expected accuracies and precision when compared to an independent standard of higher accuracy. System calibration, therefore, is an ongoing process requiring a periodic assessment of the metric performance of the total procedure.

The application of Eisenhart's concepts to the photogrammetric procedure is defined in the concept of the measurement problem by Merchant (1971, 1972) and expanded in subsequent work (Merchant *et al.*, 1988). Calibration, as it pertains to analytical photogrammetry, is the assignment of numbers to properties that describe the metric character of the measurement system and that describe the quality of its performance. The assignment of numbers to the parameters of the adopted mathematical models and the on-going assessment of the system's performance in terms of accuracy of the computed object space coordinates is clearly the task of a proper analytical photogrammetric system calibration.

The total photogrammetric measurement system is comprised of individual components which may be considered sub-systems. For example, one could consider sub-systems of the photogrammetric measurement system to be image collection, processing and printing, mensuration, compilation, etc.... The sub-system for image collection is the aerial camera, the aircraft, and camera accessories. These sub-systems each introduce a systematic error which contribute to the performance of the total measurement system.

Contrary to some beliefs, calibration of the camera itself is *not* "system calibration" in the context of Eisenhart. Although one can define the "method of measurement" of an aerial camera, one can never realize the "state of statistical control." There is *no* independent standard of higher accuracy with which one can compare the results of camera calibration, such as for the camera constant or distortion model coefficients.

However, the importance of camera calibration cannot be under estimated because it is here that numerical values assigned to the elements of interior orientation serve as the basis for numerous analytical formulations such as the General Projective Equations. Any error in the assignment of numbers to represent the "true values" of the unknown parameters will surely propagate errors into the results of all subsequent computations. Therefore, it follows that the calibration procedure for aerial cameras should be as realistic as possible, ideally under operational circumstances.

Procedures for the calibration of cameras have been developed in laboratories primarily as a tool for production quality control. The disadvantage for the photogrammetrist, of course, in laboratory calibration of aerial cameras is that it is performed under conditions which are not experienced during normal operations. Also, in contrast to an analytical approach to camera calibration, conventional laboratory methods do not explicitly treat the influence of decentering of the lens and are not able to express precise statements regarding accuracies of the calibration results.

The problem in an analytical approach to camera calibration is the determination of certain highly correlated parameters in a simultaneous least-squares solution. Certain methods of calibration, which can be conducted under circumstances similar to those found operationally and which suppress these high correlations between parameters as well as parameter functional groupings, have been established. Most notably are Brown (1969) and Merchant (1972) in the aerial case and Torlegard (1967) and Kenefick (1971) in the close-range case.

The first method for the aerial case, termed Simultaneous Multi-frame Analytical Calibration (SMAC), was established by Brown and the second, by Merchant, was termed the Method of Mixed Ranges (MMR). However, both these methods of aerial camera calibration require special circumstances. In Brown's case considerable field support was needed and in the Method of Mixed Ranges the "ideal mountain" is required.

In the absence of such ideal geometry and the inability to create elaborate test fields, other methods for calibration are needed. Kenefick, in conjunction with Duane Brown Associates

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(DBA), developed a method of camera calibration termed "Analytical Self-Calibration." Employing this method, Kenefick used highly convergent photography mixed with exposures having a 90 degree rotation of Kappa (k) to successfully calibrate a Hasselblad camera during the Apollo Lunar Mapping program (Kenefick *et al.*, 1971). It still remained to apply this method of camera orientation changes and convergent photography to the aerial case. Consequently, as part of the mandate of the ISP-Working Group on Image Geometry formed in 1973, a series of oblique and vertical photographs was taken over the Sudbury (Canada) test range in August of 1974. This was one of 12 camera calibration methods, both field and laboratory, that were to be implemented to assess consistency in the determination of lens distortion of two reseau cameras (Ziemann and Merchant, 1980). However, to the knowledge of the authors, no analytical analysis or discussion of the results of calibration from the Sudbury photography has been attempted or published.

Realistic values for elements of interior orientation (IO) are not only needed to increase the accuracy of traditional photogrammetric techniques (i.e., aerotriangulation) but their accurate determination takes on further significance in an adjustement where constraints on elements of exterior orientation (EO) are introduced. As shown by Brown (1969), in connection with tests of the USQ-28 System, tight constraints on elements of EO from external sensors (SHIRAN, HYPERNAS) sharply limit the effectiveness of projective compensation for errors in elements of IO. Again, the importance of an accurate IO model is of concern today as applications of the Global Positioning System (GPS) are integrated into photogrammetry. Studies (Lucas, 1987) have shown that the potential for aerotriangulation without ground control is a realistic goal for the "photogeodesist" if the exposure stations can be determined with sufficient accuracy. The photogrammetric significance is that far more attention must be paid to the complete and accurate recovery of elements of IO as they exist under operational circumstances if the goal is to be achieved.

As part of a research project for the Ohio Department of Transportation—Aerial Engineering (ODOT-AE), the calibration of the total photogrammetric measurement system has been undertaken (Merchant *et al.*, 1988). One of the components in a sub-system of the total measurement process is the aerial camera; more specifically, in this case, a Zeiss RMK-AR aerial camera. The development of a scheme to calibrate an aerial camera under operational circumstances, given the terrain restrictions in Ohio, was the purpose of this research. This paper reports on the theory, procedures, and results of this project.

A theoretical study using synthetic data was initially carried out by utilizing the Method of Mixed Ranges (MMR). For these simulations, the hills of southeastern Ohio were used to provide the significant variations in elevation required by the MMR. It was soon evident that, although the slope of the hills toward the river provided considerable elevation differences, they were, in fact, flat (planar) surfaces within any one photograph. This fact reduced the spatial geometry to that of an oblique exposure over level terrain, a circumstance that does not provide a sharp separation of parameters. Additional numerical simulations using highly convergent photography, as suggested by Kenefick (1971) but designed specifically for the aerial case, were conducted. After numerous configurations of photography were tested, a final photographic scheme for the complete calibration of the aerial photo collection system was adopted and implemented at the test range constructed at the Transportation Research Center of Ohio (TRC). This range possesses *no* significant elevation differences.

The calibration of the Zeiss RMK-AR aerial camera was accomplished by using a configuration combining highly convergent and vertical photography. The overall RMS error of photo coordinate residuals after adjustment was 3.0 micrometres.

TECHNICAL DISCUSSION

Photogrammetric and Adjustment Theory—General

In order to represent physical phenomena in a way to facilitate analysis and interpretation, it is necessary to adopt specific mathematical models. The definition of analytical photogrammetry is in itself the adaptation of various models to describe the physical character of the photogrammetric measurement system. The degree to which the physical reality is determined will depend, among other things, on the accuracy of the mathematical models, both functional and stochastic.

Photogrammetric Theory

The concept of collinearity is used to describe the idea that an image forming ray of light is undisturbed when passing from the object, through the lens, to the corresponding image location on the negative. The condition of collinearity is usually introduced in the derivation of the general projective equations. These equations, when used to relate image and object space, have been termed "First Order Theory" (Bender, 1971).

Continuing from Bender, the term "Second Order Theory" then accounts for the factors which cause a disturbance or departure from the ideal path of the light ray in "First Order Theory." The three major sources of departure normally treated in photogrammetric computations of "Second Order Theory" are atmospheric refraction, film deformation, and lens distortion. It follows that "Third Order Theory" comprises all the systematic departures from collinearity which are not addressed in "Second Order Theory." Attempts to model sources of "Third Order Theory" come from Brown (1969) in the function termed "Anomalous Distortion Error Model" and also from Gruen (1978) through the use of orthogonal polynomials. It is admitted without question that factors categorized in "Third Order Theory" have been demonstrated to cause significant systematic errors to the photogrammetric measurement process (e.g., platen unflatness-Brown, 1977). However, in this research the mathematical models adopted represent First and Second Order Theory only.

The observed photo-coordinates used in computation were corrected for both atmospheric refraction and film deformation as part of numerical pre-processing. For corrections due to atmospheric refraction, the model suggested by Saastamoinen (1973) was adopted for the vertical photography. In the case of the oblique photography, Merchant (1984) describes a procedure which will projectively transform coordinates from the tilted photo to a synthetic vertical, apply corrections to the transformed coordinates, and then project back to the plane of the real photo. Image distortion due to film deformation can be modeled effectively through the use of reseau photography. This was the approach adopted in this research as the photography was taken by a Zeiss RMK-AR (reseau) aerial camera. The transformation of comparator observations into the reference reseau system was accomplished through the use of a General Affine Transformation.

Adjustment Theory

The formation of the observation equations and normal equations in photogrammetry are presented by Brown *et al.* (1964) and extended for the calibration of aerial cameras from photography taken over a targeted calibration range in Brown (1969).

The camera calibration program (CALIB) used in this research was developed in-house at the Department of Geodetic Science and Surveying at The Ohio State University. It is essentially a

bundle block aerotriangulation program extended in the mathematical formulation to include parameters describing interior orientation. It follows the mathematical framework (algorithm) developed in Brown (1964, 1969). For this specific application of camera calibration, the model adopted is the General Projective Equations-Gimbal Form, augmented with selected parameters describing interior orientation—principal point coordinates (x_p,y_p), camera constant (c), and both monochromatic and decentering distortion, respectively (K_1, K_2, K_3, P_1, P_2, P_3). Again, these models are described in Brown (1969).

CALIBRATION RANGE DESIGN AND SIMULATION

General

In the application of least squares when linear dependency can be demonstrated between parameters or groups of parameters, it is not possible to separate such parameters in an adjustment. In photogrammetry, the geometric circumstances which yield such a linear dependency among parameters are termed Critical Surfaces. It is crucial in the design of aerial calibration schemes that geometric conditions exist that suppress these high correlations. If the parameters of interior orientation can be sharply separated from parameters of exterior orientation in a simultaneous solution, the numerical values assigned to describe physical circumstances can be assumed to be accurate provided that they are not contaminated by systematic errors or other highly correlated parameters.

Method of Mixed Ranges

For this research, the Method of Mixed Ranges (MMR) provided the theoretical framework for a potential aerial camera calibration range in Ohio. This method of camera calibration exploits the concept of three-dimensional (3-D) control fields originally investigated by Merchant (1967) for the aerial case and Torlegard (1967) for the close-range case. A review of the general terrain characteristics in Ohio led to several possible areas for the 3-D range along the Ohio River in Belmont and Monroe Counties. Complete calibration utilizing the 3-D range alone is possible, but it is very costly to establish a dense network of control. Therefore, to obtain a dense distribution of imaged targeted control to determine the parameters describing distortion requires photography of a second range. The TRC in East Liberty was chosen as the location for the high density target range.

Site Selection and Simulations—The Southeast Ohio Experiment

Analytical Self-Calibration. In an extensive review of the literature for a solution to improve the existing geometry of the Southeast Ohio calibration range, a series of articles by Kenefick (1971) and Kenefick *et al*. (1971, 1972) describing a procedure termed "Analytical Self-Calibration" were reviewed. Analytical Self-Calibration is a camera calibration technique developed by DBA Systems Inc. which employs certain configurations of highly convergent photography and minimal object space control. This calibration technique exploits the geometry afforded by highly convergent photography for the suppression of high correlations which exist between parameter pairs, such as camera constant with flying height. Unfortunately, even though the use of such highly convergent photography suppresses these primary projective correlations, secondary correlations exist between elements of IO with angular elements of EO (Kenefick *et al.*, 1972). However, this situation can be circumvented by incorporating the use of exposures having nominally orthogonal Kappa angles and equal exposure stations in the same reduction. Therefore, this mixture of certain configurations of highly convergent photography supplemented with exposures from common exposure stations having nominal orthogonal Kappa rotations provide a scheme for complete recovery of the elements of IO.

Simulation. In order to study the effect convergent photography would have on the calibration solution, it was decided to include one highly convergent photograph in the existing synthetic scheme. The new adjustment consisted of four vertical photographs and one convergent photograph of the three-dimensional range, mixed with one photo of the TRC high density range (Figure 1). The main objective of the experiment was to analyze the effect convergent photography had on the suppression of high correlations between selected parameter pairs and the degree of separation between these functionally dependent unknowns. The details of this simulation are reported by Tudhope (1987). Upon extensive investigations with additional vertical-convergent combinations, a scheme of photography was developed which could allow for any one of the selected Southeast Ohio sites to function as an aerial camera calibration range.

TRC Simulation. During the investigations into using certain configurations of highly covergent photography for aerial camera calibration, there arose numerous questions concerning different calibration solutions. Again, reviewing the literature concerning "Analytical Self-Calibration," a procedure developed and used by Brown (1971) for camera calibration was investigated further. The successful calibration of three Hasselblad cameras was accomplished by photography, requiring quite deliberate structure and procedure, of 100 targets (10 by 10 grid) at 2 1/2-inch intervals on a granite surface plate.

It was realized that the geometry of the calibration solutions from the Southeastern Ohio sites was essentially the same as Brown's. Recalling the comparison of the ideal mountain to the general terrain character of the potential sites, the slopes of the sites towards the Ohio River are seen as planar surfaces. It was realized that the exact duplicate of the "close-range" range used by Brown was available for the aerial case at the TRC high density range. It was decided to attempt a calibration simulation using

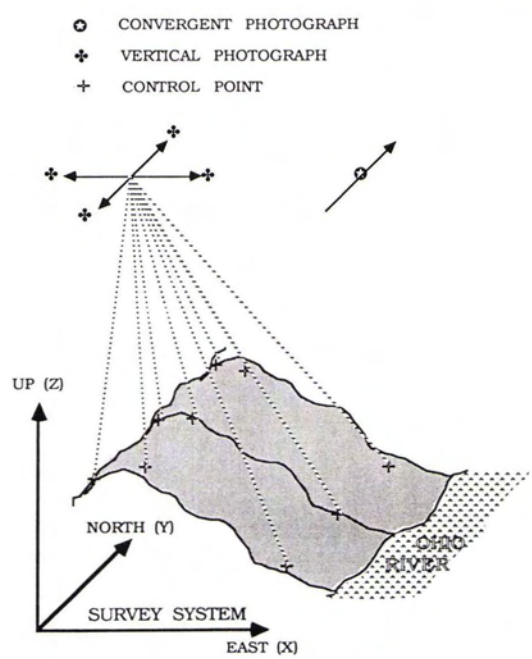

FIG. 1. Typical terrain character in southeast Ohio range sites and photographic configurations.

the TRC range exclusively. Accordingly, a selected number of vertical/convergent photographic schemes based on synthetic data were generated for the TRC range and processed through CALIB. The results were extremely successful and prompted the decision to abandon the Southeast Ohio range and concentrate on the development of one complete calibration range at the TRC, using highly convergent photography mixed with vertical photography having nominally orthogonal rotations of Kappa. It remained to develop a procedure which could be adopted for aircraft operations and which most closely approached operational circumstances.

The photographic configuration designed to generate a successful aerial calibration consisted of a module of photography comprised of four vertical and two 45° oblique photographs (Figure 2). The vertical photography consisted of one strip of three photos with 50 percent endlap. The center photo was exposed in the middle of the target array for complete coverage of the range in one photo. The fourth vertical photo was also to be exposed at the center of the target array except with a 90 degree rotation in Kappa. The two 45° oblique, or convergent, photographs were taken on each side of the single strip near the center of the target array boundaries in the same direction of flight as the vertical strip. They were positioned as such to accomplish complete coverage of the target array per exposure.

RESULTS OF SIMULATIONS

The synthetic data for survey and photo were computed and processed through CALIB. With respect to the adjustment, first approximations for certain unknown parameters were perturbed slightly. The coordinates of the principal point (x_p, y_p) and the camera constant were altered by 20 micrometres each. The exposure stations of all photos were displaced by 5 metres in position ((X_0, Y_0, Z_0) and by 5 degrees in orientation angles. In addition, the elements of eo were not constrained during the solution. The results obtained are excellent and are presented in Tables 1, 2, and 3.

Neither the synthetic survey coordinates nor the photo coordinates were perturbed from their synthetic computed value (true value). This accounts for the exceptional fit of the adopted mathematical model to the observations as demonstrated by the small *a posterior* variance of unit weight (0.12).

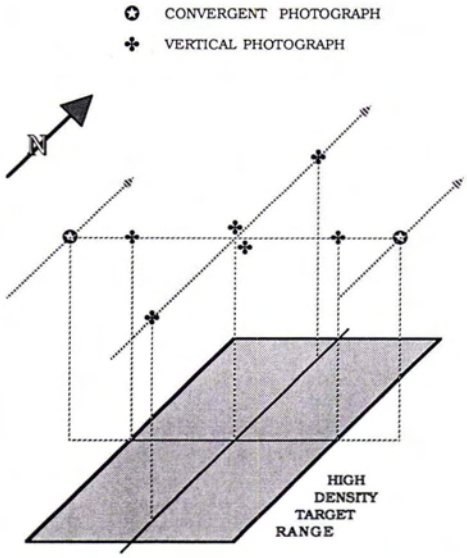

FIG. 2. Photographic configuration for oblique TRC calibrtaion.

TABLE 1. CORRELATION COEFFICIENTS OF SELECTED PARAMETER PAIRS AFTER ADJUSTMENT OF DATA BASED ON THE TRC RANGE

Exposure		Correlation Coefficients				
		c/Z_o	x_p/X_o	y_p/Y_o	x_p/ϕ	y_p/ω
TRC	1	0.74	0.01	0.00	0.01	0.00
Range	2	0.74	0.01	0.02	0.00	0.02
	3	0.80	0.60	0.82	0.59	0.01
	4	0.80	0.01	0.01	0.01	0.00
*	5	0.62	0.03	0.03	0.01	0.01
*	6	0.62	0.01	0.01	0.00	0.00

* denotes convergent photograph
A posteriori variance of unit weight = 0.12
Note: Correlation coefficient between x_p/P_1 = 0.85.

As illustrated in tests done by Merchant (1972) and as evidenced from examining the results of interior orientation in Table 2, a false distortion model is presented. Some parameters of distortion are unconstrained, partially constrained, or not exercised during the adjustment. Numerical values of distortion are computed for unknowns that physically do not exist. The erroneous results, though insignificant in magnitude, demonstrate the inherent problem of evaluating the numerical results in terms of their physical significance and interpretation when high correlations exist between parameter pairs. Again, as is illustrated in the correlation coefficient between principal point coordinate (x_p) and first order term in the decentering distortion model P_1 (i.e., x_p/P_1) = 0.85), there still exists a strong correlation. Both these results are attributed to the fact that the number of targets used in the simultion analysis was only 15. It is thought that a significant increase density of the target array (i.e., 100) would sharply separate the parameter combination x_p/P_1, and yield an accurate model describing the distortion characteristics of the camera.

However, what reinforces confidence in the calibration solution is the reduction of correlation coefficients of primary parameter pairs, especially between camera constant (c) and flying height (c/Z_0) (Table 1), the principal point location (x_p, y_p) and (c) (Table 2.) and the agreement in position of computed and true values for EO (Table 3).

Therefore, it has been demonstrated that, following certain configurations of photography, a complete calibration of an aerial camera can be accomplished over virtually any terrain. This, in principal, demonstrates the successful achievement of the research goal, i.e., to develop a method for aerial camera calibration in Ohio for ODOT - Aerial Engineering. However, in order to verify the calibration simulation method described, a photo mission was scheduled for the collection of photography based on the configuration illustrated in Figure 2 with some modification.

IMAGE COLLECTION, PROCESSING, AND OBSERVATION

GENERAL

In order to verify the favorable geometry provided by the simulated photographic configurations, it was necessary to implement the calibration scheme under operational circumstances. This required the collection of aerial photography over the targeted control range located within the TRC. Through the cooperation of the ODOT, the OSU Zeiss camera (RMK-AR) was flown over the TRC and the required photography was collected.

As mentioned, the photographic scheme used in simulation consisted of four vertical and two oblique photos. Again, one vertical photograph having an orthogonal rotation in Kappa with respect to the vertical strip of three photos and oblique

TABLE 2. RESULTS OF PARAMETERS FOR INTERIOR ORIENTATION FROM ADJUSTMENT OF TRC SIMULATION RANGE DATA

	x_p (mm)	y_p (mm)	c (mm)	$K_1 \times 10^{-7}$	$K_2 \times 10^{-12}$	$P_1 \times 10^{-7}$	$P_2 \times 10^{-11}$
Computed Value	−0.003	0.002	152.40	−0.196	0.676	0.400	0.100
True Value	0.0	0.0	152.40	0.0	0.0	0.0	0.100
σ	±0.005	±0.005	±0.005	–	–	–	–
weight	0.0	0.0	0.0	0.0	10.0	0.0	10.0

TABLE 3. RESULTS FOR EXTERIOR ORIENTATION ELEMENTS FROM ADJUSTMENT OF THE TRC SIMULATION RANGE DATA

Exposure		Metres			Radians		
		X_o	Y_o	Z_o	κ	ϕ	ω
	CV	1524.001	1386.831	1228.603	1.570801	0.00004	−0.000134
1	TV	1524.000	1386.840	1228.600	1.570796	0.0	0.0
	σ	±0.008	±0.008	±0.008	±0.000011	±0.000022	±0.000039
	CV	1523.999	1661.152	1228.596	1.570791	0.000000	−0.000136
2	TV	1524.000	1661.160	1228.600	1.570796	0.0	0.0
	σ	±0.007	±0.008	±0.008	±0.000011	±0.000020	±0.000038
	CV	1523.988	1524.005	1228.600	−0.000001	0.000119	−0.000015
3	TV	1524.000	1524.000	1228.600	0.0	0.0	0.0
	σ	±0.006	±0.008	±0.008	±0.000007	±0.000024	±0.000017
	CV	1524.000	1523.992	1228.599	1.570797	0.000003	−0.000205
4	TV	1524.000	1524.000	1228.600	1.570796	0.0	0.0
	σ	±0.006	±0.004	±0.008	±0.000007	±0.000016	±0.000026
	CV	1324.001	1523.996	1228.601	1.570660	−0.785391	−0.000205
* 5	TV	1324.000	1524.000	1228.600	1.570796	−0.785398	0.0
	σ	±0.007	±0.006	±0.006	±0.000028	±0.000031	±0.000043
	CV	1724.000	1523.996	1228.599	1.570933	0.785404	−0.000205
* 6	TV	1724.000	1524.000	1228.600	1.570796	0.785398	0.0
	σ	±0.007	±0.005	±0.006	±0.000028	±0.000031	±0.000043

CV = computed value TV = true value σ = standard deviation
* – denotes convergent photograph

exposures, all having the same flight direction, were collected. This total of six photographs constituted one photographic module. The flight plan called for two of these modules to be flown. This would provide the ability to perform two separate calibration adjustments.

With respect to the individual components of the photogrammetric system, the image collection subsystem is now considered in detail. The three major elements of image collection identified in this research are (1) the aircraft, (2) the aerial camera, and (3) the camera accessories (i.e., mount, magazine filters, etc.).

AIRCRAFT

The aircraft used by ODOT-AE for aerial photography is a modified C-45 Twin Beech. This twin-engine aircraft is capable of slow flight while maintaining good stability required for lower altitude photography. This is an extremely important requirement in order to suppress image motion and to allow time for the film to recycle properly and the vacuum to develop fully.

CAMERA

The aerial camera used in the collection of the photography for this research was a Zeiss RMK-AR 15/23, No. 21197, which is the property of The Ohio State University - Department of Geodetic Science and Surveying (OSU-DOGSS). This specific aerial camera contains an internal (shadow) reseau. The primary function of the reseau is to provide a calibrated grid which is superimposed on the negative at the instant of exposure. As a result of considerable study done in-house at OSU-DOGSS on this specific internal reseau, an adjusted set of coordinates for each reseau cross has been established. This coordinate control can be used to determine the parameters chosen to model film deformation and also provide the reference photo coordinate system.

ACCESSORIES

The film magazine and lens filters used were part of the equipment provided by OSU-DOGSS and are original with the aerial camera. The magazine and filters were inspected and serviced prior to the photo mission.

TRC AERIAL TARGET RANGE

As mentioned , the TRC was chosen as the location for the aerial camera calibration range. The TRC has controlled access; consequently, the aerial targets and survey monuments are less subject to disturbance. The area known as the Vehicle Dynamics Area (VDA) is essentially flat and clear and thus provides an excellent location for a dense array of targets. Typical low altitude photography required by ODOT-AE is flown at an altitude of approximately 1200 ft above mean ground level. Therefore, assuming a focal length of 6 inches and 9-inch format photography, the approximate ground coverage of one photo is 1800 feet. Consequently, a 10 by 10 target array, at 200-foot centers, was constructed within the VDA comprising an area of approximately 1800 ft by 1800 ft. In areas where targets could not be sprayed directly on the asphalt, a 3 ft by 3 ft concrete "patio" pad was used as a base for the target.

TARGET

In considering the design of aerial targets, one concern is how the target will appear as an image. If the image of the target is

well-defined, the precision of measurement will naturally increase. Also of consideration is the type of measurement device used and the physical character of the measuring mark. For example, most comparators and analytical plotters have circular measuring marks. Therefore, it follows that the aerial target should be circular and have high contrast. This can be achieved by constructing a white circle on a flat background in the dimensions illustrated in Figure 3. To ensure precision of pointing, the diameter of the image of the white circle should be slightly larger than the diameter of the measuring mark. For target locations on the VDA, the flat black background disk was sprayed directly on the surface while the white circle was sprayed on the flat black on both asphalt and concrete pads.

Ground Control

In order to provide control data for the calibration adjustment, it was necessary to survey a select number of targets from the array of 100 at the TRC range. Figure 4 illustrates the targets that were surveyed both horizontally and vertically. The equipment used to conduct the field survey was a Wild T-2000 total station and a Wild DI-5 EDM. The EDM was calibrated twice during recent survey projects and the results of calibration did not significantly change from the manufacturer's specifications of performance. The Wild T-2000, although not tested against an absolute standard, exhibited good precision and acceptable differences in direct and reverse pointings in all field work.

The Wild T-2000 total station and DI-5 EDM were set up on a "PK" nail in the center of target 9. Three repetitions of directions were taken on targets 1 through 8 as well as on each forward horizontal distance. The instrument was then set up over a "PK" nail in the center of target 4 and the same procedure was repeated. These measurements provided sufficient redundancy for a least-squares adjustment of the control network.

A Wild N-2 instrument was used for the leveling. Local level loop was performed around the perimeter of the VDA which included targets 1 through 8. A second level line passed through the center of the VDA and picked up point 9 in the process. The result of the field work is an adjusted set of local horizontal coordinates (X, Y) and local elevation (Z) for targets 1 through 9.

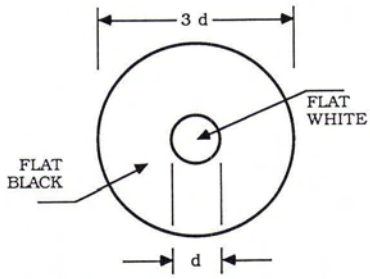

FIG. 3. Aerial target design ($d = 1/3{,}000$ of flight height for a 6-inch focal length camera).

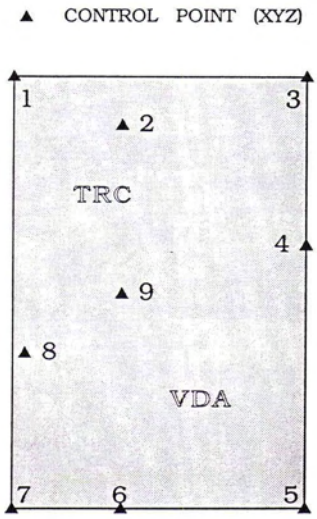

FIG. 4. Control network (vertical and horizontal) at the TRC.

Photo Mission

The photography was carried out on two successive days under excellent atmospheric conditions. On 13 October and 14 October 1987 at approximately 1400 hours EST the two vertical strips and oblique exposures were collected. In order to allow for the possibility of redundancy in the selection of particular exposures, the vertical strips were each flown twice while the oblique exposures were each taken three times.

The exposure data used throughout were aperture, $f/8$; shutter, 1/500 sec; filter, type B; and film, Kodak type 2405 (Double-X).

The aerial maneuver to position the aircraft at 45° inclination at the point of exposure was termed "a standard coordinated turn." It is admitted that at 45° the force in the direction of the camera axis exceeded a positive 1g to approximately 1.5g. This does raise concern about the additional systematic error present at the instant of exposure. It was decided that the problem of >1g force could be alleviated at higher altitudes with a modification in flying technique and the corresponding monitoring of a "g-meter" in the aircraft. However, the flying height during the TRC oblique mission was only 1000 feet above mean ground level. Accordingly, for safety reasons, the modification in flying technique, which required a slight nose forward thrust of the controls prior to the instant of exposure, was not attempted. These fine tunings of technique were not the objective of this initial "pilot" calibration procedure but are areas for further development and refinement.

Processing

Negatives from the two photo missions were processed at ODOT-AE in Columbus utilizing their automatic developing facilities. Upon indexing and referencing the two photo missions, the negatives were delivered to the authors at OSU-DOGSS for contact diapostive printing and subsequent photo coordinate observations.

Photo Coordinate Observation

The Wild BC-1 analytical plotter located at OSU-DOGSS was used to observe photo coordinates. In keeping with the general concept of systems calibration, the Wild BC-1 was tested to ensure that the observations were not subjected to any unknown systematic errors. It was concluded that the Wild BC-1 could be used as a standard of comparison for photo coordinate observations.

Because the individual exposures all have redundancy, it was necessary to select individual frames that best approached the synthetic configuration desired. Unfortuately, the number of exposures discarded was significant and essentially eliminated photo module #2 and slightly depleted photo module #1. The remaining negatives were organized for observation.

Confident that the Wild BC-1 was performing up to standards, the seleced negatives were placed on the instrument for target and reseau cross observations. For each negative, the imaged control points that could be clearly identified were observed.

For each targeted point that was observed, a corresponding observation on an adjacent reseau cross was made.

Data Pre-Processing

Recalling the parameters of IO that are carried as unknowns in the calibration, there are only those describing lens distortion, camera constant, and principal point coordinates. Therefore, it is necessary to correct for sources of error not modeled in the adjustment, thereby eliminating known systematic influences on the images. Accordingly, the photo coordinates were corrected for film deformation and atmospheric refraction. This was accomplished following the procedures and models described earlier.

Also as described earlier, the comparator coordinates of a target image on each photo were transformed into the reseau coordinate system. The parameters of transformation were computed by least squares using the General Affine model. The model was used to fit a large number of reseau point images over the entire format. From Table 4 it is evident that there is good agreement of the general affine model to the observed reseau coordinates.

Parameter Approximations

All values for IO were set to zero except camera constant. The value assigned to camera constant for first approximations was 152.40 mm. Values of EO were estimated in a conventional manner.

THE TRC DATA ADJUSTMENT

General

The photographic modules described earlier were not obtained during the first photo mission. Photo Module #2 could not be included in the initial adjustment because the oblique exposures were not satisfactory in position or attitude. Photo Module #1 was reduced in the number of vertical exposures from four to three while the angle of intersection at the center of the target range of the oblique photos was approximately 77° and not the ideal of 90°. From the adjustments done in simulation during the course of this research, it was concluded that the closer the convergent ray intersections were to 90°, the better the suppression of high correlations. This was also the conclusions drawn by Kenefick (1971) and Kenefick *et al.* (1971, 1972). However the subsequent adjustment of Photo Module #1 was quite acceptable for the first attempt ever at such an aerial camera calibration procedure.

Results of Calibration Adjustment

A broad summary of the characteristics of the TRC/VDA calibration adjustment of the Zeiss RMK-AR 15/23 #21197 is shown in Table 5. The unknown parameters in the adjustment and their *a priori* weights are presented in Table 6.

Table 4. Average Residuals After Using the Genreal Affine Model to Fit Reseau Point Images Over the Entire Photograph

Photo ID	Number of Observed Reseau Crosses	Average Residual V_x (μm)	V_y (μm)
311	46	1.8	2.6
310	61	2.6	2.6
28	58	1.9	2.0
13	71	3.4	3.5
039	65	2.0	1.6
027	72	2.4	2.2
012	63	2.7	1.9

Table 5. Summary of Characteristics of TRC/VDA Calibration Adjustment

No. of Photographs	5
– oblique	2
– vertical	3
No. of Target Points	81
– control points (X Y Z)	9
– control point (Z)	17
– non-control	55
Degrees of Freedom	601
Variance of Unit Weight	1.1
No. of Iterative Cycles	8

Table 6. Parameters of the Calibration Adjustment and Their Assigned Weights

Parameters	Assigned Weights	*A-priori* Estimates of Standard Error
c, x_p, y_p	0	–
K_1, K_2, K_3	0	–
P_1, P_2, P_3	0	–
x, y	111,111	0.003 mm
X_o, Y_o, Z_o	0	–
κ, ϕ, ω	0	–
X, Y control	10^6	0.001 metres
Z control	40,000	0.005 metres
X Y Z targets	0.01	10 metres

Table 7. Correlation Coefficients Of Select Parameter Pairs After Adjustment of TRC Data

Exposure		Correlation Coefficients c/Z_o	x_p/X_o	y_p/Y_o	x_p/ϕ	y_p/ω
TRC	1	0.88	0.08	0.19	0.04	0.02
Range	2	0.90	0.09	0.19	0.07	0.08
	3	0.89	0.52	0.69	0.59	0.46
*	4	0.69	0.19	0.08	0.04	0.01
*	5	0.73	0.07	0.02	0.15	0.06

* denotes convergent photograph

In examining the calibration results (Tables 7, 8, and 9), the *a postriori* variance of unit weight is 1.1. Therefore, the adopted mathematical model fits the observations well. Statistical testing based on the chi-square distribution gave an acceptable result of the adjustment with 601 degrees of freedom at the 5 percent significance level.

There is good separation of all parameter pairs in Table 7, although the correlation coefficients for camera constant and flying height are still moderately high. However, the correlation coefficients of the convergent photos are significantly smaller than the correlation coefficients of the vertical exposures. This again illustrates how convergent photography can suppress certain high correlations that exist.

From Table 8 the value obtained for P_2 (coefficient of decentering distortion) is the same order of magnitude as P_1. Initially this would appear in question but this is not surprising to indiviuals familiar with this particular camera. The Zeiss RMK-AR No. 21197 has always exhibited an abnormally high degree of decentering distortion (Hakkarainen, 1976). The standard errors of camera constant and principal point are slightly higher than expected but are quite acceptable. It is thought that the standard error of these parameters would decrease if the convergent angle of the oblique exposures was closer to 90°.

TABLE 8. RESULTS OF PARAMETERS OF INTERIOR ORIENTATION FROM ADJUSTMENT OF THE TRC DATA

	x_p (mm)	y_p (mm)	c (mm)	$K_1 \times 10^{-7}$	$K_2 \times 10^{-11}$	$K_3 \times 10^{-18}$	$P_1 \times 10^{-6}$	$P_2 \times 10^{-6}$	$P_3 \times 10^{-18}$
Computed Value	0.011	0.058	−152.057	−0.2622	0.1451	1.0	−0.4170	−0.6234	1.0
σ_o	±0.007	±0.008	±0.007	±0.0308	±0.0126	±0.3366	±0.0806	±0.0752	±0.3366
weight	0.0	0.0	0.0	0.0	0.0	10.0	0.0	0.0	10.0

A-posteriori variance of unit weight = 1.1

TABLE 9. RESULTS FOR EXTERIOR ORIENTATION ELEMENTS FROM ADJUSTMENT OF THE TRC DATA

	Metres			Radians		
Exposure	X_o	Y_o	Z_o	κ	ϕ	ω
1	1744.882 ±0.015	1303.900 ±0.011	707.576 ±0.017	1.599630 ±0.000008	0.003278 ±0.000038	0.046867 ±0.000038
2	1753.463 ±0.015	1509.960 ±0.016	711.202 ±0.017	1.606030 ±0.000011	−0.007414 ±0.000039	0.021798 ±0.000048
3	1691.036 ±0.016	1365.739 ±0.020	724.541 ±0.017	0.005299 ±0.000010	−0.007005 ±0.000048	−0.033102 ±0.000040
* 4	2154.471 ±0.017	1283.112 ±0.010	661.055 ±0.014	1.520482 ±0.000014	0.715651 ±0.000048	0.174665 ±0.000059
* 5	1430.382 ±0.016	1370.002 ±0.007	617.611 ±0.012	1.468805 ±0.000039	−0.634811 ±0.000049	0.058305 ±0.000054

* – denotes convergent photograph

Table 9 shows the results for the EO elements from the adjustment. The accuracy of these unknown parameters depends on the geometry of the solution and the degree to which the corrlation between certain unknown parameter pairs can be suppressed. The positional values are quite acceptable in examining the standard errors of each unknown parameter.

The final photo coordinate residuals are presented in Table 10 in terms of RMS error after the TRC/VDA calibration adjustment. There seems to be good consistency between the results of the photography, despite its being taken on two separate days. The images of the targets on the negatives were quite distinct (even at the edge of the format) which helps explain the consistency of the residuals with the precision of pointing. The estimated standard error of one observation was taken as 3 micrometres. The precision of the photo coordinate observations and the fit of these observations to the adopted model is gratifyingly close.

CONCLUDING REMARKS

With the advent of practical methods of positioning the exposure station by using the GPS, the old question of adequacy of laboratory calibration of the camera is reopened. On the one hand, Lucas (1987) demonstrated by use of GPS phase observations in differencing mode the potential of positioning the aircraft to accuracies of 5 centimetres. On the other, it has been shown that a 7-metre difference existed at 17,000 feet over the Casa Grande range for the resected position between solutions based on laboratory calibrations and the solutions provided by an aerial calibration based on the method of mixed ranges (MMR) (Merchant, 1974). Clearly, the old crutch of "projective compenstion" can no longer be leaned on to account for the systematic errors introduced by the laboratory calibration's neglect of such things as decentering, platen unflatness, and, possibly, window distortions. The concept of "measurement system calibration" as described by Eisnehart (1963) provides the necessary basis for including all systematic influences on the aerial photogrammetric system. An underlying premise of the systems approach is that the characteristics of the measurement system be assessed under operational conditions. It follows that the components of the system must also be assessed under operational conditions. It was the purpose of this investigation to develop and demonstrate such a procedure for the aerial photo collection component of the aerial photogrammetric system.

TABLE 10. PHOTO COORDINATE RESIDUALS AFTER THE TRC/VDA ADJUSTMENT (RMS ERROR) IN MICRONS

Photo	Number of Targets	RMS (μm) x	RMS (μm) y
1	71	3.3	2.7
2	58	3.5	2.8
3	61	2.6	2.9
* 4	63	3.9	3.5
* 5	66	3.0	2.3
	Component Avg.	3.26	2.82
	Group Avg.	3.04	

* denotes convergent photograph

The first attempts to recover IO from aerial photography during this investigation were based on an adaption of the MMR over the hills of southeastern Ohio. Numerical simulations indicated that a solution was possible but would not permit a sharp separation of parameters as indicated by the correlation coefficients. After further literature review, an adaptation of the method termed "analytical self-calibration" (Kenefick, 1971; Kenefick *et al.*, 1971,1972) was developed during this research for the aerial case. This method will work over flat terrain and requires only a minimum of ground control. It does require some precision flying on the part of the air crew to achieve target field coverage at 45 degree bank attitudes. The method provides reasonably good separation of interior from exterior parameters.

The oblique method was demonstrated over the TRC, a flat area 1800 feet on a side and containing 100 targets. Only nine of the targets were employed as control points in the procedure. Results of the oblique method demonstrated excellent internal precision of 3.0 micrometres RMS error (601 degrees of freedom) for the photo coordinate residuals after adjustment.

It remains to investigate the spatial quality of the resected coordinates of the exposure station coordinates based on the models determined by means of the oblique method of aerial calibration. Plans are underway to collect photography over the test fields of the TRC and compare resected results from those determined by a GPS recevier located in the aircraft operating in a differencing mode with a ground receiver located in the vicinity of the TRC.

If the spatial quality of the IO determined by this method of aerial oblique calibration can be verified through comparison to an independent standard such as provided by GPS, it is hoped that oblique calibration will become part of aerial analytical photogrammetric measurement specifications of the future. If the full accuracy offered by GPS is to be exploited by aerial photogrammetry, laboratory methods of camera calibration must be replaced by methods that recognize contributions introduced into the image geometry during aerial operations of the photo collection system. This new oblique method of aerial collection appears to be a candidate for such a systems approach to calibration.

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