# Precision Rectification of SPOT Imagery 

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#### Abstract

A model for describing the geometry of the SPOT satellite imaging process is formulated and evaluated. From this model, collinearity equations are derived. A least-squares solution to these equations is then formulated. To evaluate the method, it is applied to panchromatic scenes in two cases. The results of the tests show that subpixel accuracy can be achieved with as few as one control point, while with more control it is possible to achieve RMS errors lower than $1 / 2$ pixel relative to maps, and lower than $1 / 4$ pixel relative to a reference scene.


## INTRODUCTION

TOPOGRAPHIC MAPPING FROM SPACE IMAGERY has become possible on an operational basis using the SPOT system. Key features are high resolution and stereo viewing capabilities, coupled with unrestricted commercial availability. Achievable geometric accuracy is also of great importance.

The raw SPOT scene suffers from a number of geometric distortions. These are due to factors such as satellite orbit and attitude variations, Earth shape and rotation, sensor geometry, ect. A priori estimates of most of these factors are available and are used to produce bulk corrected scenes (level 1B). The intrinsic geometric quality of the bulk corrected scene is rather good (length distortion $\approx 0.15$ percent) but scene localization RMS error is around 800 m (Boissin et al., 1986). Other drawbacks are that errors due to attitude variations during the recording of the scene remain uncorrected. New distortions are also introduced because the bulk corrected scene is always produced in a local scene-dependent map projection instead of the mapping system in which the data are to be used. Applications demanding subpixel accuracy thus call for a special precision correction procedure.

The rectification procedure can be divided into two steps: modeling and resampling. During the modeling phase, a priori data and ground control points are used to establish a mathematical model that relates each raw image pixel to the desired coordinate system. Intensity values for the pixel locations in the ouput system are then calculated by the resampling process.
A commonly used modeling technique is to apply low order polynomials to the bulk corrected image. The modeling is thus made in two steps (see Figure 1), the first step being performed using a priori data at the receiving station. The advantage of this approach is one of simple implementation. No knowledge of the satellite system is necessary in the second step. The disadvantages are, however, several. The image will be resampled twice, causing unnecessary radiometrc degradation. As the polynomial is an improper model of the physical causes of the distortions, a large number of well distributed ground control points are necessary to avoid degradation of the model in some part of the image. Ortho images cannot be produced because of the high frequency nature of the terrain displacements.
A better result is achieved by combining a priori data with ground control point measurements in a simultaneous adjust-


FIG. 1. Polynomial modeling approach.
ment, updating the satellite model parameters (see Figure 2). The data then only need to be resampled once. The required number of ground control points will be lower, which is important when the control is acquired by expensive geodetic measurements. As the viewing geometry is determined in the adjustment, a digital elevation model (DEM) can be used to eliminate terrain displacements, making ortho image production possible. When not using a DEM, the image can be projected onto a reference surface, leaving the terrain displacements unaltered. This last aspect is of great importance for further processing of stereo imagery for parallax measurements.

A number of papers have been published on different approaches to satellite modeling of Landsat imagery (Sawada et al., 1981; Friedman et al., 1983; Salamonowicz, 1986). Konecny et al. (1987) treat the correction of SPOT data, but use a somewhat different approach, adapted to analytical photogrammetric instruments.

In this paper a method will be developed to determine the satellite imaging geometry for SPOT imagery. The method will then be applied in two cases of correction of panchromatic scenes, and the resulting accuracy of the resampled images will be evaluated.

## SPOT IMAGING CHARACTERISTICS

SPOT 1 was launched on 22 February 1986 into a near circular sunsynchronous orbit. It carries two instruments (HRV1 and HRV2) which independently image the Earth in either panchromatic $(\mathrm{P})$ or multispectral (XS) mode. The sensors are constructed using CCD arrays. The panchromatic sensor uses an array of 6000 detectors, while the multispectral sensor has 3000 detectors per spectral band. In vertical viewing, the detector arrays image a $60-\mathrm{km}$ long line of the Earth surface perpendicular to the satellite ground track. The sensor is sampled at $1.5-\mathrm{ms}$ intervals in P mode, and at 3-ms intervals in XS mode. This gives a nominal pixel size of 10 m in a panchromatic image and 20 m in a multispectral image.

A SPOT scene covers a nominal area of 60 km by 60 km . The stream of data is segmented into scenes by combining 6000 lines of P data or 3000 lines of XS data around predefined latitudes.


FIG. 2. Satellite modeling approach.

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These latitudes are chosen so that there will be a small overlap between consecutive scenes.
Each instrument has a pointable mirror, allowing for off nadir viewing in the crosstrack direction. The mirrors can be tilted in steps of 0.6 degrees, up to a maximum of 27 degrees from the vertical. The mirror position is, however, always kept fixed during the recording of one scene. At 27 degrees of nadir viewing, the scene size will increase to approximately 60 km by 80 km .

The attitude control system was designed to keep the attitude within 0.15 degrees of the nominal. The satellite also carries sensors measuring the attitude angular velocities. These angular velocities are downlinked together with the image data and are available for use in the correction process. However, as only velocities are measured, there still remains a constant unknown attitude offset.

Orbit determinations are regularly performed, and predicted ephemeris are available on the CCT. The combined effect of the unknown attitude offset and errors in the predicted orbit results in an RMS deviation of around 400 m between the true and the predicted scene positions.

## MODELING APPROACH

The elliptic Keplerian orbit can be described by six independent parameters. One possible set of these parameters is (see also Figure 3)
$a$, half major axis
$e$, eccentricity
$i$, inclination
$\Omega$, right ascension of the ascending node
$\omega$, argument of the perigee
$M$, mean anomaly
In reality, the orbit deviates from the elliptic form due to disturbing noncentral forces. These forces generate time dependent variations in the orbital parameters. The second-degree zonal component $J_{2}$ of the Earth gravitational potential contributes the major part of these forces. The first order perturbations
caused by $J_{2}$ are given by (Kaula, 1966)

$$
\begin{align*}
\dot{\Omega} & =\frac{-3 n J_{2} a_{e}^{2}(\cos i)}{2\left(1-e^{2}\right)^{2} a^{2}}  \tag{1}\\
\dot{\omega} & =\frac{3 n J_{2} a_{e}^{2}\left(5 \cos ^{2} i\right)}{4\left(1-e^{2}\right)^{2} a^{2}}  \tag{2}\\
\dot{M} & =\frac{3 n J_{2} a_{c}^{2}\left(3 \cos ^{2} i-1\right)}{4\left(1-e^{2}\right)^{\frac{3}{2}} a^{2}}+\mathrm{n} \tag{3}
\end{align*}
$$

where $a_{c}$ is the Earth's semi-major axis and $n$ is the mean motion. The first-order perturbations of $a, e$, and $i$ are all zero. The orbit can then be described by the osculating elements

| $a$, | half major axis <br> $e$, <br> eccentricity <br> inclination |
| :--- | :--- |
| $\Omega=\Omega_{0}+\dot{\Omega} t$, | right ascension of the ascending node. |
| $\omega=\omega_{0}+\dot{\omega} t$, | argument of the perigree. |
| $M=M_{0}+\dot{M} t$, | mean anomaly |

None of these parameters is explicitly given on a SPOT CCT. Instead, a predicted ephemeris is given, describing the position and velocity of the satellite during the timespan of the acquisition of a few scenes. This ephemeris is used to estimate an initial orbit.

In this study, a simplified orbital model is used. It is assumed that, during the timespan of one scene, the orbit can with sufficient accuracy be approximated by a plane, circular orbit. The orbital radius is, however, allowed to vary with time to account for the elliptic form of the orbit. The radial shape of the orbit is determined by fitting a third-order polynomial in time to the orbital radii derived from the ephemeris. This shape is considered fixed, except for the constant term. The set of orbital elements to be estimated will thus be reduced to four:
$i$, inclination
$\Omega, \quad$ right ascension of the ascending node
$t_{0}$, time at the ascending node
$r_{0}$, orbital radius at $\mathrm{t}=\mathrm{t}_{0}$
The central travel angle is derived by

$$
\begin{equation*}
\nu=\left(t-t_{0}\right) 2 \pi / P \tag{4}
\end{equation*}
$$

where $P$ is the orbit period. Initial values for all these parameters are easily derived by least-squares adjustment to the ephemeris data.

In addition to the orbital parameters, satellite attitude also needs to be estimated. As the attitude angular velocities are measured on board, relative attitude angles can be calculated by integration. The approach take here is to assume that these measurements are of sufficient accuracy, leaving only the constant terms to be estimated.

$$
\begin{array}{ll}
\omega(t)=\omega_{0}+\Delta \omega(t) & \text { (roll) } \\
\rho(t)=\rho_{0}+\Delta \rho(t) & \text { (pitch) } \\
\kappa(t)=\kappa_{0}+\Delta \kappa(t) & \text { (yaw) }
\end{array}
$$

The orbit parameter model chosen will thus result in seven elements ( $i, \Omega, t_{\mathrm{o}}, r_{\mathrm{o}}, \omega_{\mathrm{o}}, \rho_{\mathrm{o}}, \kappa_{\mathrm{o}}$ ) to be adjusted.

## INVESTIGATION OF APPROXIMATION ERRORS

To verify that errors caused by the approximations inherent in the simplified orbit model are insignificant in the timespan of one scene, their sizes will be estimated using a simulated orbit with the following mean orbital parameters and Earth constants:
$a=7200550 \mathrm{~m}$
$e=0.0011$
$i=98.72$ degrees
$a_{c}=6378153 \mathrm{~m}$
$J_{2}=1.08264 \times 10^{-3}$
Inserting these into Equations 1 through 3 gives the following orbital perturbations:

$$
\begin{aligned}
& \dot{\Omega}=0.199605 \times 10^{-6} \mathrm{~s}^{-1} \\
& \dot{\omega}=-0.582649 \times 10^{-6} \mathrm{~s}^{-1} \\
& \dot{M}=n-0.612910 \times 10^{-6} \mathrm{~s}^{-1}
\end{aligned}
$$

As the sizes of the errors vary with the position in the orbit and with the position of perigee, they were calculated at regular intervals during a complete descending pass. The approximation errorrs can be separated into a number of categories. There are three main sources of errors that were investigated:

## Effect 1

The model orbit is plane, while the true orbit deviates from a plane due to the osculating element $\Omega$. This causes the true satellite position to deviate in a direction perpendicular to the model plane. The size of this deviation can be estimated by adjusting an orbit plane to the first and last state vectors and then measuring the distance from the true satellite position to the orbit plane at the scene center. This distance is projected to the Earth is surface to give the error in the scene.

## Effect 2

For the same reason as above, the satellite heading will deviate from that of the model plane. This will cause a small error in yaw. Using an orbit plane adjusted as above, we get a yaw error which has its maxima at the scene start and stop. The position error on the ground is calculated at the east and west borders of the scene, as the effect of a yaw error is largest there.

## Effect 3

The circular orbit approximation also means that we replace the true anomaly by the mean anomaly, as the central travel angle is linear with time (Equation 4). The true anomaly is not linear with time, which will result in an along track position error. If the mean anomaly is fitted to the first and last state vectors, there will be a perfect fit at the scene start and stop, while the largest along track position error will occur close to the scene center. (The values calculated refer to an argument of perigee equal to zero.)

The results of these calculations are shown in Figure 4. The conclusion drawn is that, as none of these errors exceeds 0.2 m


Fig. 4. Orbit approximation errors as a function of latitude.
anywhere in the orbit, the simplified orbit model is a sound approximation within one SPOT scene.
One other source of error which is worthwhile investigating is that due to the assumption of error-free attitude angular velocity measurements, leaving only the constant attitude offset to be estimated. The angular velocities are sampled every 125 ms with an accuracy corresponding to a standard error of $6.25 \times 10^{-5}$ degrees/s. Calculating the attitude change over the scene by integrating the angular velocities, assuming their errors to be independent, results in an estimated RMS error in the total attitude change amounting to $\sqrt{ } n(125 \mathrm{~ms})\left(6.25 \times 10^{-5}\right.$ degrees $\left./ \mathrm{s}\right)=6.6$ $\times 10^{-5}$ degrees (where $n$ is 72 , which is the number of attitude measurements in a scene). If we transform this to a relative position error on the ground, they will be largest between the scene top and bottom borders. There they will correspond to 0.96 m for roll and pitch and to 0.03 m for yaw.

The error caused by yaw is clearly insignificant. The roll and pitch errors are much larger, but still small enough to be practically impossible to model even in extreme cases.

## REFERENCE SYSTEM

A number of coordinate systems need to be defined for use in a mathematical formulation of the adjustment problem.

## The Earth Centered Inertial Coordinate System (ECI)

This is the system to which the orbital parameters are referred. It has its origin at the Earth center of mass. Its $X$-axis points to the Vernal Equinox, the Z-axis points to the celestial north pole, while the $Y$-axis completes a right-handed system.

## The Local Orbital Reference System

This is a moving coordinate system with its origin defined at the satellite's center of mass. The Z-axis is pointing in the same direction as the satellite position vector, given in the ECI system. The $X$-axis is perpendicular to the $Z$-axis and in the orbital plane, pointing in the direction of satellite motion. The $Y$-axis completes a right-handed system.

## The Attitude Measurement Reference System

This system is fixed in relation to the satellite body. It coincides with the local orbital reference system when the attitude angles are all zero.

## The Sensor Coordinate System

The coorinates of the detectors are referred to this system. The origin is positioned at the perspective center. The $y$-axis is parallel to the array of detectors, pointing eastward in a descending pass. The $z$-axis is perpendicular to the $y$-axis, directed from the array towards the perspective center. The $x$-axis completes a right-handed system (pointing in the direction of satellite motion). As the detector array is one-dimensional, the detector coordinates will always have a zero $x$-coordinate, and a constant $z$-coordinate.

## The SPOT Ephemeris Reference System

The ephemeris from the SPOT CCT is given in an Earth-centered, Earth-fixed system (ECEF) using the International Ellipsoid 1980. The ephemeris has to be transformed to the ECI system before it can be used in orbital parameter calculations.

## The Ground Control Point Reference System

This is the local geodetic system in which the ground control is measured. This is related to some datum whose relation to the SPOT ephemeris reference system is not usually exactly known. The unknown offset will, however, be absorbed by the orbital parameters in the adjustment.

A set of orthogonal rotation matrices are used in transformations between these systems.

The sensor-body transformation $\mathbf{R}_{B S}$ relates the sensor coordinate system to the attitude reference system. It accounts for the mirror angle, as well as for constant alignment offsets between the individual sensors. It is thus constant within one scene for one particular sensor.

The body-flight transformation $\mathrm{R}_{\mathrm{FB}}$ relates the attitude reference system to the orbital reference system. It accounts for the attitude deviations. It is thus a function of time within one scene. It is defined (when using the shorthand $c$ for $\cos$ and $s$ for $\sin$ ) by

$$
\begin{gather*}
\mathbf{R}_{F B} \\
=\left[\begin{array}{rrr}
\mathrm{c}(\kappa) \mathrm{c}(\rho) & \mathrm{c}(\kappa) \mathrm{s}(\rho) \mathrm{s}(\omega)+\mathrm{s}(\kappa) \mathrm{c}(\omega) & -\mathrm{c}(\kappa) \mathrm{s}(\rho) \mathrm{c}(\omega)+\mathrm{s}(\kappa) \mathrm{s}(\omega) \\
-\mathrm{s}(\kappa) \mathrm{c}(\rho) & -\mathrm{s}(\kappa) \mathrm{s}(\rho) \mathrm{s}(\omega)+\mathrm{c}(\kappa) \mathrm{c}(\omega) & \mathrm{s}(\kappa) \mathrm{s}(\rho) \mathrm{c}(\omega)+\mathrm{s}(\kappa) \mathrm{s}(\omega) \\
\mathrm{s}(\rho) & -\mathrm{c}(\rho) \mathrm{s}(\omega) & \mathrm{c}(\rho) \mathrm{c}(\omega)
\end{array}\right] \tag{5}
\end{gather*}
$$

The flight-inertial transformation $\mathbf{R}_{l /}$ relates the orbital reference system to the ECl systems. It is a function of the orbital parameters and thus a function of time within one scene. It is defined by

$$
\begin{gather*}
\mathbf{R}_{i \mathrm{~F}} \\
=\left[\begin{array}{ccl}
-\mathrm{c}(\Omega) \mathrm{s}(\nu)-\mathrm{s}(\Omega) \mathrm{c}(i) \mathrm{c}(\nu) & \mathrm{s}(\Omega) \mathrm{s}(i) & \mathrm{c}(\Omega) \mathrm{c}(\nu)-\mathrm{s}(\Omega) \mathrm{c}(i) \mathrm{s}(\nu) \\
-\mathrm{s}(\Omega) \mathrm{s}(\nu)+\mathrm{c}(\Omega) \mathrm{c}(i) \mathrm{c}(\nu) & -\mathrm{c}(\Omega) \mathrm{s}(i) & \mathrm{s}(\Omega) \mathrm{c}(\nu)+\mathrm{c}(\Omega) \mathrm{c}(i) \mathrm{s}(\nu) \\
\mathrm{s}(i) \mathrm{c}(\nu) & \mathrm{c}(i) & \mathrm{s}(i) \mathrm{s}(\nu)
\end{array}\right]
\end{gather*}
$$

where $\nu$ is defined by Equation 4.

## DERIVATION OF COLLINEARITY EQUATIONS

Each observation of a ground control point will give rise to a set of two collinearity equations. These will be derived from the following relationship between the satellite position and the ground control point in the ECI system (see Figure 5):

$$
\begin{equation*}
\mathbf{x}=\mathbf{p}+d \cdot \hat{\mathbf{u}} \tag{7}
\end{equation*}
$$

where
$\mathrm{x}=$ ground control point position vector,


FIG. 5. The geometry of a control point observation.
$\mathrm{p}=$ satellite position vector, and
$\hat{\mathbf{u}}=$ the unit vector pointing from the satellite to the control point.
This can be developed further using the rotation matrices defined in the previous chapter. The equation relating the control point in the ECI system to its image in the sensor system is

$$
\begin{equation*}
\mathbf{x}=\mathbf{p}+d \cdot \mathbf{R}_{I F} \cdot \mathbf{R}_{F B} \cdot \mathbf{R}_{B S} \cdot \mathbf{x} \tag{8}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathbf{x}_{s}=\text { vector in the sensor system to the detector imaging the } \\
& \text { control point, and } \\
& d=\text { scaling factor. }
\end{aligned}
$$

If we invert Equation 8 to express the sensor coordinate as a function of the ECl ground control coordinate, we get

$$
\begin{equation*}
\mathbf{x}_{s}=d^{-1} \cdot \mathbf{M} \cdot(\mathbf{x}-\mathbf{p}) \tag{9}
\end{equation*}
$$

or

$$
\left[\begin{array}{c}
0  \tag{10}\\
y- \\
-f
\end{array}\right]=d^{-1}\left[\begin{array}{lll}
m_{11} & m_{12} & m_{13} \\
m_{21} & m_{22} & m_{23} \\
m_{31} & m_{32} & m_{33}
\end{array}\right]\left[\begin{array}{l}
x-x_{p} \\
y-y_{p \prime} \\
z-z_{p}
\end{array}\right]
$$

where

$$
\begin{aligned}
\mathbf{M} & =\left(\mathbf{R}_{l F} \cdot \mathbf{R}_{F B} \cdot \mathbf{R}_{B S}\right)^{\top} \text { and } \\
f & =\text { principal distance. }
\end{aligned}
$$

Dividing the first and second equation by the third in Equation 10 results in the collinearity equations:

$$
\begin{align*}
0 & =-f \frac{\left[m_{11}\left(X-X_{\psi}\right)+m_{12}\left(Y-Y_{t}\right)+m_{13}\left(Z-Z_{\psi}\right)\right]}{\left[m_{31}\left(X-X_{p}\right)+m_{32}\left(Y-Y_{t}\right)+m_{33}\left(Z-Z_{\psi}\right)\right]}  \tag{11}\\
y_{*} & =-f \frac{\left[m_{21}\left(X-X_{\psi}\right)+m_{22}\left(Y-Y_{p}\right)+m_{23}\left(Z-Z_{\psi}\right)\right]}{\left[m_{31}\left(X-X_{t}\right)+m_{32}\left(Y-Y_{t}\right)+m_{33}\left(Z-Z_{k}\right)\right]} \tag{12}
\end{align*}
$$

The most important difference from the ordinary collinearity equations in aerial photogrammetry is that the sensor $x$-coordinate is always zero, and that all parameters on the right side $(\mathbf{M}, \mathbf{x}, \mathbf{p})$ are functions of time.

## ADJUSTMENT

Each control point measurement will give rise to a measurement vector consisting of five observations, two coordinates from the raw image and three coordinates from ground. The measurement in the image will primarily result in the fractional pixel and line number for the control point image position. The fractional pixel will be converted to the $y$, coordinate in the image coordinate system, while the fractional line is converted to time of observation. The observation vector for the $k^{\text {th }}$ control point will thus be

$$
\begin{equation*}
\mathbf{I}_{k}^{\circ \circ}=\left[y_{k}^{\circ \circ} t^{\circ \circ} \phi^{\circ \circ} \lambda^{\circ \circ} h^{\circ \circ}\right]_{k}^{T} \tag{13}
\end{equation*}
$$

where

$$
\begin{array}{ll}
y_{s} \quad= & \text { coordinate for the fractional detector position im- } \\
& \text { aging the control point, } \\
t \quad & \text { time for the event of control point imaging, } \\
\phi & =\text { latitude of the control point, } \\
\lambda & =\text { longitude of the control point, and } \\
h & =\text { control point elevation above the ellipsoid. }
\end{array}
$$

The reason for using geographic coordinates for the control point, instead of first transforming them to ECEF Cartesian coordinates, is to be able to use a proper a priori weight matrix for the measurement.

The parameter vector will consist of the four elements of the simplified orbital model, and the three elements of attitude: i.e.,

$$
\begin{equation*}
\mathbf{p}=\left[i \Omega t+\circ r^{\circ} \omega^{\circ} \rho^{\circ} \kappa^{\circ}\right]^{\mathrm{T}} . \tag{14}
\end{equation*}
$$

We will start with a set of estimated values, where the four orbital parameters are derived from the ephemeris, while the three attitude angles are set to zero: i.e.,

$$
\mathbf{p}^{\circ \circ}=\left[\begin{array}{lllll}
i^{\circ \circ} & \Omega^{\circ \circ} t^{\circ} r^{\circ} & 0 & 0 & 0
\end{array}\right]^{\mathrm{T}} .
$$

The measurements and parameters are related to each other by the collinearity equations. As this relationship is nonlinear, it has to be linearized by Taylor series expansion before we can formulate the condition equations of adjustment to be solved by iteration. First we reformulate Equations 11 and 12: that is,

$$
\begin{array}{ll}
F_{1}(\mathbf{M}, \mathbf{x}, \mathbf{p}) \equiv \mathbf{m}_{1 \bullet} \cdot(\mathbf{x}-\mathbf{p}) & =0 \\
F_{2}(\mathbf{M}, \mathbf{x}, \mathbf{p}) \equiv-f \cdot \mathbf{m}_{\mathbf{2}} \cdot(\mathbf{x}-\mathbf{p})-y \cdot \mathbf{m}_{3 \bullet} \cdot(\mathbf{x}-\mathbf{p})=0 \tag{15}
\end{array}
$$

where

$$
\begin{aligned}
& \mathbf{m}_{1 \bullet}=\left[\begin{array}{lll}
m_{11} & m_{12} & m_{13}
\end{array}\right] \\
& \mathbf{m}_{2 \bullet}=\left[\begin{array}{lll}
m_{21} & m_{22} & m_{23}
\end{array}\right] \\
& \mathbf{m}_{3 \bullet}=\left[\begin{array}{lll}
m_{31} & m_{32} & m_{33}
\end{array}\right]
\end{aligned}
$$

are the rows of the matrix $\mathbf{M}$, We now expand $F_{1}$ and $F_{2}$ into Taylor series around approximate values $\mathrm{I}^{\circ}$ and $\mathrm{p}^{\circ}$ for the measurements and parameters (using $I^{\circ \circ}$ and $p^{\circ \circ}$ in the first iteration). Second and higher order terms are skipped. The firstorder derivatives are collected in the matrices

$$
\begin{gather*}
\mathbf{A}_{k}=\left[\begin{array}{c}
\frac{\partial F_{1}}{\partial y_{s}} \frac{\partial F_{1}}{\partial t} \frac{\partial F_{1}}{\partial \phi} \frac{\partial F_{1}}{\partial \lambda} \frac{\partial F_{1}}{\partial h} \\
\frac{\partial F_{2}}{\partial y_{s}} \frac{\partial F_{2}}{\partial t} \frac{\partial F_{2}}{\partial \phi} \frac{\partial F_{2}}{\partial \lambda} \frac{\partial F_{2}}{\partial h}
\end{array}\right]  \tag{17}\\
\mathbf{B}_{k}=\left[\begin{array}{c}
\frac{\partial F_{1}}{\partial i} \frac{\partial F_{1}}{\partial \Omega} \frac{\partial F_{1}}{\partial t_{\mathrm{o}}} \frac{\partial F_{1}}{\partial r_{\mathrm{o}}} \frac{\partial F_{1}}{\partial \omega_{\mathrm{o}}} \frac{\partial F_{1}}{\partial \rho_{\mathrm{o}}} \frac{\partial F_{1}}{\partial \kappa_{\mathrm{o}}} \\
\frac{\partial F_{2}}{\partial i} \frac{\partial F_{2}}{\partial \Omega} \frac{\partial F_{2}}{\partial t_{\mathrm{o}}} \frac{\partial F_{2}}{\partial r_{\mathrm{o}}} \frac{\partial F_{2}}{\partial \omega_{\mathrm{o}}} \frac{\partial F_{2}}{\partial \rho_{\mathrm{o}}} \frac{\partial F_{2}}{\partial \kappa_{\mathrm{o}}}
\end{array}\right]  \tag{18}\\
\mathbf{A}=\left[\begin{array}{l}
\mathbf{A}_{1} \\
\mathbf{A}_{2} \\
\cdot \\
\varnothing \\
\mathbf{A}=\left[\begin{array}{l}
\mathbf{A}_{n} \\
\mathbf{B}_{2} \\
\cdot \\
\cdot \\
\mathbf{B}_{n}
\end{array}\right]
\end{array} .\right. \tag{19}
\end{gather*}
$$

The condition equations for the first iteration could then be formulated as

$$
\begin{equation*}
\mathrm{A} v+\mathrm{B} \Delta=\mathbf{f}^{\circ} \tag{21}
\end{equation*}
$$

where
$\mathbf{f}^{\circ}=-F\left(\mathbf{I}^{\circ}, \mathbf{p}^{\circ}\right)$,
$\mathbf{v} \quad=$ measurement residuals, and
$\Delta \quad=$ corrections to the approximate parameters $\mathbf{p}^{\circ}$.
With this formulation we would, however, find the system impossible to solve due to singularity caused by the high correlation among some of the parameters. To stabilize the system we have add $\mathbf{p}^{\circ \circ}$ as a set of fictitious observations on the unknown parameters. The condition equations (for the first iteration) can then be reformulated as

$$
\left[\begin{array}{cc}
\mathbf{A} & 0  \tag{22}\\
0 & \mathbf{I}
\end{array}\right]\left[\begin{array}{l}
\mathbf{v} \\
\mathbf{v}_{p}
\end{array}\right]+\left[\begin{array}{c}
\mathbf{B} \\
-\mathbf{I}
\end{array}\right] \Delta=\left[\begin{array}{l}
\rho \\
0
\end{array}\right]
$$

The least-squares solution to this system can be found in Mikhail (1976): i.e.,

$$
\begin{equation*}
\Delta=\left(\mathbf{B}^{\mathrm{T}}\left(\mathbf{A Q A}^{\mathrm{T}}\right)^{-1} \mathbf{B}+\mathbf{W}_{p p}\right)^{-1} \mathbf{B}^{\mathrm{T}}\left(\mathbf{A Q A}^{\mathrm{T}}\right)^{-1} \mathbf{f}^{\mathrm{o}} \tag{23}
\end{equation*}
$$

where
$\mathrm{Q} \quad=$ a priori cofactor matrix for the measurements and
$\mathbf{W}_{p p}=$ a priori weight matrix for the parameter estimates.
The solution now has to be properly iterated until it converges. After convergence, the measurement residuals are computed from

$$
\begin{align*}
& \mathbf{v}=\mathrm{QA}^{\mathrm{T}}\left(\mathrm{AQA}^{\mathrm{T}}\right)^{-1}\left(\mathbf{f}^{\circ}-\mathbf{B} \Delta\right)  \tag{24}\\
& \text { RECTIFICATION RESULTS }
\end{align*}
$$

Two tests were designed to investigate the potential accuracy of SPOT scenes corrected by the method developed. The first test evaluates the accuracy in a scene relative to maps, using maps for collecting control points. The second evaluates the accuracy in a scene relative to a rectified reference scene from which image chips were collected and used as a control in the adjustment. The scenes and maps used for the tests were selected with great care to avoid, as far as possible, the effect of errors other than those due to the rectification method and the scenes themselves. This was achieved by using maps of very high quality, and selecting the area so that good control points could be found. The scenes in the second test where selected to minimize the effect of the differences in terrain displacements. Only panchromatic scenes were used in these tests.

## Rectification to Maps

For this test, the scene 061-229 from 18 June 1986 was selected. It was registered by HRV1 at an off nadir viewing angle of 6.4 degrees towards the east. The area is covered by modern, orthophotobased topographic maps on a scale of 1:10,000. From these maps, 84 well-defined control points could be captured using a digitizing table. The marking of the position of these points in the scene was made by manual pixelpointing, using the cursor on a display. Four-times enlargments were used, making it possible to mark them within $1 / 4$ of a pixel. Using all 84 control points in the adjustment resulted in the following RMS residual errors:

$$
\begin{aligned}
\sigma_{x} & =2.9 \mathrm{~m} \\
\sigma_{y} & =2.7 \mathrm{~m} \\
\sigma_{d} & =4.0 \mathrm{~m}
\end{aligned}
$$

where $\sigma_{\mathrm{d}}$ refers to the residuals in Euclidean distance. Due to the high redundancy in the measurements, these figures should give a good estimate of the accuracy of the scene relative to the maps. The distribution of the control points and their residual vectors are shown in Figure 6.

It would be interesting to investigate how dependent the accuracy is on the number of control points used in the


FIG. 6. Residual error distribution in the scene 061-229.


FIG. 7. RMS $X$ errors in checkpoints as a function of the number of control points used in the scene to map adjustment.
adjustment. To do this, the adjustment was repeated with a successively reduced set of control points. With decreasing redundancy, the RMS residuals are, however, no longer a good measure of the accuracy. To get a better accuracy estimate, the resampled images were evaluated using the complete set of 84 control points as check points. A chip control point library was first created by extracting small image chips, of the size 32 by 32 pixels, around each control point. Their positions in each of the resampled images were then determined by digital correlation. The subpixel position of a correlation peak was located by fitting a quadratic surface to a neighborhood around the peak. The resulting RMS error in these 84 check points is plotted as a function of the number of control points used in the adjustment in Figures 7 through 9. Another plot of interest is the comparison of RMS residual errors in the adjustments to the RMS errors in the check points. This is shown in Figure 10.


FIG. 8. RMS $Y$ errors in checkpoints as a funciton of the number of control points used in the scene to map adjustment.


FIG. 9. RMS position errors in checkpoints as a function of the number of control points used in the scene to map adjustment.


Fig. 10. RMS position errors in checkpoints, compared to the residual errors in the adjustment, as a function of the number of control points used in the scene to map adjustment.

## Registration to a Reference Scene

For this test, two acquisitions at 077-254 were used, one from 20 June 1986 and the other from 16 July 1986. They were both registered by HRV1 and at the same off-nadir viewing angle, 2 degrees to the west.
The scene from 16 July was selected as the reference scene. It was first resampled to the UTM projection using only a priori
data. As no terrain information was available for the area, the scene was projected at the ellipsoid. Then 41 image chips of the size 32 by 32 pixels were extracted from the resampled scene to be used as control points. The chips were assigned coordinates from the (local) coordinate system of the scene. The fact that the control points were assigned zero elevation does not prevent a rigorous solution in this particular case. As the two scenes were imaged at exactly the same off nadir viewing angle, the terrain displacements will be identical in both scenes, allowing us to model a perfect fit.
The object scene, from 20 June, was now rectified using these control point chips. The manual marking of control point positions could now be replaced by a digital correlation method, as described above. Using all 41 control point chips in the adjustment resulted in the following RMS residual errors:

$$
\begin{aligned}
\sigma_{x} & =1.6 \mathrm{~m} \\
\sigma_{y} & =1.6 \mathrm{~m} \\
\sigma_{d} & =2.3 \mathrm{~m}
\end{aligned}
$$

As in the test of rectification to maps, the dependency on the number of control points was evaluated by successively decreasing the number of points used in the adjustment. The resulting images were then checked by the same method, using all 41 chips as check points. The results show a similar graphical representation as in Figures 7 through 9.

## DISCUSSION OF RESULTS

The two tests were designed to investigate how accurately it is possible to model the geometry of a SPOT scene and give some clues as to where the limiting factors lie.

The first test evaluates accuracy relative to maps, using control points from maps in the adjustment. The resulting RMS residual errors ( $\sigma_{x} \approx \sigma_{y} \approx 2.8 \mathrm{~m}$ ) are not only due to errors in the image, but also affected by errors in the control points used for evaluation.

The maps used for collecting control points were orthophoto based, on a scale of $1: 10,000$. The orthophotos are produced by photogrammetric methods with an estimated radial RMS eror of 1.3 m (Kaasila et al., 1976). The control point elevations were interpolated from the $5-\mathrm{m}$ contours on the maps, with an estimated RMS error of 2 m .

Errors are also introduced when digitizing the control point position from the digitizing table. The size of these errors were estimated by repeated digitizing of a number of points. The estimated RMS errors were 1.3 m in $X$ and $Y$.

As we are marking the control points in a four-times enlarged image, we can only mark the position at the closest $1 / 4$ th of a pixel. This will cause an RMS pointing error of 0.7 m in $X$ and Y.

A significant part of the residuals in the adjustment may thus directly be caused by the limited accuracy of the control points. It is quite clear that the correction benefits from using large scale maps or geodetic measurements with control point errors not larger than 1 m .

The second test, evaluating accuracy relative to a reference scene, gives similar results (total RMS errors $\sigma_{y} \approx \sigma_{y} \approx 1.6 \mathrm{~m}$ ).

Here the control point coordinates are error free, as they are derived from the coordinate system of the reference scene. The errors in the chip correlation are, however, affecting the results. This correlation error was estimated by the following procedure. Two translated versions of the reference scene were created by resampling it with a shift of 0.2 and 0.5 pixels. Then the 41 chips were correlated to the translated scenes. The resulting RMS error, corrected for the shift, was 1.0 m in $X$ and $Y$. The real correlation error is probably slightly larger, due to radiometric differences between the two scenes, caused by almost one month separating the scenes in time.

## CONCLUSIONS

The modeling approach presented in this paper uses a simplified orbital model. Simulations of the effects of approximation errors showed that the accuracy of the model is adequate for SPOT imagery. It was shown that deviations from the Keplerian orbit due to perturbing noncentral forces are insignificant within one SPOT scene.

Formulae for least-squares estimation of the parameters of the model were derived. Two tests were then designed to investigate how accurately it is possible to model the geometry of a SPOT scene. The conclusion drawn by the results is that the potential geometric accuracy of a rectified SPOT scene is very high. Accuracies better than half a pixel can be achieved with only a small number of control points, if control point errors are kept unsignificantly small.

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