First Experiments in Viewshed Uncertainty: Simulating Fuzzy Viewsheds

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ABSTRACT: Calculation of the viewshed from a digital elevation model (DEM) is a common capability of a GIS packages. Two mistaken assumptions are made, however: first, that the DEM is accurate, and second that the viewshed is a Boolean phenomenon. The inaccuracy of the DEM data is acknowledged in reporting of the root-mean-squared error. The mistake of the Boolean viewshed is recognizable in the real-world experience of most individuals. In this paper multiple realizations of an error simulation algorithm are used to generate multiple Boolean viewsheds which are then cumulated to yield an image of the viewshed with all the properties of a fuzzy set, i.e., a fuzzy viewshed. Alternative parameterizations of the basic algorithm are presented, and the differences are explored.

INTRODUCTION

THE USUAL VIEWSHED OPERATION within a GIS presents the user with a Boolean product. It takes the digital elevation model (DEM), and a particular viewing point, at a particular elevation above the land surface, and identifies for each successive element in the DEM database whether the land surface rises above the line of sight at any intervening locations. If none does, as defined by the particular algorithm, then the result is normally reported as 1, indicating that the point is within the viewshed, while a 0 indicates that it is not.

This simple binary representation is widely used, although research by Fisher (1990, in press) and Felleman and Griffin (1990) shows that, if an error term is added to the elevations stored in the DEM, alternative viewsheds may result. Fisher (1990, in press) focuses on the area of the viewsheds found in each of the DEMs with simulated error as compared with the area of the viewshed in the original DEM. The results show that the original viewshed is invariably an overestimate of the viewshed, and in some cases can be significantly different from the viewsheds where simulated error is added to the DEM ($p=0.05$). This implies that those locations where the elevation is raised by the simulation process decrease the area of the viewshed by masking portions of the landscape from view, while those elevations that are lowered generally do not open new vistas. It also means that many cells are only occasionally included in the viewshed derived from the DEMs with simulated error.

The objective in the research reported here is to further develop the method suggested by Felleman and Griffin (1990) who demonstrate one approach to the generation of "fuzzy" viewsheds where any cell in the viewshed image has a measure of the likelihood of the cell being within view, and to examine some of the properties of those viewsheds.

BACKGROUND

DEM ERROR

Error in standard USGS DEM products is reported as the root-mean-squared error (RMSE) (USGS, 1987), and is based on comparison of elevations between the DEM and the published map at a minimum of 20 test points. The test points may be contour lines, bench marks, or spot elevations (USGS, 1987, p. 8).

Many DEMs are derived from contour maps, which themselves are prepared to particular standards and so incorporate some error (Thompson, 1988). Other DEMs are derived directly from photogrammetry which is an error prone process. The research reported here examines only the error as reported by the RMSE, not the error inherent in the map, whether the map is used only for comparison with the finished DEM, or as a source document for DEM production as well. Consideration of the earlier phase of error generation involves increased complexity in the noise algorithm, and will be the subject of future research.

THE FUZZY VIEWSHED

Previous research (Fisher, in press) has outlined an algorithm whereby it is possible to generate noise drawn from a normal distribution with a specific standard deviation and a mean of zero, and to generate a spatial autocorrelation (SA) of any desired level in that noise (1.0 to -1.0, as measured by Moran's I (Goodchild, 1986)). The noise field is then added to the original DEM to yield a realization of the simulation process. If the viewshed is found in a realization, then it represents one possible view from the viewing location, and so the viewsheds found in all realizations form a set of possible viewsheds.

In each viewshed, the numbers 1 and 0 are used to indicate that a cell is within or without the viewshed; it is a binary image of the Boolean viewshed. If the sets of binary images are summed together, then the value in a particular cell is a direct measure of the rate of occurrence of that cell in the viewsheds of the DEMs with simulated error. Thus, in the summed image each cell contains a number between 0 and the number of realizations. In research reported below 20 realizations are analyzed, and so the maximum score for any cell in the sum image is 20, which indicates that the cell is within the viewsheds of all realizations. Any smaller value indicates that number of times that the cell occurs within the viewshed, and so the value may then be viewed as an index of how likely a cell is to be in the viewshed.

The final image has a number of interesting properties. If

$$x'_{ij} = x_{ij}/n \tag{1}$$

where x_{ij} is the sum value at row i, column j, and n is the number of simulations, then the value of x' has a range for 0 to 1, where 0 indicates that the cell is very unlikely to be outside the viewshed, 1 indicates that it is certainly within the viewshed, and other values of x' indicate the degree to which a cell is likely to be within view (where the degree of certainty is limited by n). In fact, x' has the properties of the membership function for the fuzzy set of those cells within view of a particular location, where

$$x' = \{0,1 : x \in X\}, \tag{2}$$

$$0 \le x' \le 1, \tag{3}$$

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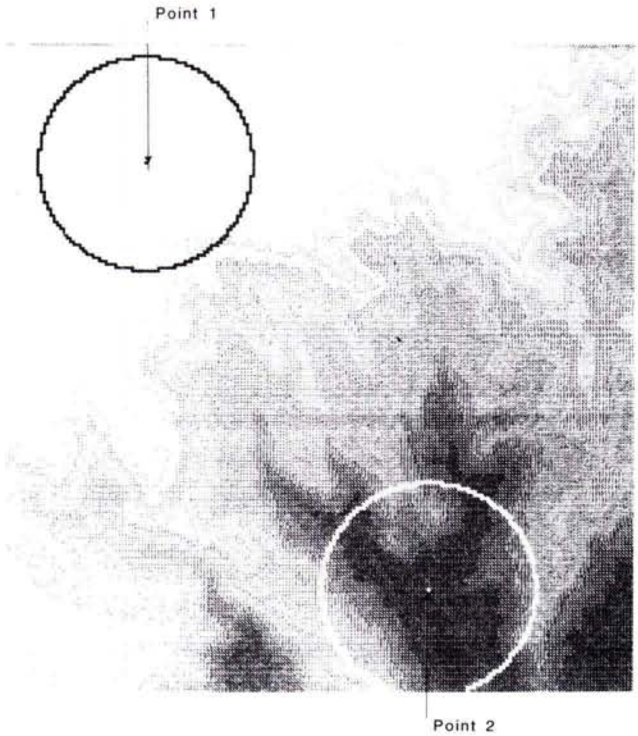

FIG. 1. A contour map of the general study area, showing the two test viewing locations. The area depicted is 6 km on each side, north is to the right, and elevation increases with lightness.

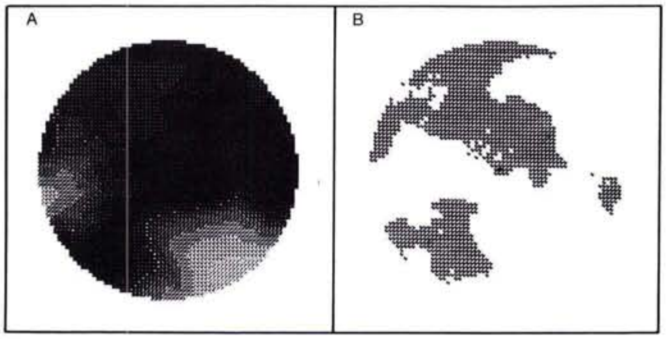

FIG. 2. (A) The DEM within 1 km from Point 1 (elevation increases with darkness), and (B) the Boolean viewshed from Point 1.

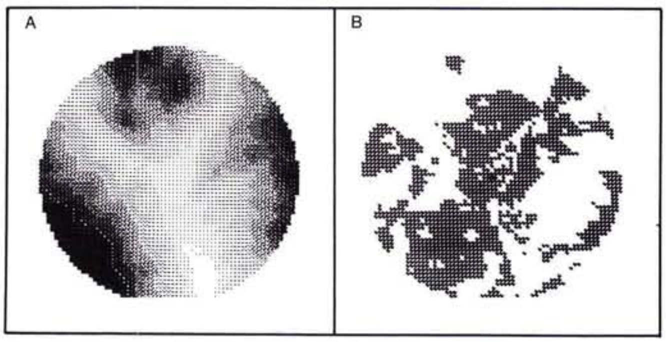

FIG. 3. (A) The DEM within 1 km from Point 2 (elevation increases with darkness), and (B) the Boolean viewshed from Point 2.

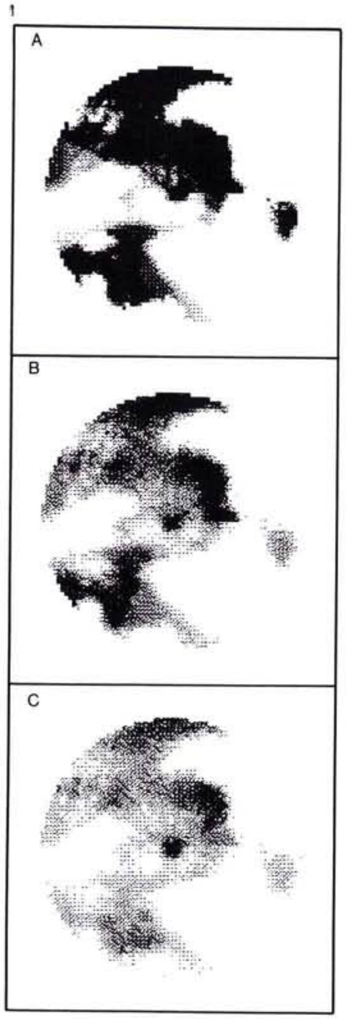

FIG. 4. Fuzzy viewsheds from Point 1 with (A) RMSE = 2, (B) RMSE = 7, and (C) RMSE = 10. Note that Figure 4b is the same as Figure 8a. Darker areas have higher fuzzy memberships.

(Zadeh, 1965; Kauffman, 1975; Leung, 1988). This is in contrast with the usual Boolean set resulting from the usual viewshed operation where

$$x = \begin{cases} 1 : x \in X \\ 0 : x \notin X. \end{cases} \qquad (4)$$

It follows that the full set of fuzzy operators (Leung, 1988) is available to manipulate these resulting viewsheds as a part of a fuzzy geographic database. Many researchers are working on aspects of such a system. Robinson (1988) gives a general overview of the theory of fuzzy sets, and a minor example. Robinson and Thongs (1975) and Wang (1989) argue that it is possible to extract fuzzy land-cover information from Landsat imagery, and Fisher and Pathirana (in press) have demonstrated that the values yielded by a fuzzy classifier may be related to the proportion of a pixel covered by a particular land-cover type. Burrough (1989) showed further handling of fuzzy spatial soil data, and Wang *et al.* (1990) discuss the manipulation of fuzzy attributes for land evaluation. Except for the remote sensing and soils examples, however, there seem to have been few attempts to define fuzzy spatial phenomena. The research reported here

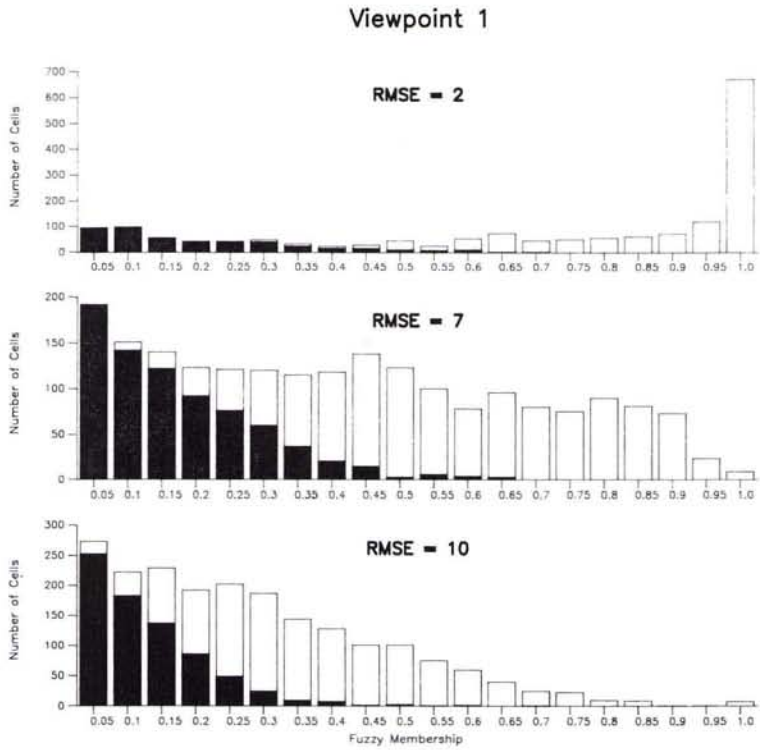

FIG. 5. Histograms of the number of cells in fuzzy viewsheds of Point 1 having fuzzy membership values from 0.05 to 1 and generated using variable RMSE.

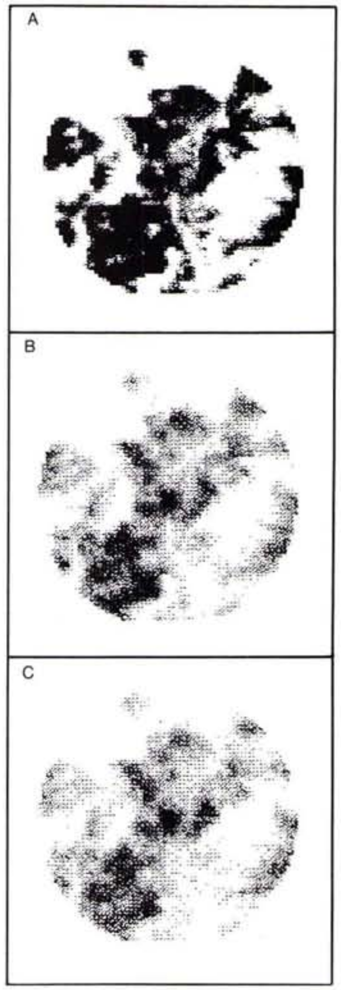

FIG. 6. Fuzzy viewsheds from Point 2 with (A) RMSE = 2, (B) RMSE = 7, and (C) RMSE = 10. Note that Figure 6b is the same as Figure 10a. Darker areas have higher fuzzy memberships.

explores a possible method to generate a fuzzy viewshed, and so is a novel aspect of GIS research. In the absence of other fuzzy data layers, however, the fuzzy viewshed cannot readily be used.

METHODS

STUDY AREA

A 200 by 200 subset of the Prentiss, North Carolina USGS 7.5-minute DEM was used in the current study. Two test viewing points were established and viewsheds were calculated to 1 km away from the point, with a viewing altitude of 2 m (approximately eye height) (Figure 1). This is all consistent with the previously reported experiments (Fisher, in press), and the 1 km viewing distance is used because that is a close approximation to the foreground (Felleman, 1986).

SOFTWARE

As in previous work, the Idrisi Viewshed program was used throughout the research reported here (Eastman, 1989). It had been found to perform well in test situations, and the file format for implementing the simulation algorithms was convenient.

RESULTS

Two sets of results are presented for each point. In one three different values of the root-mean-squared error (RMSE) are used, while the spatial autocorrelation (SA) is held constant at 0. In the second, the RMSE is held constant at the published value, 7, while three different values of the SA in the noise are used. In selecting intervals for the three values of RMSE, the concern was to span the published value, and 2, 7, and 10 were used. Values of $I = 0$, 0.7, and 0.9 were used for the *SA* in the noise, again to give a spread of increasing *SA*, as might be expected in the data. Both sets of three analyses are presented for the two viewing locations. The results of varying the RMSE is presented first, followed by those for variable SA.

For comparison, the original local elevation models and the Boolean viewsheds calculated in them are given in Figure 2 for Point 1 and Figure 3 for Point 2. The results of all analyses are presented as both grey-scale diagrams showing the fuzzy viewsheds (Figures 4, 6, 8, and 10), and in histograms (Figures 5, 7, 9, and 11). For a particular RMSE/SA combination, the histograms distinguish between cells which are in or out of the viewshed in the original DEM.

VARIABLE RMSE

Figures 4 and 6 show the results for Points 1 and 2 where RMSE is varied but SA is held constant at zero. Examination of both histograms shows that, as the RMSE is varied from 2 to 7 and 10, the modal value of the frequencies moves from 1.0 when RMSE = 2 to 0.05 when RMSE = 10. Indeed, the number of cells with fuzzy membership 0.05 reduces to only 8 when RMSE = 7 and 10 for Point 2 (Table 2) and to 9 and 8 for Point 1 (Table 1). Eight is in fact the minimum value in this category, because that is the number of immediate neighbors of the viewpoint, and those are always visible.

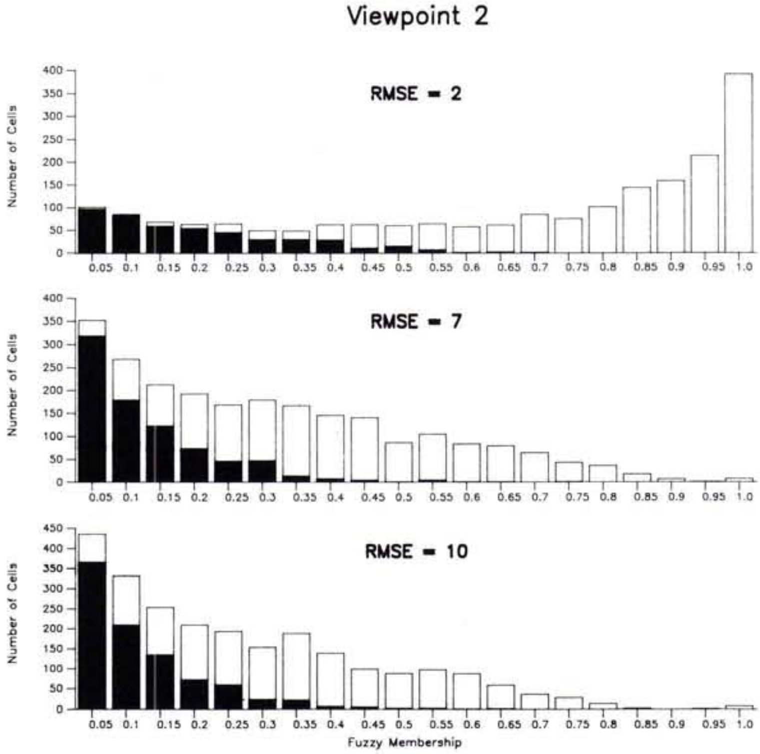

FIG. 7. Histograms of the number of cells in fuzzy viewsheds of Point 2 having fuzzy membership values from 0.05 to 1 and generated using variable RMSE.

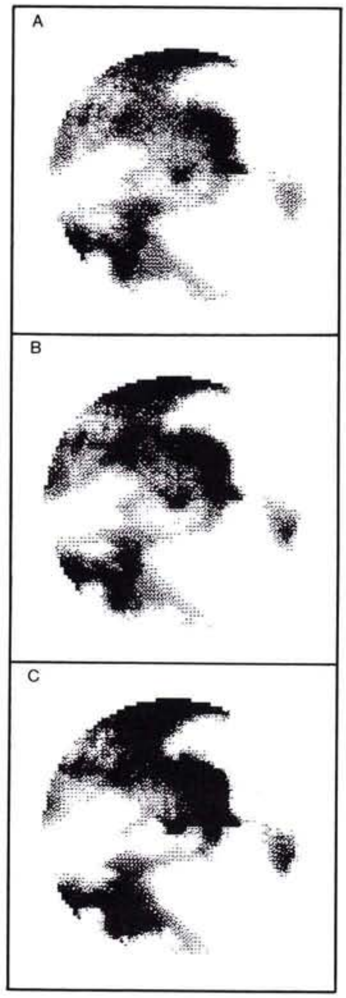

FIG. 8. Fuzzy viewsheds from Point 1 with variable SA, (A) I = 0, (B) I = 0.7, and (C) I = 0.9. Note that Figure 8a is the same as Figure 4b. Darker areas have higher fuzzy memberships.

TABLE 1. THE NUMBER OF CELLS THAT ARE WITHIN 1 KM OF THE VIEWPOINT BUT HAVE 0 FUZZY MEMBERSHIP OF BEING WITHIN THE VIEWSHED

	Point 1	Point 2
RMSE = 2	1774	1468
RMSE = 7	1457	1125
RMSE = 10	1474	1053
I = 0	1457	1125
I = 0.7	1426	921
I = 0.9	1577	898

The total number of cells certain to be outside the viewshed (the number of cells with fuzzy membership 0) decreases with RMSE (Table 1), although the number outside the viewshed of Point 1 when RMSE is 7 is lower than when RMSE is 10. This overall decrease swells the number of cells with low fuzzy membership that were not in the original viewsheds. The number of cells that were within the original DEM, but have little possibility of being within the fuzzy viewsheds (membership 0), increases from 0 for both points 1 and 2, to 5 and 31, respectively, when RMSE = 10 (Figures 5 and 7).

Finally, it is of interest to note that, irrespective of the RMSE or the viewpoint, the fuzzy membership of the cell most likely to be within the viewshed, but which was not within the viewshed of the original DEM, is between 0.65 and 0.75 in every case and with no particular pattern (Figures 5 and 7). The overall effect of increasing the RMSE in the noise then is to make the fuzzy viewsheds increasingly chaotic, and less certain.

Maps of the fuzzy viewsheds (Figures 4 and 6) show that in the simulations with RMSE = 10 it is possible to identify core areas of high membership which expand in those with RMSE = 7 and 2. In the case of both viewpoints 1 and 2, all fuzzy viewsheds shadow the Boolean viewsheds (Figures 2b and 3b) with variable memberships. In addition, with increasing RMSE the fuzzy viewsheds break up into a greater number of separate groups of contiguous cells (Table 2 and Figures 4 and 6).

TABLE 2. THE NUMBERS OF GROUPS OF CONTIGUOUS CELLS IN THE VIEWSHEDS OF POINTS 1 AND 2. GROUPS IN FUZZY VIEWSHED WERE FOUND FOR ALL NON-ZERO CELLS

Number of Polygons		Point 1	Point 2
In Original Viewshed		6	12
In Fuzzy Viewsheds	RMSE = 2	4	9
	RMSE = 7	5	11
	RMSE = 10	7	11
	I = 0	5	11
	I = 0.7	5	5
	I = 0.9	2	3

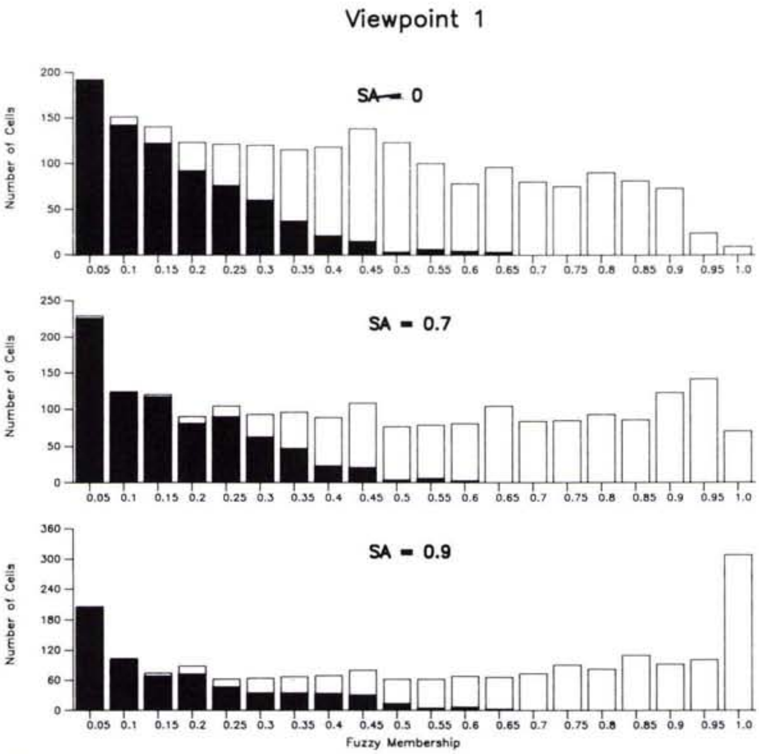

FIG. 9. Histograms of the number of cells in fuzzy viewsheds of Point 1 having fuzzy membership values from 0.05 to 1 and generated using variable SA.

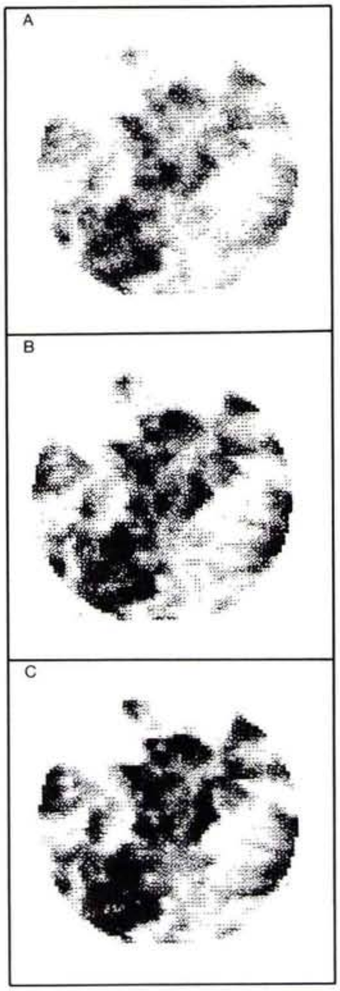

FIG. 10. Fuzzy viewsheds from Point 2 with variable SA, (A) I = 0, (B) I = 0.7, and (C) I = 0.9. Note that Figure 10a is the same as Figure 6b. Darker areas have higher fuzzy memberships.

VARIABLE SA

The results of applying noise with variable SA (Moran's I) and with RMSE = 7, the value specified for this DEM, are reported in Figures 8 and 9 for Point 1, and Figure 10 and 11 for Point 2. The results with I = 0 are repetitions of those with RMSE = 7 in Figures 4 to 7.

With increasing SA in the noise, the number of cells with high fuzzy memberships increases from 9 to 71 and 308 in the case of Point 1 (Figure 9). At the same time, the total number of cells with 0.05 fuzzy membership also increases from 192 to 229 and 206 (Figure 9). With increasing SA, therefore, the histogram of fuzzy memberships appears to become more multimodal. The pattern is, however, inconsistent between view points. Thus, when I = 0.9, fuzzy memberships of cells within the viewshed of Point 1 have modal values of 0.05 and 1 with the single most frequent fuzzy membership being 1. In the case of Point 2, the modal membership is 0.05 while only minor modes occur at 0.50 and 0.9 (Figure 11). Viewsheds for DEMs with highly autocorrelated noise also show a general evening of the extremely skewed distributions that are found when SA = 0 (compare Figures 5 and 9 and 7 and 11). Increasing SA therefore produces a very different, and more complex, response than varying the RMSE.

As the SA increases, the number of cells that are identified as not being within the viewshed increases (Table 1). Furthermore, the number of cells that may be within the viewshed (>0 in the fuzzy viewshed), but are not in the Boolean viewshed, increases with autocorrelation at Point 2 (822, 1013, and 1034 for I = 0, 0.7, and 0.9, respectively). At Point 1 they decrease (733, 804, and 655, respectively). The upper frequencies of cells outside the original viewshed changes very little, either between or within viewpoints (memberships of 0.6 to 0.8 can be noted) (Figures 9 and 11).

The spatial distribution of these fuzzy values are shown in Figures 8 and 10. In both areas, the application of increasing SA in the noise has a similar effect as decreasing RMSE (Figures 4 and 6). The fuzzy memberships of similar areas within the viewshed progressively increases. Thus, the zones with high fuzzy membership, which are identifiable in those viewsheds where the noise term had I = 0 (Figures 2b and 3b), are also identifiable in images where the noise had I = 0.7 and 0.9. Furthermore, the fuzzy viewshed with higher SA in the noise (Figures 8c and 10c) have relatively fewer separate groups of contiguous cells in the viewshed than do the fuzzy viewsheds found from lower SA noise. All the noisy viewsheds appear to have fewer groups of contiguous cells than the Boolean viewshed (Table 2).

DISCUSSION

DIFFERENCES BETWEEN VIEWING POINTS

The results show a number of differences between the effects of varying noise parameters on the different viewing locations. The two points were purposely chosen to be in very different terrain positions. Point 1 is on a ridge top which might be expected to have a large viewable area, while Point 2 is not only

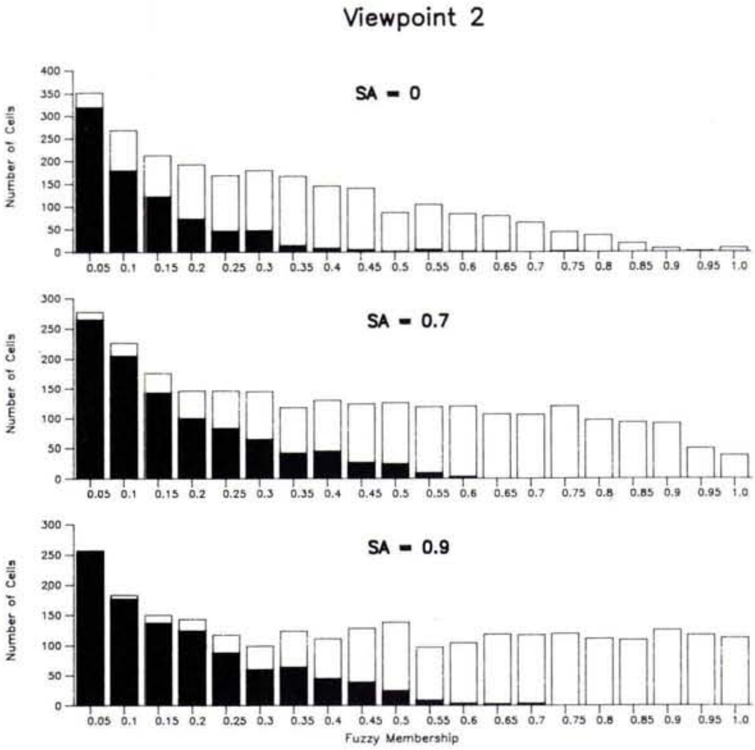

FIG. 11. Histograms of the number of cells in fuzzy viewsheds of Point 2 having fuzzy membership values from 0.05 to 1 and generated using variable SA.

TABLE 3. THE RATIO OF THE AREAS OF THE VIEWSHED WITH FUZZY MEMBERSHIP 0.5 TO THOSE WITH MEMBERSHIPS 0.05

Area Ratios	Point 1	Point 2
RMSE = 2	0.73	0.70
RMSE = 7	0.40	0.22
RMSE = 10	0.17	0.17
I = 0	0.40	0.22
I = 0.7	0.49	0.42
I = 0.9	0.58	0.49

in a valley bottom location, but is also at the mouth of a tributary valley, and so has a more limited view.

The basic difference in the viewsheds from the two locations can be seen in Figures 2b and 3b and Table 2. Most notable is the splintering of the viewshed of Point 2 (Figure 3b) as compared to that of Point 1 (Figure 2b); there are 12 distinct contiguous groups of cells in the former and six in the latter. All fuzzy viewsheds of Point 2 show some reduction in this number, and the most extreme reduction is down to three groups when I = 0.9. The fuzzy viewshed for Point 1 where RMSE = 10 actually yields one further group and in three cases there is only a reduction of 1; the lowest number of contiguous groups again when I = 0.9 is 2. Thus, the effects on the total possible viewshed are more dramatic on the valley bottom location than on the high viewing point.

A further difference between the results for the two points are the extremely bimodal distribution of fuzzy memberships of the viewsheds developed with noise with high spatial autocorrelation at Point 1, where a much less well developed multimodality is apparent in the fuzzy viewsheds of Point 2 (Figures 9 and 11). The variability in the change in the frequency of cells outside the viewshed may be a feature related to local terrain effect (Figures 5, 7, 9, and 11), particularly the increase noted for Point 1 with increasing SA, and the decrease for Point 2 (Figures 9 and 11).

The viewshed of the higher, ridge-top location (Point 1) appears to be somewhat more stable than that of Point 2. The ratio of the number of cells with fuzzy membership 0.5 or greater to the number of cells with 1 or greater (Table 3) varies less for Point 1 than for Point 2, when the RMSE is manipulated between 2 and 7. The ratios for variable SA in the noise are consistently less for Point 2 and for Point 1. The values for RMSE = 2 and 10, on the other hand, are remarkably similar.

Table 2 shows that the ridge-top location, Point 1 (Figure 2), has a higher frequency of high memberships in the image, probably due to its being in a more exposed location. The areas of high memberships of the fuzzy viewshed are more contiguous for this location than for Point 2 (see Figure 3); most of the high membership values are in three blocks of land, one immediately to the west of the viewpoint, one to the southeast, and the other to the north. From Point 2 (Figure 3), the areas of greater membership are by contrast highly disjoint, although one large area does exist to the southeast.

With only two test locations, it is not possible to state whether the observed differences between the fuzzy viewsheds are due to the locations of the two points, or to terrain that is within the viewshed. It is likely to be both to some extent, but terrain control should form a focus for further work.

LIKELY VIEWSHEDS

From the fuzzy viewsheds presented above, it is possible to derive versions of the viewshed with any desired level of fuzzy membership. The algorithm used here enables multiple parameterization of the noise. Although it is very likely that a high degree of SA occurs in the noise in the DEM data, there is no empirical evidence of this, and no basis for actually using simulations with anything but I = 0. Thus, in examining the possible applications of the fuzzy viewsheds generated here, only those with I = 0 and the published 7 RMSE are examined further. In much fuzzy logic work, the membership value of 0.5 is taken as a threshold value upon which to harden a phenomena (e.g., Burrough, 1989), and so, using this threshold, it is possible to harden the fuzzy viewsheds back to a Boolean set (Fig-

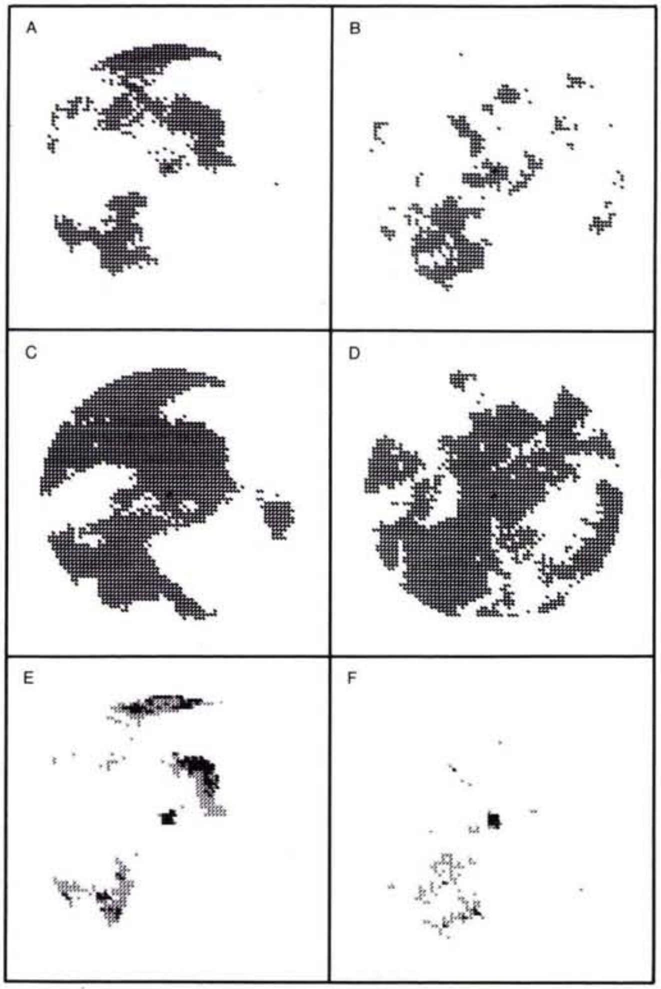

FIG. 12. Alternative realizations of the Boolean viewshed using different thresholds. (A) and (B) show viewsheds from Points 1 and 2, respectively, with threshold membership 0.5; (C) and (D) show viewsheds with threshold membership 0.1; and (E) and (F) show viewsheds with threshold membership 0.75 (grey) and 0.9 (black).

TABLE 4. CROSSTABULATIONS OF THE ORIGINAL BOOLEAN VIEWSHEDS WITH THE BOOLEAN VIEWSHEDS WHERE FUZZY MEMBERSHIP IS 0.5, GENERATED WITH RMSE = 7 AND SA = 0, FOR POINTS 1 AND 2.

		Point 1		Point 2	
		In	Out	In	Out
Viewshed Membership	In	520	9	813	16
> 0.5	Out	1025	1922	462	2213

ures 12a and 12b). The area of each viewshed is dramatically altered from its original area and distribution (Figures 2a and 3a; Table 4).

A more flexible approach to generating Boolean viewsheds may be adopted, however. The threshold value of p required in any particular situation is likely to depend on the application. If the viewshed is to be used to identify the site for a construction project which is likely to receive an unfavorable response from those living in the neighborhood, then a relatively low membership might be used because it is necessary to define a liberal viewshed to minimize the locations that might even possibly be impacted. Thus, a membership value of 0.1 might be selected (Figures 12a and 12b). Alternately, if a look-out position is to be established for forest-fire or military observation, then a conservative viewshed might be more desirable, and a fuzzy membership of 0.9 might be selected as a threshold. In the fuzzy images with RMSE = 7 and I = 0, few to very few cells have memberships 0.9 or greater (106 and 16; Figures 5, 7, 12c, and 12d), and most users would therefore feel that this image is almost useless. Lower membership values might therefore be explored, such as 0.75 (Figures 12c and 12d). The user may, however, select the value they wish to use in the particular context of their work.

CONCLUSION

Most personal experience shows that the viewshed is not really a Boolean phenomenon. Consider an individual standing at a particular viewing location. Small movements on the part of the individual may change the viewable objects. Thus, any particular point has some likelihood of being within the viewshed; for many it is small and for others it is large. Thus, a more valid representation of the viewshed should contain some measure of the certainty of each location in the database being within the viewshed. The research reported here presents a method for producing a number of different realizations of such a measure of certainty for a viewshed: membership of the fuzzy set of the locations visible from a point.

The variability in the frequency of the fuzzy memberships of the viewshed shows that the exact parameterization of the simulation is very important to the fuzzy viewshed derived, and to the possible products (alternative viewsheds, ratios, etc.). Varying the value of the standard deviation (RMSE) causes merely a shift in the modal fuzzy membership value (Figures 5 and 7), but increasing the spatial autocorrelation is seen to cause less predictable results (Figures 9 and 11). As long as a model of error is used, however, there seems no logical reason to use anything except zero spatial autocorrelation in the simulated noise, because of an absence of either empirical evidence to the contrary, or reporting of levels of autocorrelation.

The fuzzy viewshed may be used to derive alternate Boolean viewsheds to be used in usual GIS operations, depending on the applications and the thresholds that may be acceptable. The fuzzy viewshed has a more interesting future, however, as one data layer of a system designed for handling fuzzy spatial data; a fuzzy GIS. In conclusion, two points can be noted:

- the methods for deriving the fuzzy spatial phenomena may be very complex and require considerable parameterization and research; and
- from a consideration of the error contained in a particular type of spatial data, a derivative product (the fuzzy viewshed) has been formed, which is a better reflection of the real-world experience of human beings, and so considerably enhanced over the normal product (the Boolean viewshed).

ACKNOWLEDGMENTS

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