Derivation and Applications of Probabilistic Measures of Class Membership from the Maximum-Likelihood Classification

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ABSTRACT: The maximum-likelihood classification of remotely sensed data involves considerable computational effort, in the process calculating a large amount of information on the class membership characteristics for each case (e.g., pixel). Little of this information, however, is made available in the conventional output, which consists simply of the most likely class of membership for each case. More of the information generated in the classification can be output, specifically the *a posteriori* probabilities and typicalities of class membership. Each of these measures conveys different information on the class membership characteristics of a case. They may therefore be used to improve substantially the value of the classification. They also indicate the quality of the classifier's allocations on a per-case basis, a valuable supplement to the classification accuracy statement. Two case studies are discussed which show examples of how the probability measures can enhance the value of a classification. First, for a classification of agricultural crops where the probability measures can be used to identify potentially misclassified cases. By directing ground surveys to these areas and, if necessary, modifying the class allocations accordingly, the classification accuracy can be increased. The second case study focuses on a considerably different environment, heathland. In this case some of the semi-natural heathland vegetation classes do not exist in relatively discrete classes as do the crops but instead lie along continua. The probability measures here were related to the composition of the vegetation canopy and could be used to model more realistically the manner in which these vegetation classes inter-graded than would be apparent from the conventional maximum-likelihood classification output.

INTRODUCTION

MAXIMUM-LIKELIHOOD CLASSIFICATION is one of the most used image processing routines in remote sensing. Considerable research has focused on this method of classification and the factors that influence its performance and value. For instance, issues such as the number, size, and location of training sites (Campbell, 1981; Labovitz, 1986; Foody, 1988), the nature of the discriminating variables (Swain and Davis, 1978), and the evaluation of classification accuracy (Aronoff, 1982; Congalton *et al.*, 1983; Rosenfield and Fitzpatrick-Lins, 1986) have been investigated and may influence the actual and perceived quality of a classification. One issue which often appears to be taken for granted, however, is the nature of the classification output which usually comprises only an image depicting the most likely class of membership for each case. Not only is this inappropriate for environments that display gradual changes (Robinove, 1981; Allum and Dreisinger, 1987; Wood and Foody, 1989) but it is wasteful of information generated within the classification (Trodd *et al.*, 1989). This information can be used to calculate two measures related to the strength of class membership for each case, which will be referred to throughout as probability measures. These measures are the typicality and *a posteriori* probability. Each of these measures provides different information on the possible membership of a case to a class. The aim of this paper is to briefly review these measures and, with reference to two case studies, illustrate how and when this information may be exploited to improve the classification from a user's perspective.

MAXIMUM-LIKELIHOOD CLASSIFICATION AND THE DERIVATION OF PROBABILITY MEASURES RELATING TO CLASS MEMBERSHIP

The Gaussian-based maximum-likelihood classification is based on an estimated (Gaussian) probability density function for each of the reference classes under consideration; the class statistics are obtained from the training data. Assuming equal prior probabilities (Strahler, 1980) this can be expressed as Equation 1: i.e.,

$$P(\mathbf{x}_k|i) = \frac{1}{(2\pi)^{n/2}|\mathbf{V}_i|^{1/2}} \exp[-1/2(\mathbf{x}_k - \mathbf{u}_i)^T \mathbf{V}_i^{-1} (\mathbf{x}_k - \mathbf{u}_i)] \quad (1)$$

where $p(\mathbf{x}_k|i)$ is the probability density function for a pixel $\mathbf{x}_k$ as a member of class i, n is the number of wavebands, $\mathbf{x}_k$ is the data vector for the pixel in all wavebands, $\mathbf{u}_i$ is the mean vector for class i over all pixels, and $\mathbf{V}_i$ is the variance-covariance matrix for class i (Thomas *et al.*, 1987). The term $(\mathbf{x}_k - \mathbf{u}_i)^T \mathbf{V}_i^{-1} (\mathbf{x}_k - \mathbf{u}_i)$ is the Mahalanobis distance between the pixel and the centroid of class i which is a measure of how typical the pixel is of that class; typicality is negatively related to the distance between the pixel and class centroid. Typicality can be perceived as the tail area probability associated with a case for a particular class. This is illustrated in Figure 1 which shows the typicality probabilities associated with a pixel $\mathbf{x}_a$ for a simple one-band two-class situation. Typicalities can be derived from the Mahalanobis distance measurements with reference to an F distribution although a chi-squared approximation is often used (McKay and Campbell, 1982; Campbell, 1984).

In the maximum-likelihood classification, pixels are allocated

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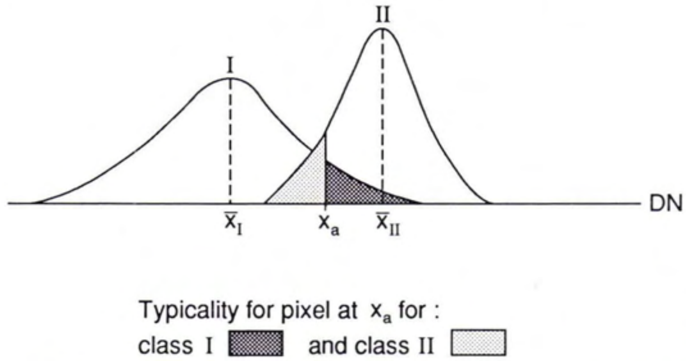

FIG. 1. Frequency histogram illustrating the typicalities of $\mathbf{x}_a$ with respect to classes I and II in a one waveband example.

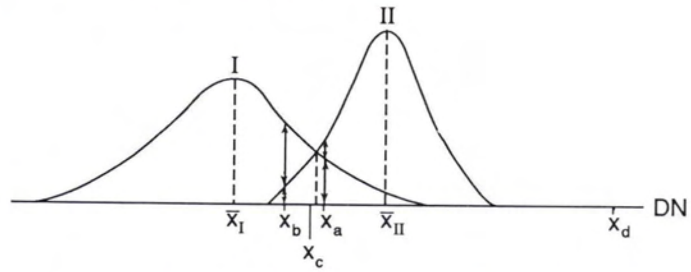

FIG. 2. Forced labeling with Gaussian probability density functions for one waveband to two classes (see text for explanation).

to their most likely class of membership. Given equal *a priori* probabilities, this can be achieved by allocating each case to the class with the highest probability density function, or equivalently, by allocating each pixel to the class with which it has the highest *a posteriori* probability of membership. For equal *a priori* probabilities, the *a posteriori* probabilities are assessed as the probability density of a case for a class relative to the sum of the densities. The *a posteriori* probability of a pixel $\mathbf{x}_k$ belonging to class i, $L(i|\mathbf{x}_k)$, may be determined from Equation 2: i.e.,

$$L(i|\mathbf{x}_k) = \frac{P_i \, p(\mathbf{x}_k|i)}{\sum_{r=1}^{t} P_r \, p(\mathbf{x}_k|r)} \tag{2}$$

where i is the class number, t is the total number of classes, and P_i is the *a priori* probability of membership of class i. The *a posteriori* probabilities sum to 1.0 for each pixel. Knowledge of the magnitude of the *a posteriori* probabilities displayed by a pixel to a group of classes may be of value because they reflect the confidence a user may place in the allocation. For instance, in the one-waveband two-class situation illustrated in Figure 2, the evidence for allocating pixel $\mathbf{x}_b$ to class I is stronger than that for allocating pixel $\mathbf{x}_a$ to class II; the ratios of the densities are 4:3 in favor of $\mathbf{x}_a$ belonging to class II and 5:1 for $\mathbf{x}_b$ belonging to class I. This information, together with that on the similarity of a pixel to a class conveyed by the typicality, is a valuable addition to the standard classification output which conveys no information other than the single most likely class of membership. The standard classification output therefore would not provide any distinction between a pixel which is spectrally similar to a single reference class (e.g., $\mathbf{x}_b$ in Figure 2) and one which displays a high level of similarity with a number of classes (e.g., $\mathbf{x}_c$ in Figure 2) or even one dissimilar to all reference classes (e.g., $\mathbf{x}_d$ in Figure 2). The *a posteriori* probabilities give the relative probabilities of a case belonging to each class in turn, on the assumption that the case belongs to one or the other of the classes, while the typicality probabilities indicate whether it is reasonable to assume that a case actually belongs to a class. These two probability measures encountered in the course of the maximum-likelihood classification should therefore be able to convey useful information on the class membership properties of a pixel to the user.

APPLICATIONS

The probability measures relating to class membership may be used in a variety of ways to improve the quality of a classification. They can, for instance, provide information on the confidence a user can place in the classification – are the *a posteriori* probabilities all close to 1.0, or are they only marginally greater than those for another class?, and are the labeled pixels typical of the nominal class of allocation or are they spectrally dissimilar? One widely used approach associated with the latter issue is to modify the display of the classified image to leave unclassified those pixels which have a typicality probability to their most likely class of membership below a pre-set threshold; many digital image processors include this function in their maximum-likelihood classification routines. Additional possibilities exist once functions which display images depicting typicalities and *a posteriori* probabilities are available, as with some image processing systems. The *a posteriori* probabilities and/or typicalities could be used, for example, to modulate the intensity of the color guns of an image display (Wallace and Campbell, 1988; Hobbs *et al.*, 1989). This would indicate cases that appear, for instance, satisfactorily classified (high *a posteriori* probability and high typicality to allocated class) and those less satisfactorily classified. The latter could include cases that are spectrally closest to their allocated class but atypical of that class (high *a posteriori* probability, low typicality). This may indicate that the training sets are inadequate descriptors of the class, that this particular case has unusual properties (e.g., diseased crop or lodged crop) or the presence of a mixed pixel, among other factors. Cases displaying a low typicality to their most likely class of membership would represent those which are spectrally distinct from the specified class.

The probability measures may be used in other ways to improve a classification, and two case studies to illustrate potential applications are given below.

CLASSIFICATION OF AGRICULTURAL CROPS

Crop maps are required for a variety of applications ranging from general inventory requirements to the enforcement of quota limits (Jewell, 1989). Considerable attention has therefore focused on the classification of agricultural crops from remotely sensed data. This section will show how the value of a classification can be improved by outputting information on the probabilities associated with class membership in addition to the most likely class of membership.

STUDY AREA

A 100 km² area centered approximately on the village of Feltwell, Norfolk, U.K. (Figure 3) was selected as the test site. The predominantly flat land of the test site was used mainly for arable agriculture with spring barley, sugar beet, and winter wheat as the main crops grown here with a lesser, but significant, proportion of the land planted to crops such as potatoes, carrots, and spring wheat.

DATA AND METHODS

X-band HH polarized synthetic aperture radar (SAR) data were acquired on four dates through the 1986 growing season for this test site as part of the European AgriSAR campaign (Anon, 1986). The data sets were co-registered and corrected radiometrically prior to classification (Foody *et al.*, 1989). Ground data for the site comprised a crop map, depicting crop type on a per-field basis, which was compiled from aerial photograph interpretation and field survey.

The aim of the investigation was to classify on a per-field basis the three major crops in the region; spring barley, sugar beet, and winter wheat. Twelve fields of each of these three crops were selected to train a maximum-likelihood classification in which the mean tone for each field from each of the four images was used as the discriminating variable. The latter was estimated from a sample area of at least 10,000 pixels located in the central portion of each field to avoid boundary effects. These data were used to classify a further 90 fields sampled from the area. These latter fields were used to evaluate the classification accuracy using Cohen's Kappa coefficient of agreement, $\hat{k}$ (Rosenfield and Fitzpatrick-Lins, 1986) expressed as a percentage.

Results and Discussion

Of the 90 fields classified, ten were misclassified and the overall accuracy was $\hat{k}$ = 83.33 percent (Table 1). This may be inadequate for some applications and also does not indicate classification quality for individual fields. The classification output could be enhanced by plotting the probability of correct allocation (Skidmore, 1989). Skidmore and Turner (1988) show how this can be achieved with a non-parametric classification. An alternative could be to plot the *a posteriori* probabilities derived in the maximum-likelihood classification. A problem with this, however, is that a case can be dissimilar to all classes and still exhibit a high *a posteriori* probability of membership to one of the classes. Furthermore, while there are differences in the magnitudes of the *a posteriori* probabilities of cases correctly and incorrectly allocated by the classification, they are sometimes small and insignificant (Table 2).

The typicalities can, however, be used to improve the classification. Of the several ways in which they can be used (see, for example, Campbell (1984) and Mather (1987)), the only commonly used approach is to set thresholds below which an observation is left unclassified (Schowengerdt, 1983). While this can increase classification accuracy, it can also result in large areas of the imagery being left unclassified. Furthermore, some cases that would be allocated correctly by the classification, but displayed a low typicality to their most likely class of membership, would be unclassified. This is illustrated in Table 3 which gives the results of a classification of the data with a 0.05 typicality probability threshold; cases with a typicality of <0.05 to their most likely class of membership are left unclassified. The cause of this is illustrated in Figure 4 which shows that, while cases are generally more typical of the class to which they belong than to other classes, the distributions are not as skewed as might be desired. Of the 13 cases which exhibited typicalities below the threshold and were consequently left unclassified, eight would have been correctly allocated in the absence of the threshold.

The typicalities may, however, be used to direct field surveys

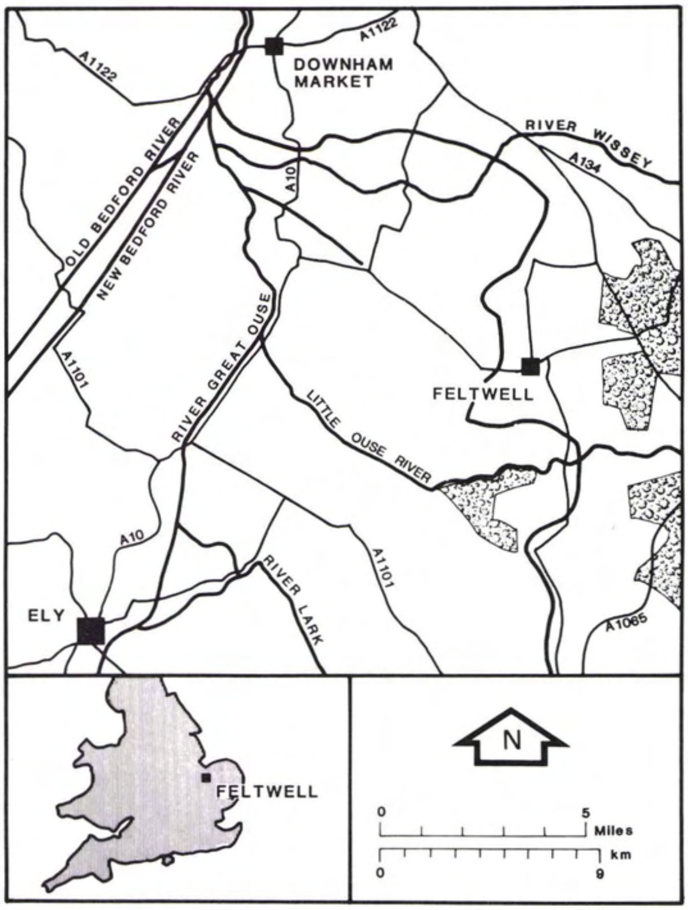

Fig. 3. Location map of the Feltwell test site.

Table 1. Classification Error Matrix for Maximum-Likelihood Classification with no Typicality Thresholds.

		Predicted Spring Barley	Sugar Beet	Winter Wheat	Total
	Spring Barley	27	0	3	30
	Sugar Beet	2	28	0	30
Actual	Winter Wheat	5	0	25	30
	Total	34	28	28	90

Cohen's $\hat{k} \times 100 = 83.33\%$.

Table 2. Variations in the *a posteriori* Probabilities of Class Membership for Cases Correctly and Incorrectly Allocated for each Class (where $\bar{x}$ = mean, σ = standard deviation, and min = minimum).

Crop	Allocated Class Correct/Incorrect	n	Posterior Probability $\bar{x}$	σ	min
Spring Barley	Correct	27	0.925	0.126	0.596
Spring Barley	Incorrect	3	0.747	0.110	0.649
Sugar Beet	Correct	28	0.994	0.027	0.854
Sugar Beet	Incorrect	2	0.991	0.012	0.981
Winter Wheat	Correct	25	0.859	0.131	0.552
Winter Wheat	Incorrect	5	0.700	0.169	0.547
		90			

Table 3. Classification Error Matrix for Maximum-Likelihood Classification in which Cases with a Typicality < 0.05 to their Most Likely Class of Membership are Left Unclassified.

		Predicted Spring Barley	Sugar Beet	Winter Wheat	Total
	Spring Barley	25	0	2	27
	Sugar Beet	0	22	0	22
Actual	Winter Wheat	3	0	25	28
	Total	28	22	27	77

Cohen's $\hat{k} \times 100 = 90.21\%$.

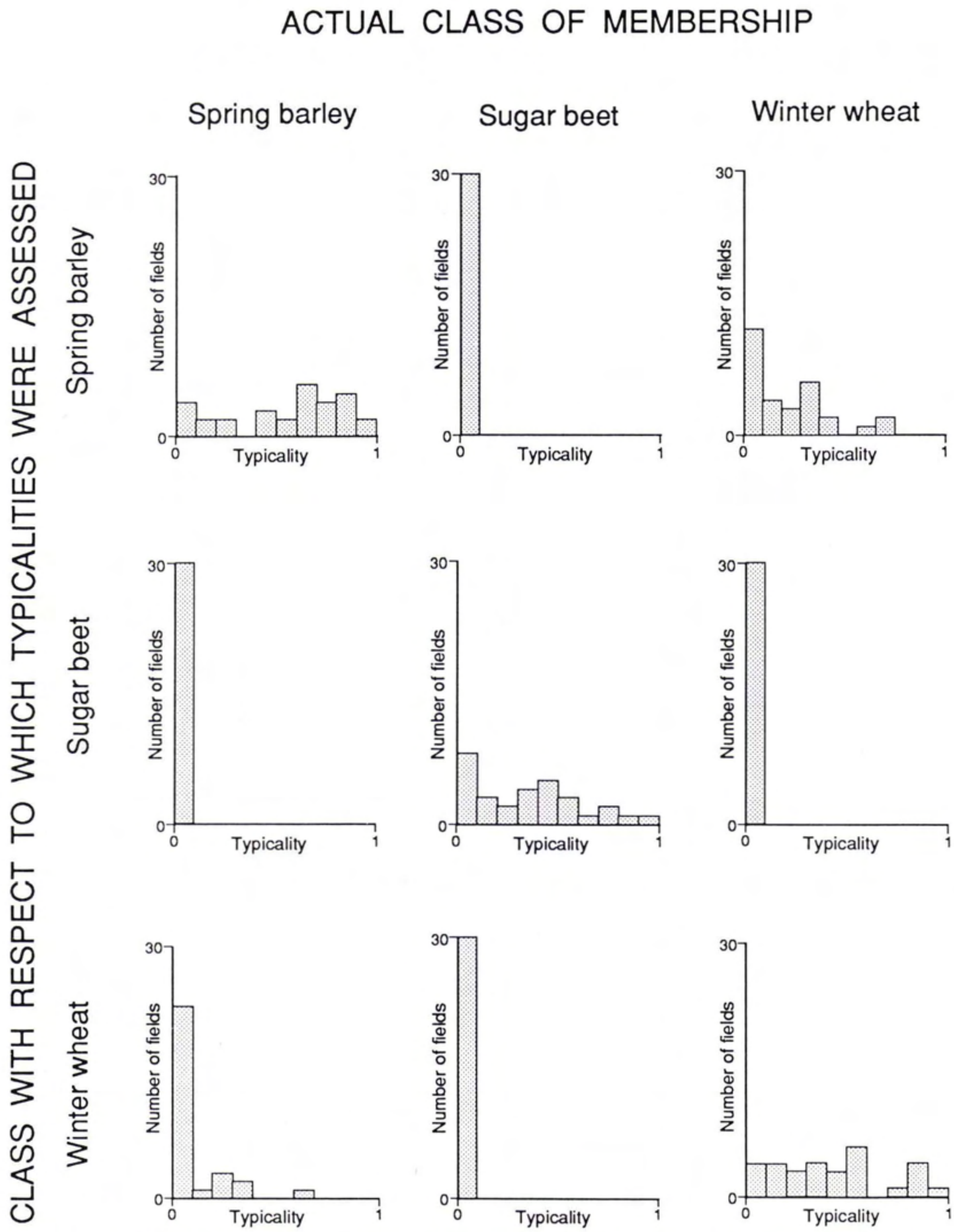

FIG. 4. Observed typicalities of class membership for all cases to each of the three classes.

to those regions where the classification may be inaccurate. Thus, cases displaying a low typicality of membership to their most likely class of membership may be visited in the field to have their true class membership determined. This does increase classification accuracy significantly (Table 4) but at the expense of an increase in the amount of ground survey required. This additional effort is, however, focused on those cases that have a high possibility of being misclassified as they appear atypical of their most likely class of membership. Larger and more significant increases in classification accuracy are observed if some cases are members of untrained classes (Foody, 1990). The latter may display high posterior probabilities of class membership but low typicalities, similar to the situation with pixel $\mathbf{x}_d$ in Figure 2.

MAPPING CONTINUA

The agricultural crops in the previous example can, to a large extent, be considered to exist in discrete classes. Semi-natural vegetation, however, display a more complicated spatial distribution because they exhibit continua, often lacking well-defined inter-class boundaries. Consequently, semi-natural vegetation classes tend to inter-grade gradually, rather than exist as a mosaic of geographically well-defined units as might be found, for instance, in agricultural regions or forest plantations. Furthermore, the zones where classes inter-grade, ecotones, are often of interest themselves (Johnston and Bonde, 1989). In these circumstances, application of conventional classification techniques is often less than ideal (Wood and Foody, 1989); however, such techniques have often been used to provide the user with a simplified map despite the acknowledged difficulty in placing a boundary in an image where in reality a gradient of change exists (Robinove, 1981; Allum and Dreisinger, 1987). Consequently, low classification accuracies are often obtained from attempts to classify such features. Felix and Binney (1989), for instance, noted that the continuous nature of the vegetation types they were mapping was to some extent responsible for the low accuracy – 37 percent for 13 classes – of their classification.

The conventional maximum-likelihood classification of a region of semi-natural vegetation would provide only the most likely class of membership for each pixel. No information on the gradients between classes or the confidence of the class allocations would be conveyed by this output. A pixel which came from a large area of a definable end point of a continua would, for instance, appear the same in the output as another close to the point where two, or more, classes intergrade. Thus, in Figure 5 a pixel at A would be as firmly allocated to woodland as one at B. Furthermore, a pixel at C, which possesses some woodland characteristics, would be allocated to grassland as firmly as a pixel at D. Therefore, in a classified scene, the pixel at C appears as different to B as D is from A.

The implications of continua on classification have been examined for an area of semi-natural vegetation. Particular attention focused on the relative ability of a maximum-likelihood classification and the probability measures which may be derived from it to model the continuous character of the vegetation.

Study Area

Pirbright Common, part of the Ash Ranges, Surrey, U.K. (Figure 6), was selected as the test site. It contains communities of wet and dry heathland that are of national importance (Harrison, 1970) and an area for which accurate up-to-date data are required for environmental monitoring. In regions where burning has been of suitable frequency and intensity, extensive areas of *Calluna vulgaris* and *Molinia caerulea* heath are found. On the steeper slopes, dry heath communities with *Ulex minor* are frequently observed and gulleys often contain flushes of wet heath. Wet heath/bog communities are found in the lower lying areas and these tend to be floristically variable in composition. Poor management and neglect of these areas permits the invasion by species of *Pteridium*, *Betula*, and *Pinus* and eventually the development of scrub woodland (NCC, 1985).

Data and Methods

Airborne thematic mapper (ATM) data were acquired in 11 wavebands within two hours of solar noon with a Daedalus 1268 scanner for the test site in March 1989.

At three periods, near the time of the ATM data acquisition and approximately one year before and after, ground data were recorded at the site. Because the latter related only to land cover, which may be considered intransient at this site and scale, all the ground data were amalgamated; care was taken to ensure that areas which had undergone change (e.g., burn regeneration) were excluded from the investigation. Ground data were acquired for a total of 107 sample sites. At 92 of these only the land-cover class was recorded because these sites were located in regions thought to be representative of a single class or end

Table 4. Classification Error Matrix for Maximum-Likelihood Classification in which Cases with a Typicality <0.05 to their Most Likely Class of Membership Have Their Class Membership Determined by Ground Survey.

		Predicted Spring Barley	Sugar Beet	Winter Wheat	Total
Actual	Spring Barley	28	0	2	30
	Sugar Beet	0	30	0	30
	Winter Wheat	3	0	27	30
	Total	31	30	29	90

Cohen's $\hat{k} \times 100 = 91.66\%$.

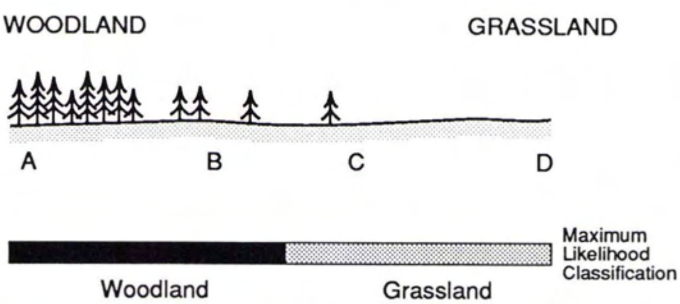

Fig. 5. The inappropriateness of classification for the representation of continuous classes (see text for explanation).

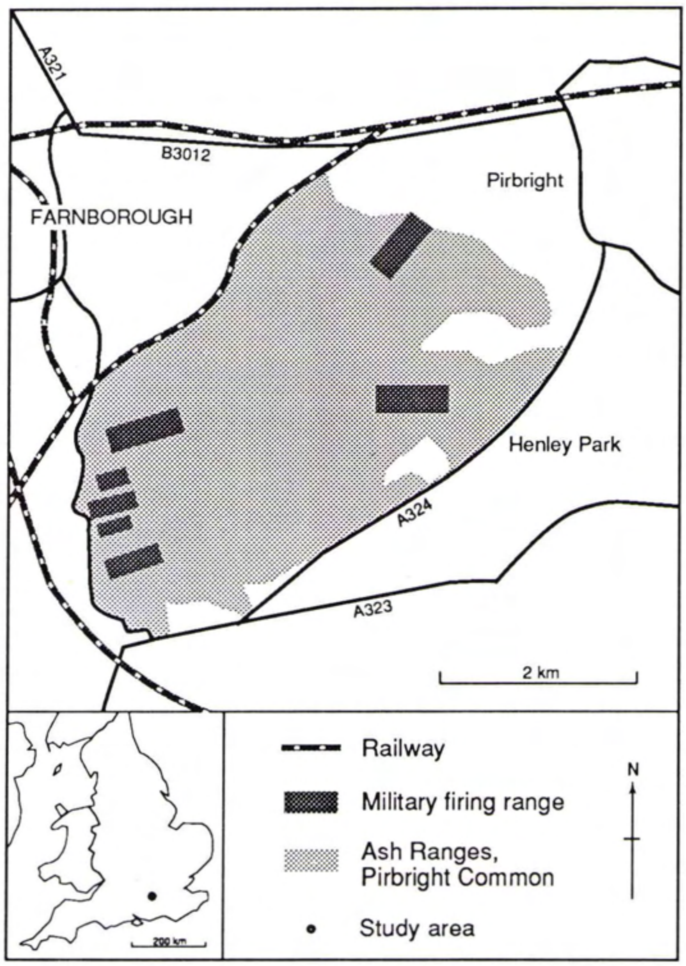

Fig. 6. Location map of the Ash Ranges test site.

point. In total, four classes were sampled – dry heath, wet heath/bog, coniferous woodland, and mixed woodland – and the data were used to assess the spectral separability of the class end points. The remaining 15 ground data sample points were located along a transect that extended from a region of dry heath to one of wet heath/bog. At these sample sites detailed data on the species composition of the heathland were acquired from 16m^2 quadrats which, for the purpose of this investigation, were degraded to show the proportion of dry heath and wet heath/bog species present.

After smoothing the imagery with a 3 by 3 mean filter to reduce possible mislocation errors, the pixels corresponding to the ground data sample sites were identified. For the 92 sites representative of the class end points, the DN in 10 of the 11 ATM wavebands recorded were extracted and used to assess the spectral separability of the classes; the 1.55- to 1.75-μm waveband was excluded from all analyses because of severe radiometric distortion in that data set. The DN of the samples representative of the dry heath and wet heath/bog classes in the 2.08- to 2.35-μm waveband were also used to define the spectral responses of these classes in that waveband. These were used to calculate the probability measures relating to class membership for the 15 sample sites located on the transect, the DN of which had also been extracted in the 2.08- to 2.35-μm waveband. Only the latter waveband was used in this part of the investigation because it is particularly sensitive to moisture content, the transect traversed effectively a moisture gradient, and because this waveband provides a high degree of interclass separability (Wilks' Lambda, negatively related to class separability (Klecka, 1980) = 0.079).

Results and Discussion

To represent the heathland environment more appropriately than a classification, the output of an analysis should model the continuous character of the vegetation classes. Probability mapping aims to achieve this by indicating the spatial variations in the relative strengths of class membership. This should enable the distinction between areas which appear to be representative of the class end points and those where the classes intergrade. It is therefore essential that the analysis discriminates accurately between the class end points, and that the probabilities of class membership are related to canopy composition. The initial analysis therefore aimed to evaluate the separability of the class end points. This was achieved with a maximum-likelihood classification of the DN of the 92 pixels in the ten ATM bands sampled over the test site encompassing the four main land-cover classes. All the data were used to both train and test the analysis. While inter-class separability will be inflated by this approach (Swain and Davis, 1978), the result – 100 percent correct allocation – indicated that the end points of the main heathland classes may be discriminated to a high level of accuracy from these data.

The extent to which the measures of the probability of class membership were sensitive to canopy composition was investigated using the data recorded along the transect. The results showed that there was a significant relationship between the probabilistic measures of class membership and canopy composition. Table 5 shows the correlation coefficients observed between the probability of dry heath membership and the proportion of dry heath vegetation for each of the measures of probability. Because the probabilistic measures are strongly related to the composition of the canopy, probability mapping should therefore be an appropriate technique for the representation of heathland from remotely sensed data (Foody and Trodd, 1990); an example of such a representation which concurs with field observation can be found in Wood and Foody (1989). Furthermore, a spatial presentation of results illustrates that variations in the *a posteriori* probability values, the measure most strongly correlated with canopy composition, modeled the heathland variations more realistically than the allocation of a maximum-likelihood classification (Figure 7). This was most apparent where the maximum-likelihood allocations were incorrect and near the middle of the transect where a small tract of dry heath disturbed the general transition from dry heath to wet heath/bog.

DISCUSSION AND CONCLUSIONS

The maximum-likelihood classification involves considerable computational effort and generates a substantial amount of information on the class membership properties of a pixel. The output of the classification, an image depicting the code of the most likely class of membership, is, however, wasteful of much of this information and inappropriate for the representation of continuous classes. Furthermore, this output does not provide any information on the confidence a user could place on the correctness of the classifier's allocation on a per-pixel basis, although the probability measures generated in the course of the classification provide valuable information on the relative similarity of a pixel to the defined classes. Making fuller use of the probability measures would therefore enable, for example, an indication of per-pixel classification quality that would be a valuable supplement to the conventional classification accuracy statement. The latter normally indicates the overall level of classification accuracy and cannot be associated reliably with individual pixels. This additional information on per-pixel classification quality may be of particular value if the results of the classification are to be integrated into a geographical information system (GIS) where information on data quality is of considerable importance (Burrough, 1986; Walsh *et al.*, 1987). The main drawback is the increased data storage required, although it would not be essential for all the probabilities derived in the classification to be incorporated into the GIS in addition to the conventional classification output. The conventional classification and its associated accuracy statement could still be calculated but a user may, for instance, generate probability maps to represent continuous classes more satisfactorily than a classified scene. Alternatively, the probability measures may be employed to identify potentially misclassified cases and exclude these from further analyses undertaken in the GIS. The probability measures therefore can provide more meaningful and useful information for users, especially in combination with other

TABLE 5. SPEARMAN RANK CORRELATIONS BETWEEN THE PROPORTION OF WET HEATH/BOG AND DRY HEATH VEGETATION AT SAMPLE SITES ALONG THE TRANSECT WITH EACH OF THE PROBABILISTIC MEASURES. ALL THE CORRELATIONS WERE SIGNIFICANT AT THE 99 PERCENT LEVEL OF CONFIDENCE.

	Correlation coefficient	
Probabilistic measure	Wet heath/bog	Dry heath
Typicality probability	0.92	0.88
A Posteriori probability	0.87	0.87

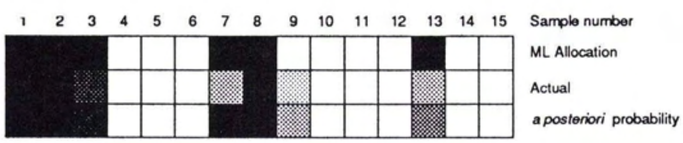

FIG. 7. A representation of the transect study results. For each of the 15 sites along the transect, the maximum-likelihood (ML) allocation, actual proportion of dry heath vegetation, and the probability density function (pdf) associated with dry heath are presented on an arbitrary intensity scale where black indicates dry heath and white wet heath/bog. Note the ML allocation is incorrect for sample numbers 7 and 13. (Not to scale).

data sets in the GIS, and provide an indication of classification quality on a per-case basis. The latter can aid the minimization of error in GIS analyses, a valuable asset, although, as noted above, it does increase the volume of data to be incorporated into the system.

In summary, three points have been made in this article:

- While a substantial amount of information on the class membership characteristics of each case is generated in a maximum-likelihood classification, very little is usually output. In the classification two measures relating to class membership are encountered: *a posteriori* probability and typicality. Outputting these measures in addition to the conventional output, most likely (forced) class of membership labels, provides considerably more information to users.
- The case studies showed examples of how the probability measures can be used to improve the value of a classification. In the case of the crop classification, typicalities were used to identify cases that were atypical of their most likely class of membership. This allowed field surveys to be directed to regions where there was a high possibility of misclassification and the classification modified accordingly, increasing classification accuracy. For the semi-natural vegetation, the probabilities were able to model more realistically the manner in which vegetation classes inter-grade than did the output of the conventional classification.
- The probability measures provide an indication of classification quality on a per-case basis. This is a valuable supplement to the conventional classification accuracy statement which can be at times only a poor indicator of the overall classification accuracy. Knowledge of the quality of the classification on a per-case basis will enable, for instance, fuller use of the classification within geographical information systems, where information on data quality is paramount to determining the system's value.

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