Applications of Digital Image Processing to Photoelastic Stress Analysis

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ABSTRACT: A two-dimensional transparent photoelastic model of multiple underground chambers was utilized to investigate changes of the stress fields around openings in response to simulated failure of an opening support and increases of room dimensions. Photographs of the photoelastic fringes were scanned by an Eikonixscan camera and processed by a computer to better define the locations of fringe lines on the images and to investigate the variation of stress fields under different engineering excavations. In the study, the gradient process, which enhanced local radiometric variability of the fringe lines, was considered an applicable technique for analysis of photoelastic stress. The method also provided information of fractional fringe orders for a comparison of the boundary stresses under various engineering conditions.

INTRODUCTION

THE PHOTOELASTIC METHOD is a widely used technique for determining the stress distribution in a structure under loading. The method has been applied in areas including establishment of engineering design criteria, improvement of product reliability, and reduction of project cost. The demand for increased use of this method has accelerated in response to the current technological advancement and special requirement for engineering designs. The photoelastic method is versatile and easy to apply. It can be used both in the field and in laboratory environments. Because the technique provides engineers with full-field visualization of stresses with a quantitative measurement capability, it plays an important role in the field of experimental stress analysis (Measurements Group, 1984).

The photoelastic stress analysis includes two common approaches: (1) reflection photoelasticity, and (2) transmission photoelasticity. In practice, reflection photoelasticity is used primarily with photoelastic coatings on an object of interest. Transmission photoelasticity is, however, restricted to transparent objects such as two- and three-dimensional models. One of the criteria in the selection of either experimental method is dependent upon the required development and progress of a project. In the early stage of an engineering design - for example, before the excavation of an underground chamber - preliminary studies with two-dimensional transmission models are often the most effective approach. For such tests, a diffused light transmission polariscope is used. The technique involves stressing a photoelastic model in a loading frame and illuminating it with polarized light. A resulting fringe pattern, which becomes visible when viewed through the polariscope, reveals the overall strain distribution and the corresponding locations of stress concentration areas. In most photoelastic experiments, photographs are often taken to establish a permanent record of the tests and to study the stress distribution in the models. With the use of digital image processing, the analysis of photoelastic stress can be improved and simplified. Huang and Aughenbaugh (1988) and Huang (1988) converted photoelastic photographs to digital images for stress analysis.

This paper introduces the potential applications of digital image processing to photoelastic stress analysis. The theory of photoelasticity is briefly summarized in the text and then the routine image processing methods which effectively enhance the fringe patterns are evaluated and discussed.

THEORY OF TRANSMISSION PHOTOELASTICITY

The theory of photoelasticity has been discussed in detail by a number of researchers (Kuske and Robertson, 1974; Dally and

PHOTOGRAMMETRIC ENGINEERING AND REMOTE SENSING, Vol. 56, No. 4, April 1990, pp. 495–499. Riley, 1978). The following briefly descirbes the basic concept of photoelastic stress analysis.

The electromagnetic waves emitted from a light source propagate in all directions and vibrate in all the corresponding perpendicular planes. However, by using a polarizing filter, only one component of these vibrations will be transmitted into the model (Figure 1). If another polarizing filter, the analyzer, is placed behind the polarizer, complete extinction of the light intensity can be obtained when the axes of these two filters are perpendicular to each other.

Photoelastic materials become optically anisotropic when stressed. The change in index of refraction is a function of the stress applied. When a polarized light with a wavelength, λ , propagates through a model of thickness, *t*, the light will be further polarized along the major principal stress directions, σ_1 and σ_2 , and both polarized beams will travel at different speeds. The relative retardation, δ , between these two beams can be described as follows:

$$\delta = N \lambda = t K (\sigma_1 - \sigma_2). \tag{1}$$

The strain-optical coefficient, *K*, characterizes a physical property of the photoelastic material and the order of fringe lines, *N*, is directly proportional to the magnitude of the principal stress difference ($\sigma_1 - \sigma_2$). It is apparent from Equation 1 that the stress difference can be determined if the relative retardation, δ , can be measured, provided the strain-optical coefficient



Fig. 1. Schematic representation of plane polariscope and the stressed photoelastic model (modified after Measurements Group, 1984).

is known through calibration. The function of a polariscope is acutally to determine the value of fringe order, N, at each point in the model.

Due to the relative retardation, the two waves are no longer in phase when emerging from the model. The analyzer will transmit only one component of each of these beams in the direction of its axis. These waves will interfere with each other and the resulting light intensity, *l*, becomes

$$I = a^2 \sin^2 (2 \beta) \sin^2 (\pi \delta \lambda)$$
(2)

where

- *a* = the amplitude of the light,
- β = the angle between the axis of polarizer and the direction of principal stresses,
- λ = the wavelength of the light, and
- δ = the relative retardation.

In a plane polariscope, the directions of principal stresses can be measured from the above equation. The light intensity becomes zero with 2β equals $N\pi$, where N = 0,1,2,3, etc. This phenomenon implies that, when β is any exact multiple of $\pi/2$, the direction of either principal stress coincides with the axis of the polarizer. When the entire model is viewed, a dark fringe pattern, known as isoclinic fringe pattern, occurs. The fringes are loci of points for which the principal stress directions coincide with the axis of the polarizer.

When the relative retardation equals $N\lambda$, the extinction of light intensity also occurs. The fringe order, N, indicates the extent of retardation and is dependent upon the magnitude of the principal stress difference ($\sigma_1 - \sigma_2$), the thickness of model (*t*), and the material strain-optical property (*K*). The fringe lines of this type are known as isochromatic fringes.

During a photoelastic experiment, isoclinic and isochromatic fringe patterns are superimposed. When the stress differences over the entire model are to be measured, the isoclinic lines have to be suppressed by placing two quarter-wavelength plates at 45° to the axes of the polarizer and the analyzer. Through this arrangement, the isoclinic lines can be eliminated from the fringe pattern in the model.

LABORATORY PHOTOELASTIC EXPERIMENT

A two-dimensional photoelastic model of underground chambers was studied under a transmission polariscope with two quarter-wave plates in place. The study investigated changes of the stress fields around rectangular openings in response to different underground excavations such as failure of a pillar support and enlargement of an opening. A model mimicking multiple underground chambers was constructed by cutting four equally spaced rectangles on a PSM-1 photoelastic sheet (Figure 2A). The plastic material, having an elastic modulus of 2.5 GPa and a Poisson's ratio of 0.38, provided a reasonable sensitivity to the change of surface strain.

The photoelastic material was loaded uniaxially, and stress concentrations created as a result of the openings were investigated and photographed. Throughout the experiment, the center support of the model was constantly modified to reflect a possible pillar failure, opening width increase, or roof enlargement (Figures 2B to 2D). These problems might be encountered during the construction of an underground power plant, underground storage chamber, or other general subsurface geotechnical excavations.

Theoretically, a fringe line should be a line of zero width. However, due to the characteristics of either visual observation or photographic device, the fringes recorded are shown as bands of dark fields (Figures 2A to 2D). The band width of the isochromatic fringe can be reduced by a mechanical technique which employs partial mirrors in a circular polariscope. It may also be achieved by a digital process which provides a better definition



FIG. 2. Digitized fringe images. (A) Four equally spaced openings, Fourrooms; (B) failure of center pillar, Pillar; (C) removal of center support, Wide-room; and (D) enlargement of opening, Large-room

of the exact locations of those fringe lines, and the approach is discussed in the text.

The photographs of the dark-field fringes were scanned by an Eikonixscan 78/99 camera at the Alaska Data Visualization and Analysis Laboratory at the University of Alaska Fairbanks. The digitized fringe images were further processed to enhance the variations of stresses around the openings under various engineering conditions, and to heighten the fractional fringe orders for measurements of the boundary stress concentrations.

IMAGE PROCESSING METHODS

Image processing was performed on a Comtal/3M image processor, with a VAX 11/750 as the host computer. The Land Analysis System (LAS) image processing software package was utilized to enhance the fringe patterns for a texture analysis, and to superimpose fringe images for a comparison of the changes of stress concentrations. A number of digital processing techniques — including contrast enhancement, image registration, signal processing, data transformation, and logical processing — were performed.

The photoelastic photographs recorded directly from the polariscope were not in the exact scale. Difficulties arose when the fringe images of different excavation models were superimposed. An alternative that circumvented this problem was the use of an image-to-image registration process, in which the positions of the corners of rectangular rooms on the other images (Figure 2B to 2D) were referenced to the coordinate system of Figure 2A.

The signal processing routines of the LAS package were used to analyze image data in the frequency domain. In this approach, an image is separated into its various spatial frequency components through application of a mathematical transformation operation. The signal processing investigated in this study included complex image processing, a 1-D and 2-D Fourier transformation, and a Gauss transformation. Among these three methods, it was noted that the Gauss transformation produced much better defined fringe lines for the cases studied (Figures 3A to 3D).

The Gauss transformation operated on the original fringe im-

age, and produced an output image according to the following alogrithm:

$$O(X,Y) = A \cdot e^{-B \cdot [I(X,Y) - MEAN]}$$
(3)

where

O(X, Y) = the output pixel value at location (X, Y), I(X, Y) = the input pixel value at location (X, Y), MEAN = the average pixel value of the whole image, and

A,B = the transformation parameters.

The coefficient A in Equation 3 was chosen as 100 for all four cases, and coefficient B was also chosen in a range from 0.005 to 0.009. The mean pixel value of each image was determined from the cumulative frequency curve of the image statistics. Table 1 lists the data used in the process.

Comparing Figure 2 with Figure 3, it is apparent that Gauss transformation functions enhance the edges of fringe lines and suppress the areas where the changes of pixel values are not significant. The exact locations of the fringe lines, which indicated areas of constant stress differences in the photoelastic model, were determined as lines in the middle of those curves outlining the fringes.

The spatial process, which emphasized or de-emphasized image pixels of various spatial frequencies, was also used to vary the tonal changes on the images (Lillesand and Kiefer, 1987). Three techniques were tested, namely (1) convolution filtering, (2) frequency filtering, and (3) texture analysis.

The convolution process involved a 3 by 3 moving average filter. An output image was derived by multiplying each coef-

TABLE 1. SUMMARY OF COEFFICEINT VALUES FOR GAUSS TRANSORMATION FUNCTION (EQUATION 3).

Figure	Image	Α	В	MEAN
3A	Four-rooms	100	0.005	125
3B	Pillar	100	0.007	144
3C	Wide-room	100	0.009	66
3D	Large-room	100	0.008	149

ficient in the filter by the corresponding pixel value in the original image, and assigning the sum of products to the pixel of the output image at the center of the filter.

Frequency filtering, a special application of convolution, was applied to different frequenies to enhance low frequency areas, including the dark or light fringes. High frequency features such as edges of fringe lines were also enhanced. Three levels of frequencies were applied, and the output images did not show any improvement of fringe patterns from the originals.

Texture analysis, a study of frequency of contrast change, generates a new image by an aggregation of features that may be too subtle to be observed individually on the image (Campbell, 1983). These features might include the center of the darkfield or light-field fringes. Among the texture analysis routines, the gradient process generated images with enhanced fringe patterns (Figures 4A to 4D). The gradient process performed an edge enhancement on the original fringe image by applying a spatial gradient function which accentuated areas of rapid changes of pixel values (Dougherty and Giardina, 1987).

In the photoelastic stress analysis, the fringe patterns were examined for the locations of critically stressed areas. The interpretation of results followed a routine procedure which involved identification of the fringe orders and determination of stress differences based upon those fringe orders. Figures 4A to 4D show enhanced isochromatic fringes at the *N* and *N*/2 positions. This gradient process was of particular significance in the stress analysis because the fractional fringe orders (*N*/2), normally obtained from two conventional dark-field and light-field photoelastic photographs, were contained in these processed images. These fractional fringe orders are valuable in defining the locations of stress concentrations around underground openings.

PHOTOELASTIC STRESS ANALYSIS

The fringe orders around the openings were determined from the processed images, and the corresponding stress differences were calculated from Equation 1. Distributions of the stress differences in terms of the fringe order are shown in Figures 5A and 5D.

The image of Figure 5A illustrates the stresses around open-



Fig. 3. Enhanced fringe patterns by Gauss transformation functions. (A) Four-rooms, (B) Pillar, (C) Wide-room, and (D) Large-room.

FIG. 4. Edge enhancement of fringe lines by Gradient process. (A) Fourrooms, (B) Pillar, (C) Wide-room, and (D) Large-room.



FIG. 5. Stress concentrations around the openings. (A) Four-rooms; (B) Pillar; (C) Wide-room; and (D) Large-room ($\Delta \sigma^*$: stress diffrence in terms of fringe order, N).

ings and the boundary stress concentrations for a row of four rectangular openings with width-to-height ratio of 4.0. From the diagram, it was observed that a relatively large positive stress difference occurred around the right-angled corners. The magnitude of the positive (compressive) stress gradually reduced as it approached the sidewalls of the openings and it changed to a negative (tensile) stress around the crown of the openings. The tensile stress within the roof extended to a distance of 5 times the opening height. An overstressed zone close to the surface of the pillar was observed, and it diminished at the center of the support.

Figure 5B illustrates the boundary stress concentrations in response to failure of the center supporting pillar. The compressive stress around the corner of the new, larger opening as compared with the curve of Figure 5A indicated the development of a much higher magnitude stress level. The increase of stresses was due to the redistribution of the load which was previously carried by the center pillar. Redistribution of the load reduced the tensile stress around the crown of the opening, and slightly increased the stress concentration within the supporting pillar. Removal of the center support, as shown in Figure 5C, increased the area of disturbance above the enlarged opening and caused a small zone of compression within the disturbed roof. It did not, however, drastically increase the concentrations of boundary stresses.

Figure 5D shows the distribution of stresses around a larger room with width-to-height ratio of 0.77. The stress pattern was appreciably different from the previous three cases discussed. Tensile stress occurred at the crown of the room and within the roof. Stress became compressive, however, towards the sides of the opening. The maximum boundary compressive stress around the ovaloidal-shaped roof was less than that around a right-angled corner. The fringe order of the highest compressive boundary stress was found to be about 2.8 and the fringe order of the maximum tensile stress was less than 2.0. It is apparent that the stress distribution is sensitive to changes in opening geometry.

CONCLUSIONS

Digital image processing techniques, especially the methods involving analysis of frequency of contrast change, were able to enhance the photoelastic fringe patterns on images. The gradient process was, among a number of image processing routines tested, an applicable technique to generate an enhanced fringe image. The method also provided additional information which allowed differentiation of fractional fringe orders and associated stress concentrations. This approach is also useful in other engineering applications for which photoelastic stress analysis is used to establish design criteria.

Photoelastic stress analysis indicated that tensile stress occurs around the crown of rectangular openings and it became compressive on moving towards sidewalls of the openings. The stress concentration near the right-angled corner increased due to the failure of the center pillar. Removal of the center support did not, however, change the stress distribution significantly as compared with the case of a failed pillar. An increase of room height caused a totally different stress pattern. Tensile stress occurred at and near the crown of the roof and diminished as it approached the sidewalls. It was observed that the stress distribution was also sensitive to changes in opening geometry.

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