First Experiments in Viewshed Uncertainty: The Accuracy of the Viewshed Area

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ABSTRACT: Digital elevation models (DEMs) are computer representations of a portion of the land surface. The elevations recorded in the DEM are not, however, without error, and the United States Geological Survey (USGS) publish a root-mean-squared error (RMSE) for each DEM. The research reported here examines how that error propagates into derivative products resulting from geographic information system (GIS)-type operations. One product from a DEM, the focus of this paper, is the viewshed. The viewshed is the area observable from a viewing location versus that which is invisible. In this research repeated error fields with varying parameters are added to the original DEM, and the viewshed is determined in the resulting noisy DEM. Results show that the area of the viewshed calculated in the original DEM may significantly overestimate the viewshed area.

INTRODUCTION

ERROR IS INHERENT IN MOST GEOGRAPHIC DATABASES (Goodchild and Gopal, 1989), and some research has examined how that error propagates into derivative products resulting from transformations of the database in a geographic information system. Most research has concentrated on error propagation in the map overlay operation (MacDougall, 1975; Newcomer and Szajgin, 1984; Vitek *et al.*, 1984; Walsh *et al.*, 1987; Chrisman, 1989; Maffini *et al.*, 1989; Veregin, 1989), but numerous other operations exist in the GIS toolbox. Some first experiments in studying the propagation of error from a digital elevation model (DEM) into the derivative product showing visible locations, sometimes known as a viewshed, are reported here (see also Felleman and Griffin (1990) and Fisher (1990)).

After discussing the viewshed operation, and the nature of error in DEM data, the general methodology of simulating error is discussed. This is followed by the use of the method in assessing the accuracy of viewsheds.

THE VIEWSHED

In many GIS the viewshed operation is a standard function. The basic algorithm used in establishing the viewshed determines whether two points are intervisible, where one point is a viewing location or viewpoint and the other is a target which may be within the viewshed (Figure 1; Travis *et al.*, 1975; Aronoff, 1989). If any land or object rises above the line of sight between the two points, then the target location is not within the viewshed of the viewing location; if no land rises above the line of sight, then the target is within the viewshed. In establishing the viewshed, either all possible targets in the area of database (Clarke, 1990), or only those within some constrained portion of the area, may be considered (Travis *et al.*, 1975; Aronoff, 1989). The actual algorithm depends on the methods of storage of the DEM as either a triangulated irregular network (DeFloriani *et al.*, 1986) or a grid (Anderson, 1982), and even within any one data structure multiple algorithms may exist (Sutherland *et al.*, 1974). Felleman and Griffin (1990) have compared the output of four different GIS-based implementations of the viewshed operation, and show that the viewsheds delimited may be very different.

The viewshed operation itself has numerous applications. Specifically, it is used for assessing the visual impact of construction projects, by finding the areas from which those developments are visible, and, at the planning stage, by identifying least visible routes and locations for the construction. It is essential in locating observation posts such as forest-fire observation towers (Travis *et al.*, 1975). It is also used extensively in landscape planning and architecture to define areas of both limited and open views. In the military domain it is used for planning least visible troop movements, and for locating radar systems for maximum visibility into the surrounding terrain. In all of these applications, knowing the reliability of the locations identified as being within and without the viewshed would greatly enhance the effectiveness of the viewshed product (Aronoff, 1989; Burrough, 1986; Star and Estes, 1990; Tomlin 1990).

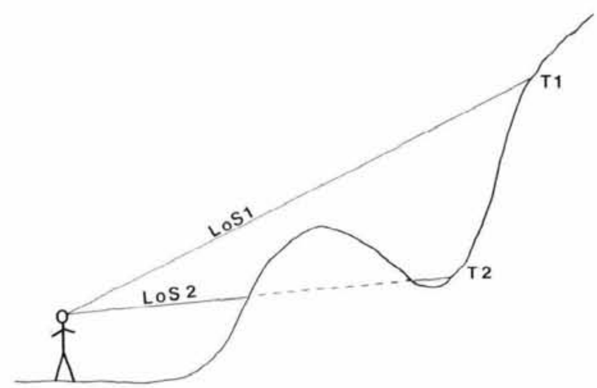

FIG. 1. The basic algorithm for finding the viewshed establishes whether one location is visible from another, given the elevations of the intervening land and the height of the observer above the ground. Thus, with respect to the observer at the Viewpoint, the Line of Sight (LoS 1) to Target T1 is not broken by the landsurface, but Line of Sight 2 is, and so Target 1 is within the viewshed, but Target 2 is not. Indeed, all the landsurface shaded is outside of the Viewshed of the observer. Various complexities can be added, including deviations in the Line of Sight, the curvature of the Earth, and the height of cultural and vegetation features above the land surface.

ERROR IN DEMs

Digital elevation models may be derived from a number of different sources, and stored in either of two formats. DEM data are derived by one of two major methods: direct photogrammetry from aerial photographs, or digitizing from contour maps

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(Aronoff, 1989). Recently the stereo viewing capability of the SPOT satellite has enabled direct DEM derivation from that imagery (Swann *et al.*, 1988).

In any particular digital elevation model (DEM) any particular point may not actually be at the elevation recorded for it, and the sources of this error may be multitudinous. Sources of inaccuracy include the original survey by field workers or photogrammetrists, the expertise of the cartographer who generated the map, and of the digitizer-operator who converted it, or, if generated directly from aerial photographs, again of the photogrammetrist. The error may be caused by faulty equipment, fluctuating power supply, precision of the data format, poor interpolation, or human problems.

The major sources of DEM data for the United States are the U.S. Geological Survey (USGS) and the Defense Mapping Agency. Two gridded DEM products are available, depending on the size of the grid: either 30 m or 3 arc seconds. Both USGS DEM products are in the same general format, and the User's Guide specifies that the root-mean-squared error (RMSE) should be published with all data (USGS, 1987). The RMSE for any one DEM is based on a comparison between the elevations of at least 20 locations on the map, and their elevations recorded in the database. In addition, it should be noted that most USGS source maps are commonly stated as conforming to National Map Accuracy Standards, which themselves state that "at no more than 10 percent of the elevations tested will contours be in error by more than one half the contour interval," as established by comparison with survey data (Thompson, 1988). In generating a DEM from a map by digitizing, therefore, at least three stages are present when error may be introduced: map compilation, DEM generation from the map, and comparison of DEM elevations with map elevations. The last of these and a combination of the first two also occur if the DEM is generated directly by photogrammetry.

Caruso (1987) and Carter (1989) identify a number of different types of error in gridded DEMs, although they present alternative taxonomies. Carter (1989) defines "relative" and "global" error, where the former refers to generally single values being inconsistent with their neighbors, while global errors are whole blocks of cells which are found to be in error. These broadly correspond to the "random" and "systematic" errors identified by Caruso (1987) who also identified more massive "blunders" which, he states, rarely get into a published DEM.

Stereo imagery from SPOT Image Corp. is now capable of supporting generation of a DEM as a standard product on a 10-m grid (Gugan and Dowman, 1988). Studies have shown the error in these products to be less than 10m RMSE in all three dimensions (Swann *et al.*, 1988).

Error in DEMs is then widely acknowledged, and has been the subject of some study, which has concentrated, however, on the nature and description of the error, rather than its propagation into derivative products. Felleman and Griffin (1990) have compared implementations of the viewshed operation, and simulated error in the DEM before calculating the viewshed, using a method similar to that reported here. They examined three viewpoints in two test areas for each of which ten error simulations were run. Results were only reported for one test location, however.

METHOD

MONTE CARLO TESTING

A Monte Carlo simulation and testing approach is taken to studying the propagation of DEM error. In this approach, a number of randomizing models of how error occurs are established and then coded as computer procedures. The resulting computer program may be used to generate multiple realizations of the random process. Many workers have used the original data in combination with the realizations to establish the statistical significance of the original data with respect to the random process (Besag and Diggle, 1977). Thus, Openshaw *et al.* (1987) were able to locate two significant clusters of incidents of childhood leukemia in northern England by analyzing 499 realizations of the random process. Hope (1968) has shown that only 19 realizations are necessary to yield a statistically useful result at the 0.05 significance level (see also Ripley, 1987).

ALGORITHMS

How the error is distributed across the area of any one DEM is currently unknown, and factors that may affect the distribution of error are largely unresearched. The inference of the error reporting used by the USGS is that the error at any point occurs independently of that at any other point (i.e., the error is not spatially autocorrelated), and some independent or random errors are established in the literature (Caruso, 1987; Carter, 1989; Hutchinson, 1989). Therefore, the following algorithm was implemented:

(1) Define a standard deviation of a normal distribution (S = RMSE);
(2) Read Original__Value for the current cell:
  (a) Using the Box-Muller (or some other) algorithm, generate a random number drawn from a normal distribution with mean = 0 and standard deviation = S;
  (b) Add the random number to the Original__Value for the current cell, to give the New__Value;
(3) Repeat step (2) for all cells in the Map__File.

This assumes that the RMSE is equivalent to the standard deviation of a normal distribution (Caruso, 1987). In the absence of any other information on error structure, this may not be unreasonable, but the error term could actually be drawn from some other distribution, and that distribution may vary from DEM to DEM, and even within a DEM.

The assumption of independence implied by the USGS error reporting is likely to contribute only a small portion of the error (Caruso, 1987; Carter, 1989). High spatial autocorrelation is probably present, and banding can often be seen in the DEM data. To accommodate the occurrence of spatial autocorrelation, a version of the algorithm given by Goodchild (1980) was implemented, using Moran's I to measure the autocorrelation (Goodchild 1986; Griffith 1987). It works as follows:

(1) Define a target autocorrelation (I_t) and a standard deviation of a normal distribution (S = RMSE);
(2) For each cell in the DEM, generate a random value with a normal distribution with mean = 0 and standard deviation = S;
(3) Calculate the spatial autocorrelation of the field (I_1);
(4) Randomly identify two cells in the DEM:
  (a) Swap the values in the two cells;
  (b) Calculate the new spatial autocorrelation (I_2);
  (c) IF I_t and $I_2 > I_1 > I_1$ THEN retain the swap, and $I_1 = I_2$
  OR
  IF $I_t < I_1$ AND $I_2 < I_1$ THEN retain the swap, and $I_1 = I_2$
  ELSE swap the two cells back to their original values;
(5) Repeat step (4) until $(I_t - I_1)$ is within some threshold.
(6) For each cell in the original DEM, add the value in the corresponding autocorrelated field.

This algorithm is simple and can be made computationally efficient. It has been criticized, however, by Haining *et al.* (1983) for not allowing any control over the resulting structure in the autocorrelated values. Those criticisms relate to a context where multivariate attributes of polygons are being explored; that is not the case here.

EXPERIMENTAL APPROACH

A 200- by 200-cell subset of the USGS Prentiss, North Carolina, 7.5-minute DEM was acquired covering the Coweeta Experimental Watershed (Figure 2). This DEM has been the subject of considerable research on DEM products (Band, 1986; Lammers and Band, 1990). Within the area of the DEM, two test viewing locations (viewpoints) were arbitrarily identified, one near an interfluve (Point 1) and one in a valley bottom (Point 2).

The DEM was read into a format compatible with Idrisi (Eastman, 1989), and all further processing was done with either Idrisi modules or with implementations of the above algorithms written by the author. The VIEWSHED module of the Idrisi package is crucial to the research reported here, and so some simple test situations were established to examine the veracity of the viewable area calculated by that module. In every test, the module performed satisfactorily. The random number generator used in programming the algorithms was also tested because, like all such implementations, it is actually only a pseudo-random number generator (Ripley, 1986). The generator included with Turbo Pascal 5.5 was used here. The runs test was used to check for serial autocorrelation, the chi-squared test was used to check for the uniform distribution, and serial autocorrelation was tested for all lags to check for cycling in the generator. The generator performed satisfactorily for all cases when number sequences up to 10,000 long were tested (corresponding to the 100 by 100 array used in the generation of autocorrelation).

The purpose in adding error is to examine the consequence of that error on the viewshed area found by the GIS function; it is not to explore the actual accuracy of the viewshed from the viewing point, as compared with the viewshed in the field. Therefore, the starting assumption of this research is that the published DEM is accurate, and that the viewshed found in the original DEM is the true viewshed.

The published RMSE for the Prentiss DEM is 7m. In the experiments discussed, four different approaches to perturbing the surface of the original DEM were used:

- The first algorithm was applied exactly as stated, with variable RMSE (S = 2, 7, and 10; I = 0);
- Only a randomly selected 50 percent of the map area had a noise term added, without autocorrelation (S = 7);
- The elevation of the viewpoint was held constant, while the remainder of the area was perturbed (S = 7, I = 0); and
- The second algorithm was applied to yield noise terms (S = 7 and I_t = 0.7 and 0.9).

Note: I and S are parameters used in algorithms for perturbing the original DEM surface.

In each experiment, 19 realizations of the perturbation process were generated, so that the following hypothesis might be tested, with p = 0.05, and using the area of the viewshed as a summary value:

H_0 : the viewshed area in the original DEM is not a member of the set of viewshed areas in elevation models generated by the algorithm listed above; and

H_1 : the viewshed area in the original DEM is a member of the set of viewshed areas in elevation models generated by the algorithm listed above.

In the original DEM and in each realization, the viewsheds of the two test locations were calculated from the approximate eye-height of an individual, 2 m above the ground, and within the approximate near-view to 1000 m away from the viewing location (Figure 2), and the elevation of the viewpoint recorded together with the maximum and minimum elevations within 1000 m of the view point (giving a measure of relative relief), and the area of the viewshed. The average of the elevations within 1000m of the viewpoint was also recorded, but they are not reported because values were found to be identical in simulations and the original for any particular viewing point.

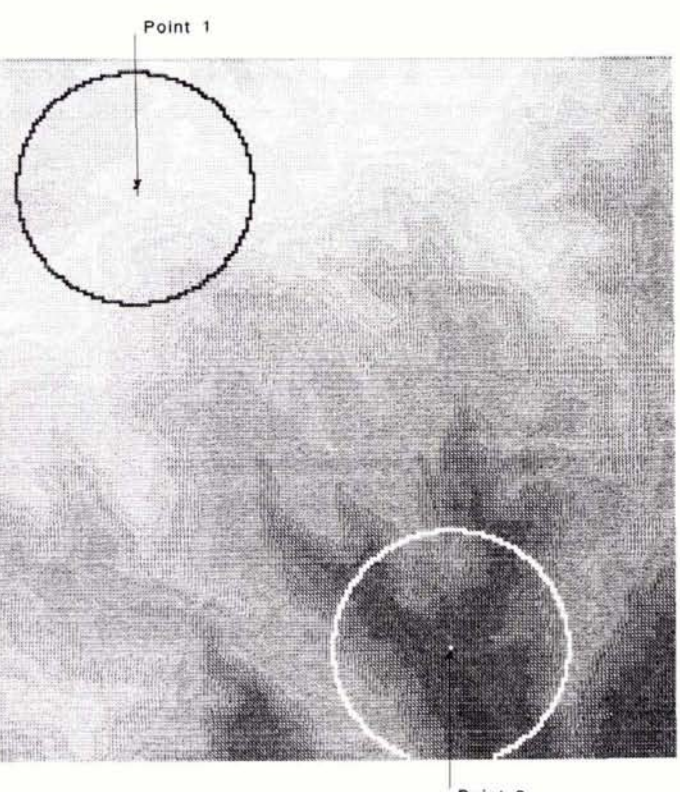

FIG. 2. Shaded map of elevations in the 200 by 200 portion of the Prentiss DEM used here. The two test locations used in this study are shown, together with the area used for viewshed testing, up to 1000m away from each.

RESULTS

All sets of realizations of the random process are summarized in Table 1. For cases in Table 1 with variable RMSE but zero autocorrelation, the elevation of the viewpoint in the simulations spread around the actual elevations in the original data, as should be expected from the algorithm. This is the only column in the table which reports the elevation at a single point

TABLE 1. SUMMARY OF VIEWSHEDS FOR THE 19 REALIZATIONS OF ELEVATIONS IN A 100 BY 100 SUBSET OF 30-M DEM DATA FOR THE COWEETA WATERSHED, N.C.

	View point Elevation (metres)	Maximum (m) within 1000m	Minimum (m)	Viewshed Area (cells)	Significance level
Point 1	1440	1549	1098	1275	
RMSE=7m	1429-1460	1549-1564	1087-1098	284-1544	0.1
RMSE=2m	1437-1447	1547-1551	1095-1101	990-1633	0.4
RMSE=10m	1425-1451	1549-1574	1081-1098	139-1131	0.05
C=50%	1427-1444	1549-1558	1083-1099	323-1283	0.1
Constant	1440	1549-1564	1087-1098	440- 993	0.05
I=0.7	1426-1460	1544-1562	1083-1105	357-1431	0.2
I=0.9	1428-1451	1542-1569	1079-1113	958-1419	0.25
Point 2	694	985	671	1545	
RMSE=7m	678-706	911-1001	653-671	23-1742	0.1
RMSE=2m	688-697	982-987	667-671	783-1539	0.05
RMSE=10m	687-714	984-1014	646-671	108-1355	0.05
C=50%	683-705	980- 994	652-667	367-1557	0.1
Constant	694	911-1001	653-671	535- 933	0.05
I=0.7	686-707	976-995	654-668	658-1481	0.05
I=0.9	669-706	977-995	650-670	673-1703	0.2

in all simulations, and so values are directly comparable. The maximum and minimum elevations of the simulated surfaces within 1000 m of the viewpoint also spread around the actual values, although in some cases the values are skewed (e.g., for RMSE = 10m, the view point elevation is from 687 to 714m, being a higher minimum than for realizations with RMSE = 7m), as again might be expected with only 19 realizations. These observations confirm that the algorithm implemented performs the simulations as desired. The simulation results relating to variable spatial autocorrelation use elevations that are no longer truly random, and so further skewing can be observed in these. The area of the viewshed calculated for each realization is also presented in Table 1, together with the significance level of the area in the original DEM, compared with the simulations.

When the viewshed area is calculated for the randomized noise over the whole area with variable RMSE (standard deviation), in three out of the six instances presented the area in the original DEM exceeds the area in any of the 19 simulations, and in two of the remaining three sets of simulations it is the second largest. Therefore, the null hypothesis may be rejected at $p = 0.05$ in three instances, and $p = 0.1$ in two. In only the case of Point 1 with an RMSE of 2m is the area in the original even approximately at the middle of the distribution, and even in that group it is higher than the median for the simulations.

When only 50 percent of the DEM area has random noise added ($C = 50$ percent, Table 1), as opposed to 100 percent of the area, the actual viewshed area is the second largest value in the two new results presented, and so the null hypothesis may only be rejected at $p = 0.1$. With the viewpoint elevation held constant (Constant, Table 1), the area of the viewshed in the original DEM is the largest by a considerable amount, and so the null hypothesis is rejected with a significance level of 0.05 for both test points.

Applying spatial autocorrelation to the noise added to the original DEM shows that the viewshed area from the original DEM is in all cases larger than the median value for those DEM with simulated noise added. In only one case is the original value larger than all realizations, however. Less extreme results are common, with significance levels of 0.2 and 0.25 occurring in three of the four sets of new results.

To explore further the relationships of the various parameters given in Table 1, correlation matrices were calculated (Table 2) and stepwise multiple regressions were conducted, taking the viewshed area as the dependent variable and the viewpoint elevation and the maximum and minimum elevations within 1000m as the independent variables. Only the elevation of the viewpoint was found to have any predictive power in determining the area of the viewshed.

TABLE 2. CORRELATIONS OF VIEWSHED AREAS AND ELEVATIONS RECORDED IN THE AREA OF THE VIEWSHED.

	View point Elevation	Maximum within 1000 m	Minimum within 1000 m
		Point 1	
RMSE = 7m	0.844	−0.400	−0.054
RMSE = 2m	0.923	0.478	−0.311
RMSE = 10m	0.835	−0.150	0.071
C = 50%	0.907	0.078	0.082
Constant	n/a	−0.623	0.495
$I = 0.7$	0.807	−0.113	0.397
$I = 0.9$	0.448	0.198	0.175
		Point 2	
RMSE = 7m	0.777	−0.200	0.285
RMSE = 2m	0.839	−0.050	0.198
RMSE = 10m	0.804	0.236	0.185
C = 50%	0.848	−0.182	0.419
Constant	n/a	−0.082	0.451
$I = 0.7$	0.794	0.374	0.387
$I = 0.9$	0.730	0.219	0.498

Figures 3 to 6 show this relationship. In all, it is possible to see the ordering of viewshed areas, yielding the significance values reported in Table 1. The usually strong linear relationship between the two variables plotted can also be seen, with only a single outlier in one graph (Figure 3, Point 2), and a more dispersed scatter in another (Figure 6, Point 1, $I = 0.9$). This last scattergram shows a fundamental change in the effect of the error with highly autocorrelated noise, a consequence that will be the subject of future investigation. The plotted point representing the original viewshed, in almost all cases, is apparently not part of the distribution plotted, being invariably to the bottom right of the general linear trend of the relationship (Figures 3 to 6).

In examining the case where the viewpoint elevation is held

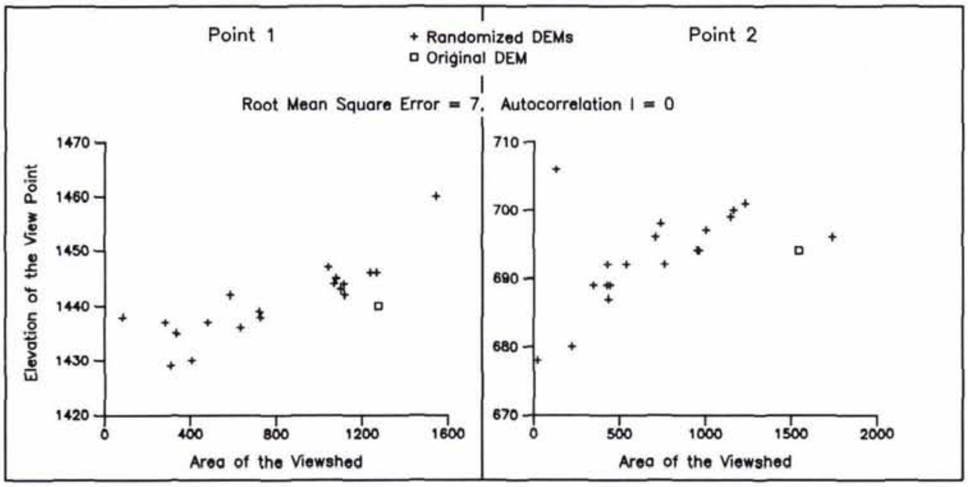

FIG. 3. Scattergrams of viewpoint elevation versus viewshed area for the two test locations when the noise has the parameters reported by the USGS, and inferred from the User's Guide: RMSE 7 and autocorrelation 0.

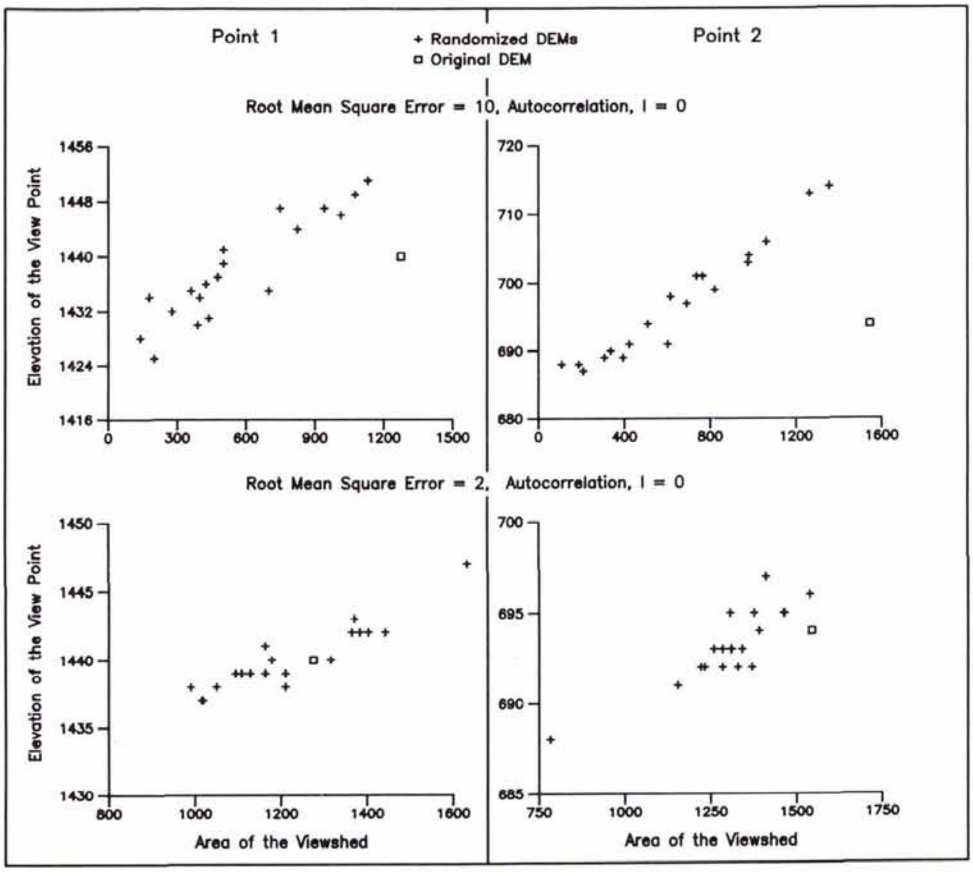

FIG. 4. Scattergrams of viewpoint elevation versus viewshed area for the two test locations when the noise has RMSE 10 and 2 but autocorrelation 0.

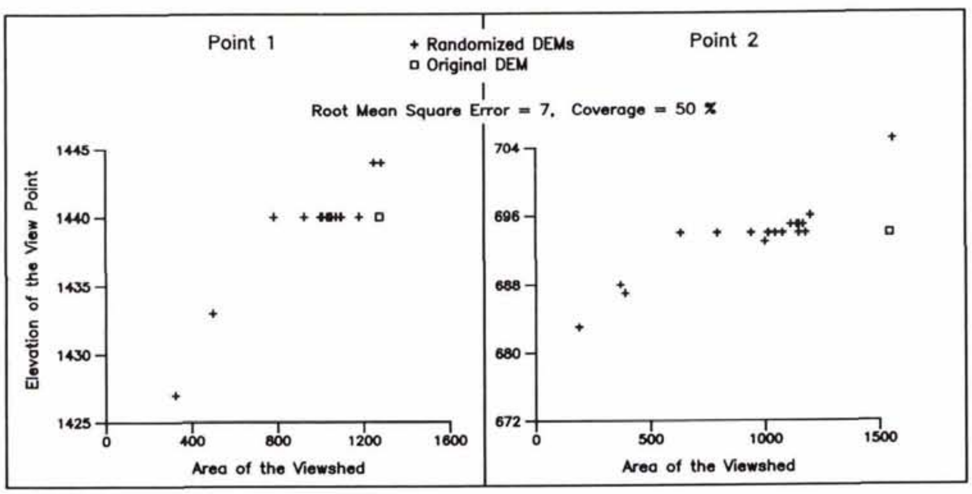

FIG. 5. Scattergrams of viewpoint elevation versus viewshed area for the two test locations when the noise has RMSE 7, but is only applied to a random 50 percent of cells.

constant, only the relationship to maximum and minimum elevations was examined. Here an inconsistent pattern was presented, with the maximum elevation in the viewshed of Point 1 having a reasonably strong negative correlation, while the maximum for Point 2 and the minimum in the viewsheds of both points one and two were not significant. The importance of the maximum elevation within the viewshed can be explained by the shielding effect.

DISCUSSION

All the results reported in Table 1 lead to one fundamental conclusion; there is consistent bias in calculation of the viewshed

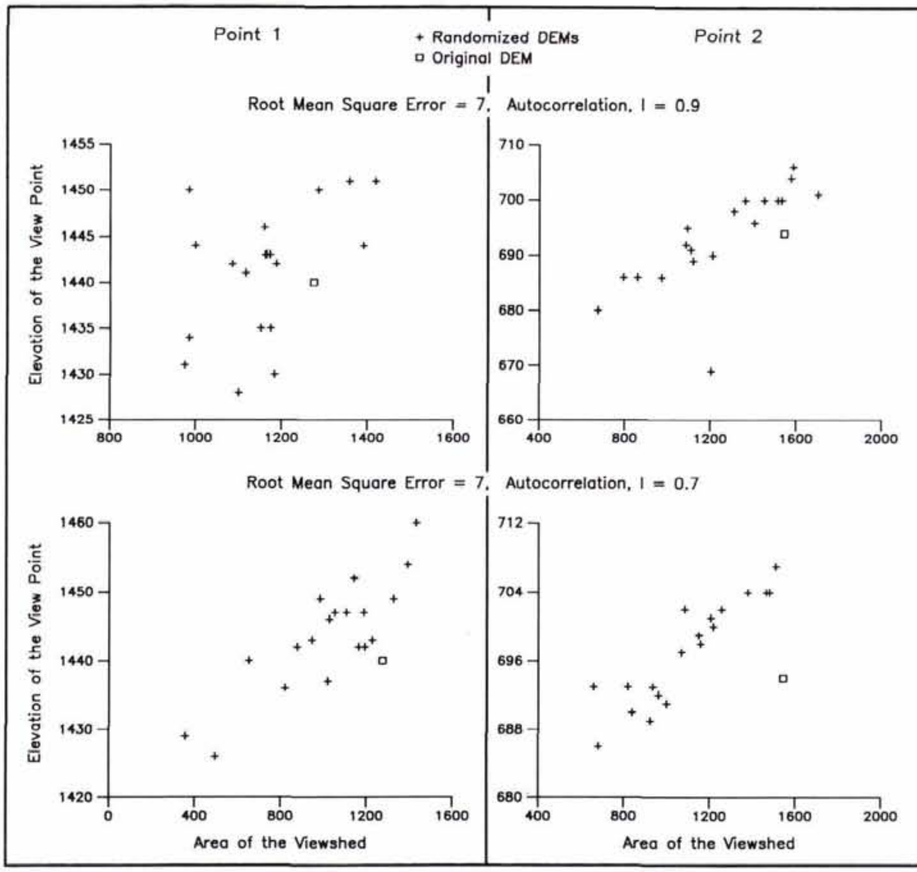

FIG. 6. Scattergrams of viewpoint elevation versus viewshed area for the two test locations when the noise has RMSE 7 (as published) and autocorrelation 0.7 and 0.9.

area from a particular viewpoint in a DEM known to contain error. Furthermore, that bias seems to lead to the area of the viewshed calculated in the original DEM being larger than the viewshed when error, generated by any of the four methods used here, is added to the original surface. Indeed, it is possible to state that in six out of the 14 different cases presented, the viewshed area in the original DEM is significantly different from those in the realizations of the random process at the 0.05 significance level (a level that is usually considered significant), and in four other cases it is at the 0.1 level (which is not usually significant but is indicative).

Only limited results are presented here, but there is an indication that for at least one viewpoint the viewshed calculated in the original DEM is more representative when the RMSE is smaller. Similarly, as the spatial autocorrelation in the perturbation increases, so it seems the viewshed in the original DEM is more representative (Table 1).

The strong positive correlation between viewshed area and elevation of the viewpoint suggests that as that elevation rises the visible area increases, and vice versa. This effect, however, does not explain the full variability (r^2 varies from 0.85 to 0.53 and 0.20 in one extreme case; see Table 2), and when the viewpoint elevation is held constant the same general phenomenon is visible, although the range of viewshed areas is reduced (Table 1), and only the maximum elevation within the viewshed of one location has any predictive power over the viewshed area.

The remaining effect must be due in part to the alternating consequence of raising or lowering elements of the landscape according to the error term modeled. Elevations that are within the original viewshed, and are raised, will cut other areas from view, while those that are lowered do not necessarily open others to being visible. The full result of this can be seen when the viewpoint elevation is held constant, and the original viewable area is larger than in all simulations by a larger margin than in any other set of simulations (Table 1). Indeed, the significant correlation of the maximum elevation in the viewshed to the area of the viewshed when the viewpoint elevation is constant (Table 2) can be taken to be collaboration of the importance of increased elevations.

Relief effects are suggested in the results presented here, but with two points tested these can only be indications. The near-interfluve location, Point 1, seems to have a more robust viewshed, with the alternate hypothesis being rejected for an RMSE of 2m, and the correlation between viewpoint elevation and viewshed area being higher than in any of the cases of the valley-bottom Point 2, as well as the high correlation of viewshed area and maximum elevation when the viewpoint elevation is constant. On the other hand, the correlation between viewshed area and viewpoint elevation is less well established for Point 1 where, as the autocorrelation is varied from 0 to 0.9, the correlation coefficient falls from 0.84 to 0.45 (Table 2), suggesting that, although with highly autocorrelated noise the viewshed area found for the original DEM may be more representative ($p=0.2$), in the same circumstance, none of the variables recorded have any predictive power.

CONCLUSION

The work reported here provides some insights as to the possible consequences of DEM error on the viewshed. Specifically, and most strikingly demonstrated, is the effect on the area of the viewshed, where the viewshed in the original DEM data is consistently greater than that found in the same DEM with the addition of simulated error. In some situations the difference found is statistically significant at the 0.05 level.

The implication of this is that the current results of viewshed operations, which show locations as either in or out of the viewshed, should be treated with the utmost caution. Clearly, further study is required to establish the certainty of particular points being within the viewshed, and to examine the role of the different possible influences over the accuracy, and the possible role of landscape characteristics in determining the accuracy.

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REFERENCES

Anderson, D.P., 1982. Hidden line elimination in projected grid surfaces. *ACM Transactions on Graphics*, 1: 274–291.

Aronoff, S., 1989. *Geographic Information Systems: A Management Perspective*. WDL Publications, Ottawa.

Band, L.E., 1986. Topographic partition of watersheds with digital elevation models. *Water Resources Research*, 22: 15–24.

Besag, J. and P.J. Diggle, 1977. Simple Monte Carlo tests for spatial patterns. *Applied Statistics*, 26: 327.

Burrough, P.A., 1986. *Principles of Geographical Information Systems for Land Resources Assessment*. Clarendon Press, Oxford.

Carter, J.R., 1989. Relative errors identified in USGS gridded DEMs. *Proceedings, Auto Carto 9*, American Congress on Surveying and Mapping, Bethesda, Maryland, pp. 255–265.

Caruso, V.M., 1987. Standards for digital elevation models. *Technical Papers, ASPRS-ACSM Annual Convention*, American Society for Photogrammetry and Remote Sensing-American Congress on Surveying and Mapping, Falls Church, Virginia, Vol. 4, pp. 159–166.

Chrisman, N.R., 1989. Modeling error in overlaid categorical maps. *The Accuracy of Spatial Databases*, (M.F. Goodchild and S. Gopal, editors), Taylor and Francis, London, pp. 21–34.

Clarke, K.C., 1990. *Analytical and Computer Cartography*. Prentice Hall, Englewood Cliffs, New Jersey.

DeFloriani, L., B. Falcidieno, and C. Pienovi, 1986. A visibility-based model for terrain features. *Proceedings of the Second International Symposium on Spatial Data Handling*. International Geographic Union, Columbus, Ohio, pp. 235–250.

Felleman, J., and C. Griffin, 1990. *The Role of Error in GIS-Based Viewshed Determination — A Problem Analysis*. U.S. Forest Service, North Central Experiment Station, Agreement No. 23-88-27.

Fisher, P.F., 1990. Simulation of error in digital elevation models. *Papers and Proceedings of the 13th Applied Geography Conference*, Vol. 13, pp. 37–43.

Goodchild, M.F., 1980. Algorithm 9: Simulation of autocorrelation for aggregate data. *Environment and Planning A*, 12: 1073–1081.

———, 1986. *Spatial Autocorrelation*. CATMOG 47. Geo Books, Norwich, U.K.

Goodchild, M.F., and S. Gopal (editors), 1989. *The Accuracy of Spatial Databases*. Taylor and Francis, London.

Griffith, D.A., 1987. *Spatial Autocorrelation: A Primer*. Association of American Geographers, Washington, D.C.

Gugan, D.J., and I.J. Dowman, 1988. Topographic mapping from SPOT imagery. *Photogrammetric Engineering & Remote Sensing*, 54: 1409–1414.

Haining, R., D.A. Griffith, and R. Bennett, 1983. Simulating two-dimensional autocorrelation surfaces. *Geographical Analysis*, 15: 247–255.

Hope, A.C.A., 1968. A simplified Monte Carlo significance test procedure. *Journal of the Royal Statistical Society B*, 30: 582–598.

Hutchinson, M.F., 1989. A new procedure for gridding elevation and stream line data with automatic removal of spurious pits. *Journal of Hydrology*, 106: 211–221.

Lammers, R.B., and L.E. Band, 1990. Automating object representation of drainage basins. *Computers and Geosciences*, 16: 787–810.

Lodwick, W.A., 1989. Developing confidence limits on errors of suitability analyses in geographical information systems. *The Accuracy of Spatial Databases*, (M.F. Goodchild and S. Gopal, editors), Taylor and Francis, London, pp. 69–78.

MacDougall, E.B., 1975. The accuracy of map overlays. *Landscape Planning*, 2: 23–30.

Maffini, G., M. Arno, and W. Bitterlich, 1989. Observations and comments on the generation and treatment of error in digital GIS data. *The Accuracy of Spatial Databases*, (M.F. Goodchild and S. Gopal, editors), Taylor and Francis, London, pp. 55–67.

Newcomer, J.A., and J. Szajgin, 1984. Accumulation of thematic map errors in digital overlay analysis. *The American Cartographer*, 11: 58–62.

Openshaw, S., 1989. Learning to live with errors in spatial databases. *The Accuracy of Spatial Databases* (M.F. Goodchild and S. Gopal, editors), Taylor and Francis, London, pp. 263–276.

Openshaw, S., M. Charlton, C. Wymer, and A. Craft, 1987. A Mark 1 Geographical Analysis Machine for the automated analysis of point data sets. *International Journal of Geographical Information Systems*, 4: 335–358.

Ripley, B.D., 1987. *Stochastic Simulation*. John Wiley, New York.

Star, J., and J. Estes, 1990. *Geographic Information Systems: An Introduction*. Prentice Hall, Englewood Cliffs, New Jersey.

Sutherland, I.E., R.F. Sproull, and R.A. Scumaker, 1974. A characterization of ten hidden-surface algorithms. *Computing Surveys*, 6: 1–55.

Swann, R., D. Hawkins, A. Westwell-Roper, and W. Johnstone, 1988. The potential for automated mapping from geocoded digital image data. *Photogrammetric Engineering & Remote Sensing*, 54: 187.

Thompson, M.M., 1988. *Maps for America*, Third Edition. United States Geological Survey, Reston, Virginia.

Tomlin, C.D., 1989. *Geographic Information Systems and Cartographic Modeling*. Prentice Hall, Englewood Cliffs, New Jersey.

Travis, M.R., G.H. Elsner, W.D. Iverson, and C.G. Johnson, 1975. *VIEWIT: Computation of seen areas, slope and aspect for land-use planning*. Technical Report PSW-11/1975, USDA Forest Service.

USGS, 1987. *Digital Elevation Models*. Data Users Guide 5. United States Geological Survey, Reston, Virginia.

Veregin, H., 1989. Error modeling for the map overlay operation. *The Accuracy of Spatial Databases* (M.F. Goodchild and S. Gopal, editors), Taylor and Francis, London, pp. 3–18.

Vitek, J.D., S.J. Walsh, and M.S. Gregory, 1984. Accuracy in Geographic Information Systems: An assessment of inherent and operational errors. *Proceedings of Pecora IX*, American Society for Photogrammetry and Remote Sensing, Bethesda, Maryland, pp. 296–302.

Walsh, S.J., D.R. Lightfoot, and D.R. Butler, 1987. Recognition and assessment of error in geographic information systems. *Photogrammetric Engineering & Remote Sensing*, 53: 1423–1430.

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