

Counting the Errors: Presentation of Aerotriangulation Residuals for Easy Evaluation

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ABSTRACT: A technique for graphically displaying the large volumes of residual error data which result from aerotriangulation adjustments is discussed. This method enables the user to view the entire error distribution, instead of just a few key statistics, when evaluating adjustment results.

INTRODUCTION

LEAST-SQUARE ADJUSTMENT COMPUTATIONS have become a routine part of contemporary photogrammetric practice. They range in application from relatively simple problems, such as interior orientation on an analytical stereo compilation instrument, to larger, more complex problems, such as aerotriangulation bundle adjustments with constraints and added parameters. The quality of these adjustments is not automatically high. Careful evaluation by the user is required to insure a sound solution through proper weighting and editing of the observations.

The information available for assessing an adjustment varies with the software being used. Some programs give very sparse diagnostics and some include sophisticated "data snooping" reports. But almost all programs list the residual errors, and they are most often the basis for accepting or rejecting an adjustment. In general, if the residuals are too big, then something needs attention.

When judging small problems, which have only a handful of residuals, a simple visual inspection of the numbers may be sufficient. By contrast, in a large aerotriangulation block, the residuals may number into the thousands, making evaluation more difficult. In fact, even a moderately sized block will yield more data than can be effectively handled manually. To help reduce the data to a more manageable level, many triangulation programs include some type of listing threshold so that only the residuals greater than a defined absolute value are printed. Another approach is to flag the larger values with characters that will catch the eye or which can be searched for in the print file with a text editor. In either case, the user must evaluate individual observations somewhat out of context with the overall error distribution.

The motivation behind the use of a printing threshold or a data flagging technique is sound. It is to get the computer to specifically point out the important items. The user may otherwise miss them, due to the sheer volume of material there is to digest. This paper will expand on that concept, within the environment of the aerotriangulation bundle adjustment.

IDENTIFYING WHAT IS IMPORTANT

With proper instruction, the computer can supply all the statistics that a user could possibly need. So one must first decide what information is critical for evaluation.

The root-mean-square error is a traditional statistic to use. It is relevant, easy to calculate, and most programs list it. There are at least three other values which are also very important, and should be listed. They are the average error, and the two extreme errors within the sets of residuals. The average, which should be near zero, indicates if there is a substantial bias in the distribution of the errors. The two extreme values represent the worst case errors in the positive and negative directions.

The above statistics should be calculated and listed separately for each of the various groups of observations, such as the ground control X , Y , and Z coordinates (three groups), and the image x and y coordinates (two groups). Some composite grouping is also valuable, such as the ground control X and Y together. Grouping the image x and y together is another possibility. In a combined adjustment of photogrammetric and survey data, the statistics for each type of survey observation should also be "broken out" in the report.

PUTTING THE IMAGES IN CONTEXT

Because the image data form a relatively large group of observations, it may not be enough to have only the critical statistics, as identified above. It may also be important to know the actual overall distribution of the error. While there may be too many residuals, making it difficult to look at each one individually, it is not difficult to look at all of them simultaneously, in a graphical representation.

Image residuals lend themselves very nicely to graphic display in the elementary format of a histogram. They can easily be classified into error intervals. The number of observations in each interval is then counted, and the results are shown in a graph having the appearance of a bar chart. Histograms are simple figures; sophisticated graphics are not needed in order to display them. They can be created using printer characters. They make it possible to view the behavior of the image observations as a group, and provide a background for judging the root-mean-square error, the averages, and the extreme values. The shape of the histogram should approximate the classic normal distribution curve expected for random errors. As with the critical statistics, it may be useful to plot separate histograms for the x coordinates, the y coordinates, and also a composite.

IMPLEMENTATION

Those photogrammetrists who write bundle adjustment software might take the above comments into consideration when designing their output reports, but the majority of practitioners use existing commercial or government programs. If such a program does not provide all of the above information in its output report, it may be possible to make modifications. However, even if the source code is available for alteration by the user, a high degree of competence is usually required to extract and include additional information from an existing program. This need not be a limitation if the program is capable of simply listing all the residuals for the adjustment.

With very simple programming skills, software can be written to read the residuals from the bundle adjustment print file, determine whatever critical statistics are desired, and list them in a customized report.

An example of such a post processing routine is the program VAN (V ANALYSIS), in use at Hammon, Jensen, Wallen and As-

sociates. This program opens a bundle adjustment print file as its input file. By searching for certain key sequences of characters, it locates and reads all image and ground control residuals. It is then trivial to calculate the root-mean-square errors

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***** ANALYSIS OF
*      *
* VAN * BLOCK ADJUSTMENT
*      *
***** RESIDUALS
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SAMPLE PROJECT

GROUND CONTROL RESIDUALS COMPUTED BY: BINGO

POINT	VX	VY	VZ
301	0.059	-0.030	0.022
302	-0.025	-0.052	-0.024
303	0.019	0.009	-0.054
304	-0.052	-0.025	0.082
305	-0.008	0.022	-0.040
306	-0.026	-0.022	0.023
307	-0.025	-0.036	-0.141
308	0.012	0.010	-0.045
309	0.042	0.005	-0.038
310	0.011	0.009	0.020
311	0.044	-0.004	-0.004
312	-0.006	0.103	0.013
313	-0.045	0.011	-0.058
114			-0.047
115			0.058
116			0.036
117			0.101
118			0.026
119			0.055
120			0.029
121			-0.024
122			0.050
123			0.009
124			-0.049

RMS ERROR 0.033 0.037 0.053
AVERAGE 0.000 0.000 0.000

NUMBER 13 13 24
MINIMUM -0.052 -0.052 -0.141
MAXIMUM 0.059 0.103 0.101

FIG. 1. Example of the ground control residual summary printed by the post processing program VAN, including root-mean-square errors, average errors, and extreme errors.

IMAGE RESIDUALS COMPUTED BY: BINGO

SAMPLE PROJECT

NUMBER OF IMAGES = 338

STATISTICS OF THE RESIDUALS IN MILLIMETERS

COMPONENT	AVERAGE	RMS	MIN	MAX
X	0.000	0.003	-0.013	0.010
Y	0.000	0.005	-0.011	0.014
X AND Y	0.000	0.004	-0.013	0.014

FIG. 2. Example of the image residual summary printed by VAN, including root-mean-square errors, average errors, and extreme errors.

and average errors, and to locate the extremes for the ground control and for the image residuals. Figures 1 and 2 illustrate the summary printed by VAN for a thirty-photograph block which was adjusted with the commercial bundle adjustment program BINGO (Kruck, 1985).

VAN also classifies the image residuals into one-micrometre categories and prints histograms, showing all residuals in a range from -25 micrometres to +25 micrometres. Absolute errors exceeding 25 micrometres are listed separately, if encountered. Histograms are printed separately for x , for y , and for x and y together, as illustrated in Figures 3, 4, and 5. Each "*" in a histogram represents one observation, unless the length of a print line is exceeded. The lengths of the lines are then scaled proportionately. The number of observations having a residual of a particular magnitude are printed at the left edge.

SUMMARY OF ADVANTAGES

Presenting the residual error data in a convenient manner enables the results of a bundle adjustment to be evaluated with greater ease and confidence. Such presentations can be created without writing or modifying a bundle adjustment program, if post processing techniques are employed. By considering histograms, in addition to the traditional statistics, a large volume of image error data can be appraised in context. In addition, having the computer locate the worst case errors is more reliable than having a person search a lengthy printed report for them. As a result, fewer significant errors will slip by undetected.

REFERENCES

Kruck, E., 1985. *BINGO User's Manual*, University of Hannover, Institute for Photogrammetry and Engineering Surveys, Hannover, West Germany, 252 p.

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X RESIDUALS (MICROMS)

NUMBER	MAGNITUDE
0	-25
0	-24
0	-23
0	-22
0	-21
0	-20
0	-19
0	-18
0	-17
0	-16
0	-15
0	-14
1	-13
0	-12
0	-11
1	-10
1	-9
3	-8
7	-7
9	-6
8	-5
18	-4
17	-3
24	-2
37	-1
70	0
45	1
14	2
20	3
17	4
7	5
9	6
5	7
2	8
2	9
1	10
0	11
0	12
0	13
0	14
0	15
0	16
0	17
0	18
0	19
0	20
0	21
0	22
0	23
0	24
0	25

FIG. 3. Example of the image x residual histogram printed by VAN.

Fig. 4. Example of the image y residual histogram printed by VAN.

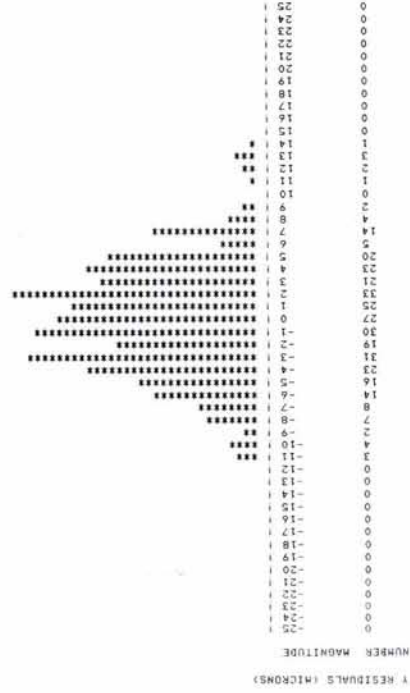


Fig. 5. Example of the image x and y composite residual histogram printed by VAN.

