# Multi-Sensor DLT Intersection for SAR and Optical Images 

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Arecent article in PE\&RS (Brill, 1987) presented a new, noniterative method of resecting multi-sensor images (syn-thetic-aperture radar (SAR) and optical) images using a direct linear transformation (DLT). The method has much in common with the DLT method of resection from optical images (AbdelAziz and Karara, 1971; Williamson, 1972). However, at the time of writing of the earlier article, it seemed that there could be no noniterative intersection for multi-sensor image sets containing SAR images, even though such intersection can be done for purely optical images by DLT. To quote directly from the earlier article, it was said that "SAR/SAR and SAR/optical intersections require iterations over the parameters of the various curves of ambiguity [because] the condition equations for a SAR image combine to yield a system of quadratic equations, which does not have a closed-form solution." The implicit assumption behind this statement is that all the available condition equations are used equally in the solution. If this implicit constraint is removed, the result is that a noniterative intersection can be done with multi-sensor image sets containing SAR images. This paper is intended to explain the method briefly, and to show that the method works with as few as two images, even though some of the condition equations are used only to check the solution obtained from the other equations.
The SAR condition equations, Equation 10 in the previous paper, with vector X in the solve state, can be rewritten as

$$
\begin{align*}
X^{2}+A X_{1}+B X_{2}+C X_{3} & =D  \tag{1a}\\
E X_{1}+F X_{2}+G X_{3} & =H . \tag{1b}
\end{align*}
$$

Here, the quantities $A$ through $H$ depend on the $L$-values in the images containing point $X$, and also on the image coordinates $q_{1}, q_{2}$ of the point in each image. If there are $N$ overlapping images, there are $2 N$ equations in the unknowns $X_{1}, X_{2}$, and $X_{3}$. Because

$$
\begin{equation*}
X^{2}=X_{1}^{2}+X_{2}^{2}+X_{3}^{2} \tag{2}
\end{equation*}
$$

it would seem that solving these equations for a series of images is a nonlinear task, and cannot be done in closed form. However, a closed-form solution of Equations 1 can be obtained, if $X^{2}, X_{1}, X_{2}$, and $X_{3}$ are placed "independently" in the solve state, temporarily disregarding the fact that $X^{2}$ is related to the components of $\mathbf{X}$. Then the equations become linear in the four unknowns $X^{2}, X_{1}, X_{2}$, and $X_{3}$, and can be solved by a single pseudoinverse operation on the right-hand-side vector. Depending on how many optical and SAR images there are, there will be more or fewer occurrences of the quantity $X^{2}$. [There must be at least one SAR image to apply the method indiscriminately, or else the first column of the left-hand-side matrix will contain all zeros.] After solving for the four "independent" unknowns, a forward check is performed to determine the difference between $X^{2}$ and $X_{1}{ }^{2}+X_{2}{ }^{2}+X_{3}{ }^{2}$. This will be a measure of the accuracy of the result.

This new multi-sensor development enables incorporation of more than two images into the intersection program, as well as allowing intersection without iteration by means of something like "Newton's method." Note that, if there are at least two images, one of which is a SAR image, there are four equations for which the four unknowns can be solved. The luxury of the extra unknown did not require use of more images.
The above procedure yields a unique solution for $X_{1}, X_{2}$, and $X_{3}$, so it might be asked why there is no possibility for double intersections between a SAR projection circle and an optical line of sight (or another SAR projection circle). The answer can be found by first examining Equations 1 for two SAR images, then examining the equations for intersecting an optical and a SAR image, and finally by generalizing to arbitrary numbers of SAR and optical images participating in the multi-sensor intersection.
Equation 1b, the SAR Doppler condition equation, is linear in the object-space coordinates; however, Equation 1a, the SAR range condition equation, is nonlinear. The range equations obtained from two SAR images define the circle of intersection between two range spheres-which also defines a plane. Taking the difference between two SAR range equations (from two SAR images containing the same object-space point) results in a linear equation for $X_{1}, X_{2}$, and $X_{3}$, with $X^{2}$ absent. This linear equation just specifies the plane containing the circle of intersection of the two range spheres. Except under certain degenerate circumstances, this plane intersects with the planes of both projection circles, defined by Equation 1b, to define a unique point. The only times this does not occur are when the planes of the projection circles and the plane of intersection of the range spheres are parallel or coincident. Under such conditions, two of the equations used for intersection will be redundant because they define the same plane (or contradictory because they define parallel planes). Only when the projection circles lie in the same plane will they intersect at two points; in that case, this DLT intersection will not work. But one could remedy such a difficulty, in the solution of two SAR images, by introducing an additional SAR image containing the unknown point. It is not likely that the introduction of a third SAR image would produce the same coplanarity for the projection circles for the same unknown point. However, this increase in the number of SAR images can be continued until enough equations are obtained to provide a multi-sensor solution using the intersecting planes without resorting to the nonlinear equations.

The situation is exactly analogous for intersecting a SAR and an optical image. The line of sight defined by the two optical condition equations (which are also linear) intersects the plane of the projection circle in exactly one point. The line of sight intersects the actual projection circle twice only when the line of sight lies in the plane of the SAR projection circle. In that case, the SAR Doppler equation is redundant with the optical condition equations, and one has only three equations left from

0099-1112/89/5502-191\$02.25/0
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which to solve for $X_{1}, X_{2}, X_{3}$, and $X^{2}$. Only by using the dependence of $X^{2}$ on the other unknowns can the intersections be found, and they will in general be double intersections. If the line of sight and the projection circle are not coplanar, the range condition equation can be ignored entirely (except as a check on accuracy), and there will be three linear equations remaining in the three unknowns $X_{1}, X_{2}$, and $X_{3}$.
When intersecting an arbitrary number of SAR and optical images, the above principles are in effect. The range spheres intersect pairwise to define planes, and these planes intersect with each other, with projection-circle planes, and with optical lines of sight, to produce a unique point. Of course, measurement errors create a scatter of points; hence, the formalism for solution involves a pseudoinverse operation to retrieve a "best fit' solution from the overdetermined system of MSDLT linear equations. This is in direct analogy with the DLT method of optical intersection.

This article, in conjunction with the previous paper (Brill, 1987), provides the conceptual foundation for applying multisensor direct linear transformation (MSDLT) procedures, in both resection and intersection. MSDLT applies simultaneous adjustment techniques to multiple SAR, mixed SAR and optical, and pure optical imagery. As in the standard application of DLT, there can be as few as two images. MSDLT resection of each optical image requires at least six control points, and MSDLT resection of each SAR image requires at least eight control points. The basis of the DLT method (Abdel-Aziz and Karara, 1971; and Williamson, 1986) is to obtain a closed-form solution to the condition equations by casting these equations into a linear form in terms of the unknown. The same DLT method is used in the application of MSDLT, which involves reevaluating the information required for a solution. In MSDLT resection, the acqui-
sition geometry is reduced to $L$-values, which determines all the information needed for a subsequent intersection but does not (and in principle cannot) determine the acquisition geometry uniquely. In MSDLT intersection, the required compromise is to use the linear condition equations with $\mathbf{X}$ in the solve state, and the other equations only as a check or possibly for refinement of the original answer. (All condition equations are linear when the $L$-values are in the solve state.) We emphasize that there are certain degenerate circumstances under which MSDLT resection cannot be done: namely, if all the control points are coplanar.

The end result of this MSDLT application is an efficient method of obtaining object-space coordinates of unknown points from convergent imagery.

## REFERENCES

Abdel-Aziz, Y., and H. M. Karara, 1971. "Direct Linear Transformation from Comparator Coordinates into Object-Space Coordinates," ASP Symposium on Close-Range Photogrammetry, Urbana, Illinois, January 1971.
Brill, M. H., 1987. Triangulating from Optical and SAR Images Using Direct Linear Transformations, Photogrammetric Engineering and Remote Sensing, Vol. 53, No. 8, pp. 1097-1102. Errata in this paper are as follows: p. 1099, line 3, $P_{i}$ should be $\rho_{i j}$ in Equation 9 b , the last $l$ subscript should be 13 ; in Equation $10 \mathrm{~b}, x_{2}$ and $x_{3}$ should be $X_{2}$ and $X_{3} ;$ p. 1100, last full paragraph, references to Equation 12 should be to Equation 10.
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