The Reversibility of Six Geometric Color Spaces

Tian-Yuan Shih

Abstract

Color coordinate systems provide a way to address, to describe, and to manipulate colors. In the field of image processing and computer graphics, several color models in common use include RGB, HVC, HSV, HLS, and IHS. In this study, the reversibility of several color coordinate systems and their numerical characteristics are studied. All these models are classified as user-oriented and related to the perceptual color space.

Introduction

Color provides a rich and informative attribute for sensory and visual communication. Color spaces provide a method to manipulate colors. For remote sensing applications, manipulating images in the perceptual color spaces is efficient and useful for image enhancement and compression (Kruse and Raines, 1984; Green, 1989). In the recent development of GIS (Geographic Information Systems), color space transformations were investigated extensively for data fusion, either for visual enhancement (Welch and Ehlers, 1987; Carper et al., 1990; Harris and Murray, 1990; Chavez et al., 1991; Grasso, 1993) or to improve the accuracy of classification (Munechika et al., 1993). Both increasing applications and the nature of color space itself deserve further exploration.

Early work on organizing colors into order includes Leonardo da Vinci’s Notebooks from about 1500. There are many ways to define, to describe, or to organize colors. One method is to use a three-dimensional coordinate system, the color model (Billmeyer and Saltznan, 1981). In a color model, each color is addressed by a coordinate pair. Based on various guiding principles, there are several coordinate arrangements that generate, in turn, color models. A color space, a color coordinate system, and a color model generally express the same meaning.

Gloss (1984) classified the color coordinate systems as:

- objective, such as CIEXYZ, and
- subjective, such as the Munsell system.

The major difference between these two categories is the underlying principle. The objective model is based on physical measurements, whereas the subjective model is based on human perceptual feeling. The subjective category is also termed “perceptual,” because in perceptual color spaces, colors are more uniform than in those objective models. Hence, color distances computed in perceptual models are closer to visual differences for a human viewer. Among RGB, YIQ, LAB, and opponent color models, the hexcone HSV model provides the best user interaction in terms of the accuracy (Schwarz et al., 1987).

This article investigates the question:

which model provides better reversibility with the 8-bit depth limitation?

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CIE-L*u*v* Model
In 1931, the Commission Internationale de l'Eclairage (C.I.E.) defined three standard primaries X, Y, and Z to replace red, green, and blue. Subsequently, the L*u*v* coordinate system became a C.I.E. standard in 1976. This color model provides a computationally simple measure of color in agreement with the Munsell color system, which is a perceptually uniform color space (Pratt, 1991).

The forward and backward transformations between RGB and L*u*v* color models are defined as follows:

The forward transformation equations for the NTSC receiver primary system are

\[ u' = \frac{u^*}{13L^*} + 0.201 \]
\[ v' = \frac{v^*}{13L^*} + 0.461 \]
\[ Y = \frac{1}{100} \left( \frac{L^* + 16}{25} \right)^3 \text{ if } L^* \geq 7.996248 \]
\[ Y = \frac{L^*}{903.3} \text{ if } L^* < 7.996248 \]
\[ X = \frac{9u'}{4v'} Y; \]
\[ Z = \left[ 12 - 3u^* - 20v^* \right] Y \]

The backward transformation equations for the NTSC receiver primary system are

\[ L^* = 25 \left[ 100Y/Y_0 \right]^{1/3} - 16 \text{ if } \frac{Y}{Y_0} \geq 0.008856 \]
\[ L^* = 903.3 \frac{Y}{Y_0} \text{ if } \frac{Y}{Y_0} < 0.008856; \]

\[ u^* = \frac{4X}{X + 15Y + 3Z} \]
\[ v^* = \frac{9Y}{X + 15Y + 3Z} \]
\[ u^* = 13L^*(u' - u_0) \]
\[ v^* = 13L^*(v' - v_0) \]

Color Bubble Model
Using a floating disk with variable radius, the color bubble, Smith (1993) derived the representation of colors in the HVC (hue, value, chroma) system. These three dimensions are defined as the dominant frequency (hue), luminance (value), and the amount of color (chroma). Although the color bubble model can be presented on plane paper, it actually defines a three-dimensional space.

As illustrated in Figure 2, a floating disk (color bubble) referred to one coordinate line has three degrees of freedom: the radius, center height, and the rotation. On this bubble, R, G, and B points are spaced evenly. The line from the center, which is normal to the horizontal axis, is the baseline. The Hue parameter is defined as the angle between the radius from the center to the green point and the baseline. The vertical coordinate of the center constitutes the color's Value, and the radius is considered as the Chroma. The transformations between this color coordinate system and RGB are

\[ V = (R + G + B)/3 \]
\[ H = \tan^{-1} \left( \frac{R - B}{\sqrt{3} \left( V - G \right)} \right) \]
\[ C = (V - G)/\cos(H) \text{ if } |\cos(H)| > 0.2 \]
\[ C = (R - B)/(\sqrt{3} \sin(H)) \text{ if } |\cos(H)| \leq 0.2 \]

and
Because Hue is measured as an angle, a singular case arises when the radius of the color bubble, the chroma, is zero.

The Color Bubble model is clearly analogous to a cylindrical coordinate system.

Hexcone Model

"Mimicking the way an artist mixes paints on his palette," Smith (1978) derived a color model based on the geometry of Hexcone (Figure 3). The three primaries are named hue, saturation, and value (HSV). The artist's tint, shade, and tone concepts (Figure 4) are intuitively incorporated:

He choose a pure hue, or pigment, and lightens it to a tint of that hue by adding white, or darkens it to a shade of that hue by adding black, or in general obtains a tone of that hue by adding some mixture of white and black, a gray (Smith, 1978).

Geometrically, the HSV model is the RGB cube tilted onto its back corner. It is formed by viewing the RGB color cube along its principal diagonal from white toward black (Figure 3). The top of this hexcone corresponds to the principal section of the color cube.

For a detailed derivation, see Smith (1978). The transformation described below is developed according to Levkowitz and Herman (1993) and Foley et al. (1990).

Let max represent the maximum value among \( R, G, B \), and min the minimum value,

\[
V = \max; \\
S = \frac{\max - \min}{\max}; \\
H = \begin{cases} 
H \text{ is undefined; otherwise,} & \text{if } S = 0, \\
\frac{G - B}{\max - \min} & \text{otherwise,}
\end{cases}
\]

Then, \( H \) can be scaled to \([0, 360]\) by multiplying by 60, or normalized to \([0, 1]\) by dividing by 6, or scaled by multiplying by \((255/6)\) to the range of \([0, 255]\). This transformation can be reversed with the following algorithm:

\[
\begin{align*}
&\text{If } S = 0, \text{ and } H \text{ is undefined; then } (R, G, B) = (V, V, V). \\
&\text{Otherwise, scale } H \text{ back to } [0, 6], \text{ let } i = \text{Floor}(H), \text{ in which } i \text{ is the sector number of the hue angle;} \\
&f = H - i; \text{ in which the } f \text{ is hue value in each sector;} \\
&\min = V(1 - S); \text{ mid1} = V(1 - S^1); \text{ mid2} = V(1 - S^1 - f); \text{ max} = V; \text{ then,} \\
i = 0, (R, G, B) = (\max, \text{ mid1}, \min), \\
i = 1, (R, G, B) = (\text{ mid2}, \max , \min), \\
i = 2, (R, G, B) = (\min, \max, \text{ mid1}).
\end{align*}
\]
i = 3, \( (R,G,B) = (\text{min}, \text{mid}2, \text{max}) \),
i = 4, \( (R,G,B) = (\text{mid}1, \text{min}, \text{max}) \),
i = 5, \( (R,G,B) = (\text{max}, \text{min}, \text{mid}2) \).

Triangle Model
Besides the hexcone model, Smith (1978) also described a triangle-based model. Utilizing the generalized brightness and weights generates a family of color models. Gonzalez and Woods (1992) presented a triangle-based model with derivations. Although there are some distinctions in the computational scheme for hue angles, this model is equivalent to the unbiased case of the triangle model of Smith (1978). A graphic illustration appears in Figure 5.

\[
I = \frac{1}{3} (R + G + B),
\]
\[
S = 1 - \frac{3}{(R + G + B)} [\text{min}(R,G,B)], \text{and}
\]
\[
H = \cos^{-1} \left\{ \frac{1}{2} \frac{(R - G) + (R - B)}{[(R - G)^2 + (R - B)(G - B)]^{1/2}} \right\}.
\]

The Hue is undefined when \( S = 0 \), and \( S \) is undefined when \( I = 0 \).

The ISH-to-RGB transformation is divided into three sectors:

\[
0 < H \leq \frac{2\pi}{3},
\]

\[
B = I(1 - S), \quad R = I \left[ 1 + \frac{S \cos H}{\cos \left( \frac{\pi}{3} - H \right)} \right],
\]

\[
G = 3I - (R + B);
\]

\[
\frac{2\pi}{3} < H \leq \frac{4\pi}{3},
\]

\[
R = I(1 - S), \quad G = I \left[ 1 + \frac{S \cos H}{\cos (\pi - H)} \right],
\]

\[
B = 3I - (R + G);
\]

\[
\frac{4\pi}{3} < H \leq 2\pi,
\]

\[
G = 3I(1 - S), \quad R = I \left[ 1 + \frac{S \cos H}{\cos \left( \frac{4\pi}{3} - H \right)} \right],
\]

\[
B = 3I - (G + B).
\]

Double Hexcone Model
When transforming a cylindrical coordinate system onto a cube, a double hexcone model results (Figure 6). The three coordinates of this color space are Hue, Lightness, and Saturation (HLS). The following transformations are developed based on Levkowitz and Herman (1993).

Let \( \text{max} \) represent the maximum value among \( (R, G, B) \), and \( \text{min} \) the minimum value; then,

\[
L = \frac{(\text{max} + \text{min})}{2}.
\]

If \( \text{max} = \text{min} \), then \( S = 0 \), and \( H \) is undefined. Otherwise,

\[
S = \frac{(\text{max} - \text{min})}{(\text{max} + \text{min})}, \text{when } L \leq 0.5;
\]

\[
S = \frac{(\text{max} - \text{min})}{(2 - \text{max} - \text{min})}, \text{when } L > 0.5.
\]

\[
H = \frac{(G - B)}{\text{max} - \text{min}}, \text{if } R = \text{max};
\]

\[
H = 2 + \frac{(B - R)}{\text{max} - \text{min}}, \text{if } G = \text{max};
\]

\[
H = 4 + \frac{(R - G)}{\text{max} - \text{min}}, \text{if } B = \text{max}.
\]

Then, scale \( H \) into the demanded range. The reverse transformation starts with rescaling the hue angles back into the range \([0, 6]\). Then,

\[
\text{if } S = 0, \text{ } H \text{ is undefined; then } (R,G,B) = (L,L,L).
\]

Otherwise, scale \( H \) back to \([0, 6]\); let
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Table 1. Range of Color Models

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>RGB</td>
<td>R [0, 255]</td>
<td>G [0, 255]</td>
<td>B [0, 255]</td>
<td></td>
</tr>
<tr>
<td>CIE-L* u* v*</td>
<td>L* [0, 720]</td>
<td>u* [-976, 1707]</td>
<td>v* [-1179, 895]</td>
<td></td>
</tr>
<tr>
<td>CIE-IHS</td>
<td>H [0, 2π]</td>
<td>S [0, 1]</td>
<td>V [0, 255]</td>
<td></td>
</tr>
<tr>
<td>Bubble</td>
<td>H [0, 2π]</td>
<td>C [0, 170]</td>
<td>V [0, 255]</td>
<td></td>
</tr>
<tr>
<td>Hexcone</td>
<td>H [0, 6]</td>
<td>S [0, 1]</td>
<td>V [0, 255]</td>
<td></td>
</tr>
<tr>
<td>Triangle</td>
<td>H [0, 2π]</td>
<td>S [0, 1]</td>
<td>V [0, 255]</td>
<td></td>
</tr>
<tr>
<td>Double Hexcone</td>
<td>H [0, 6]</td>
<td>S [0, 1]</td>
<td>L [0, 255]</td>
<td></td>
</tr>
<tr>
<td>Raines (1977)</td>
<td>H [0, 2π]</td>
<td>S [0, 208]</td>
<td>V [0, 422]</td>
<td></td>
</tr>
<tr>
<td>Pratt (1991)</td>
<td>H [0, 2π]</td>
<td>S [0, 233]</td>
<td>V [0, 255]</td>
<td></td>
</tr>
<tr>
<td>Niblack (1986)</td>
<td>H [0, 2π]</td>
<td>S [0, 208]</td>
<td>I [0, 255]</td>
<td></td>
</tr>
</tbody>
</table>

\[ i = \text{Floor}(f), \text{in which } i \text{ is the sector number of the hue angle;} \]
\[ f = H - i; \text{in which } f \text{ is the hue value in each sector.} \]

Depending on the lightness \( L \), there are two cases:

\[ L \leq L_{\text{critical}} = \left( \frac{127.5}{255} = 2 \right); \]
- \( \text{max} = L(1 + S); \)
- \( \text{mid1} = L(2(1 - 1 - S)); \)
- \( \text{mid2} = L(2(1 - f)S + 1 - S); \)
- \( \text{min} = L(1 - S); \)

\[ L > L_{\text{critical}} = \left( \frac{127.5}{255} = 2 \right); \]
- \( \text{max} = L(1 - S) + 255S; \)
- \( \text{mid1} = 2[(1 - f)S - (0.5 - f)_{\text{max}}]; \)
- \( \text{mid2} = L(1 - f) - (0.5 - f)_{\text{max}}; \)
- \( \text{min} = L(1 + S) - 255S; \)

The \((R, G, B)\) assignments are the same as those in Equation 7.

HSV Model (Raines, 1977)

Raines (1977) derived a color model based on cylindrical geometry which approximates the Munsell color system (Figure 7). The derivation was originally made for geological applications, and was applied for contrast stretching by Kruse and Raines (1984). The forward and backward transformations are listed in Equation 13 and 14: i.e.,

\[
\begin{bmatrix}
V \\
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
\sqrt{3} & \sqrt{3} & \sqrt{3} \\
3 & 3 & 3 \\
\sqrt{6} & \sqrt{6} & \sqrt{6} \\
6 & 6 & 3 \\
\sqrt{2} & \sqrt{2} & 0
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

\[ S = (V_1 + V_2)^{\frac{1}{2}} \]

If \( V_1 = 0, H \) is undefined; otherwise, \( H = \tan^{-1}\left( \frac{V_2}{V_1} \right). \)

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} =
\begin{bmatrix}
\sqrt{3} & \sqrt{6} & \sqrt{2} \\
3 & 6 & 2 \\
\sqrt{3} & \sqrt{6} & \sqrt{2} \\
3 & 6 & 2 \\
3 & 3 & 0
\end{bmatrix}
\begin{bmatrix}
V \\
V_1 \\
V_2
\end{bmatrix}
\]

in which \( V_1 = S \cos(H) \) and \( V_2 = S \sin(H). \)

For input of \( R, G, \) and \( B \) in the range \([0, 255]\), hue: \([0, 360]\), value: \([0, 442]\), saturation: \([0-256]\). There is a model of a similar formulation stated by Shettigara (1992), having exactly the same formulation but another baseline for hue angle.

IHS Model (Pratt, 1991; Niblack, 1988)

Pratt (1991) stated an IHS (Intensity, Hue, Saturation) coordinate system. The transformation between IHS and RGB spaces is

\[
\begin{bmatrix}
I \\
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
3 & 3 & 3 \\
\sqrt{6} & \sqrt{6} & \sqrt{6} \\
6 & 6 & 3 \\
1 & 2 & 0 \\
\sqrt{6} & \sqrt{6} & 0
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

\[ S = (V_1 + V_2)^{\frac{1}{2}} \]

If \( V_1 = 0, H \) is undefined; otherwise, \( H = \tan^{-1}\left( \frac{V_2}{V_1} \right). \)

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} =
\begin{bmatrix}
4 & 2\sqrt{6} & \sqrt{6} \\
3 & 9 & 3 \\
2 & \sqrt{6} & \sqrt{6} \\
3 & 9 & 3 \\
1 & 3 & 0
\end{bmatrix}
\begin{bmatrix}
I \\
V_1 \\
V_2
\end{bmatrix}
\]

in which \( V_1 = S \cos(H) \) and \( V_2 = S \sin(H). \)

According to Pratt (1991), the element \((2, 2)\) of the transformation matrix in Equation 16 has a positive sign. Because the transformation matrix in Equation 16 is actually the inverse
Table 2. Extreme Misdisclosures

<table>
<thead>
<tr>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta R$</td>
<td>$\Delta G$</td>
</tr>
<tr>
<td>CIE-L* $u^<em>v^</em>$</td>
<td>2.7978</td>
</tr>
<tr>
<td>CIE-IHS</td>
<td>3.4875</td>
</tr>
<tr>
<td>Triangle</td>
<td>4.1626</td>
</tr>
<tr>
<td>Raines (1977)</td>
<td>1.8631</td>
</tr>
</tbody>
</table>

Reverse of the transformation matrix in Equation 15, the element (2, 2) in Equation 16 should have a negative sign.

The detailed derivation of this color model was not provided by Pratt (1991). Background work revealed that it may also have been derived from cylindrical geometry. Following Niblack (1986), the $R = G = B$ axis is taken as the intensity coordinate. The vector $V_i$ is defined as orthogonal to intensity axis $l$ and lies in the plane of $l$ and the blue axis $B$. The third axis $V_3$ is the cross product of $l$ and $V_i$. The resulting transformations are

$$
\begin{bmatrix}
I \\
V_i \\
V_3
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
3 & 3 & 3 \\
\frac{\sqrt{6}}{2} & \frac{\sqrt{6}}{2} & \frac{\sqrt{6}}{2}
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
$$

$$
S = \left( V_i^2 + V_3^2 \right)^{1/2}
$$

if $V_i = 0$, $H$ is undefined; otherwise, $H = \tan^{-1}\left( \frac{V_3}{V_i} \right)$.

Relative to the HSV model developed by Raines (1977), the elements in the first row of the transformation matrices indicate another scale choice. The transformation matrix in Raines (1977) model is an orthogonal matrix, whereas the one developed following Niblack (1986) scheme is not.

Range, Singular Points, and the Validation

Because 8 bits is the image depth commonly used, the domain selected for this work is a set of integers belonging to $[0, 255]$. The final range used is also the same for each coordinate. In order to have an appreciation of rounding errors during this scaling process, the original ranges of each color space are summarized in Table 1. For the $L^*, u^*, v^*$ and $I, S$ of CIE models and $S$ of the Niblack (1986) model, the ranges are rounded to the nearest integer.

In all six models, there is a common singular situation. When saturation is zero, the hue angle is undefined. When the intensity is zero, both saturation and hue are undefined. In the practical implementation, there are different conditions.

1. **Color Bubble Model (HVC)**
   - If $R + B = 2G$, then $(V - G)$ would be zero. Together with $R - B = 0$, the singular situations occur when $R = G = B$.

2. **Hexcone Model (HISV)**
   - When $R = G = B$, $S$ is zero, then $H$ is undefined.

3. **Triangle Model (IISI)**
   - The saturation reaches zero when $(R + G + B) = 3$ (the $[R, G, B]$). This set includes all cases for which $R = G = B$.

4. **Double Hexcone Model (IISI)**

Table 3. Coordinates of Extreme Misdisclosures

<table>
<thead>
<tr>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta R$</td>
<td>$\Delta G$</td>
</tr>
<tr>
<td>CIE-IHS</td>
<td>(250, 236, 2)</td>
</tr>
<tr>
<td>Color Bubble</td>
<td>(7, 0, 252)</td>
</tr>
<tr>
<td>Hexcone</td>
<td>(235, 255, 3)</td>
</tr>
<tr>
<td>Triangle</td>
<td>(254, 254, 0)</td>
</tr>
<tr>
<td>Double Hexcone</td>
<td>(245, 255, 0)</td>
</tr>
<tr>
<td>Raines (1977)</td>
<td>(173, 255, 0)</td>
</tr>
<tr>
<td>Niblack (1986)</td>
<td>(255, 255, 0)</td>
</tr>
</tbody>
</table>
The singular condition is $\text{max} = \text{min}$, that is, all $R = G = B$ cases.

(5) HSV Model (Raines, 1977)
In the model of Kruse and Raines (1984), $R = G = B$ makes $V_1$ zero, $R = G$ makes $V_2$ zero.

(6) IHS Model (Pratt, 1991)
(a) $I = 0: R = G = B = 0$;
(b) $S = 0: R + G = 2B$ which causes $V_1$ to be zero, $2R = G$ which causes $V_2$ to be zero.

There are 45 points in the 8-bit integer $(R, G, B)$ space which satisfy the $S = 0$ condition.

(7) IHS Model (Niblack, 1986)
The situation is the same as the HSV model of Raines (1977).

**Reversibility**
Because all of these color space transformations are non-linear, converting from RGB, then transforming back with 8-bit unsigned integers fails to assure a full recovery.

Examining the procedure, the steps are

(1) to convert RGB to perceptual components, such as HSV;
(2) to scale and round the resulting perceptual coordinates into $[0, 255]$ integers for storage;
(3) to read in the perceptual coordinates of an 8-bit unsigned integer and to scale them into the proper domain;
(4) to convert perceptual coordinates back to RGB and to round; and

**Table 4a. Average Misclousures (Steps 1 through 4)**

<table>
<thead>
<tr>
<th>Method</th>
<th>Average Error</th>
<th>Average Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R$</td>
<td>$G$</td>
</tr>
<tr>
<td>CIE-L*$u^v*$</td>
<td>-0.00733</td>
<td>-0.00650</td>
</tr>
<tr>
<td>CIE-IHS</td>
<td>-0.00693</td>
<td>-0.00705</td>
</tr>
<tr>
<td>Color Bubble</td>
<td>0.001612</td>
<td>-0.001572</td>
</tr>
<tr>
<td>Hexcone</td>
<td>0.000887</td>
<td>0.000748</td>
</tr>
<tr>
<td>Triangle</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Double Hexcone</td>
<td>-0.246636</td>
<td>-0.245965</td>
</tr>
<tr>
<td>Raines (1977)</td>
<td>-0.003221</td>
<td>-0.003202</td>
</tr>
<tr>
<td>Pratt (1991)</td>
<td>0.001359</td>
<td>0.001320</td>
</tr>
<tr>
<td>Niblack (1986)</td>
<td>0.001994</td>
<td>-0.002113</td>
</tr>
</tbody>
</table>

**Table 4b. Average Misclousures (Steps 1 through 5)**

<table>
<thead>
<tr>
<th>Method</th>
<th>Average Error</th>
<th>Average Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R$</td>
<td>$G$</td>
</tr>
<tr>
<td>CIE-L*$u^v*$</td>
<td>0.000707</td>
<td>-0.00458</td>
</tr>
<tr>
<td>CIE-IHS</td>
<td>0.000603</td>
<td>-0.000991</td>
</tr>
<tr>
<td>Color Bubble</td>
<td>0.000009</td>
<td>0.001209</td>
</tr>
<tr>
<td>Hexcone</td>
<td>0.004853</td>
<td>0.005499</td>
</tr>
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<td>Triangle</td>
<td>-0.000376</td>
<td>-0.000376</td>
</tr>
<tr>
<td>Double Hexcone</td>
<td>-0.246312</td>
<td>-0.245645</td>
</tr>
<tr>
<td>Raines (1977)</td>
<td>0.000966</td>
<td>0.001240</td>
</tr>
<tr>
<td>Pratt (1991)</td>
<td>0.000278</td>
<td>0.000103</td>
</tr>
<tr>
<td>Niblack (1986)</td>
<td>0.002380</td>
<td>-0.000176</td>
</tr>
</tbody>
</table>

**Table 5a. Root-Mean-Square Errors (Steps 1 through 3)**

<table>
<thead>
<tr>
<th>Method</th>
<th>Average Error</th>
<th>Average Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R$</td>
<td>$G$</td>
</tr>
<tr>
<td>CIE-L*$u^v*$</td>
<td>0.000686</td>
<td>0.000150</td>
</tr>
<tr>
<td>CIE-IHS</td>
<td>0.000665</td>
<td>0.000029</td>
</tr>
<tr>
<td>Color Bubble</td>
<td>0.000096</td>
<td>0.000029</td>
</tr>
<tr>
<td>Hexcone</td>
<td>0.001493</td>
<td>0.001493</td>
</tr>
<tr>
<td>Triangle</td>
<td>0.000983</td>
<td>0.001256</td>
</tr>
<tr>
<td>Double Hexcone</td>
<td>0.000118</td>
<td>0.000101</td>
</tr>
<tr>
<td>Raines (1977)</td>
<td>0.000384</td>
<td>-0.000126</td>
</tr>
<tr>
<td>Pratt (1991)</td>
<td>0.000392</td>
<td>-0.000126</td>
</tr>
</tbody>
</table>
The reversibility of several popular perceptual color models was examined. These models are widely implemented for forward and backward transformation. For instance, a table of frequency of occurrences for the triangle model of Gonzalez and Woods (1992) is listed in Table 6. Other occurrence statistics are summarized in Table 7, in which the misclosures are counted and generated numbers in R, G, and B at least one different, or three completely different.

The maximum Euclidean distance of misclosures in the RGB space, and the location of the first occurrence, are tabulated in Table 8. The Euclidean distance in the RGB space is defined according to Equation 19: i.e.,

\[ AD = \sqrt{(\Delta R)^2 + (\Delta G)^2 + (\Delta B)^2} \]

\( (19) \)

Concluding Remarks

There is a strong symmetry among R, G, and B, caused not only by the geometry of models but also by the algorithm used for forward and backward transformation. For instance, a table of frequency of occurrences for the triangle model of Gonzalez and Woods (1992) is listed in Table 6. Other occurrence statistics are summarized in Table 7, in which the misclosures are counted and generated numbers in R, G, and B at least one different, or three completely different.

The maximum Euclidean distance of misclosures in the RGB space, and the location of the first occurrence, are tabulated in Table 8. The Euclidean distance in the RGB space is defined according to Equation 19: i.e.,

\[ AD = \sqrt{(\Delta R)^2 + (\Delta G)^2 + (\Delta B)^2} \]

\( (19) \)

Concluding Remarks

The reversibility of several popular perceptual color models was examined. These models are widely implemented for image processing of remotely sensed data (PCI, 1987; ERM,
1992; Khoros, 1991), and for computer graphic applications (Claris, 1991; Joblove and Greenberg, 1978; Schneier, 1993; Tajima, 1963; Torel and Smith, 1990). Among all these models, the HSV model derived by Raines (1977) provides the least error in the forward and backward transformations for 8-bit depth image representation. The reversibility error is insignificant for human visual interpretation. However, it may be worth considering when an analytic operation, such as classification, follows the color space transformation.

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References
PEER-REVIEWED ARTICLE

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FORTHCOMING ARTICLES

The following list includes only those articles scheduled for publication in PEERS through March 1996.

R. M. Batson and E. M. Eliason, Digital Maps of Mars.
Michael F. Baumgarter and Albert Rango, A Microcomputer-Based Alpine Snow Cover Analysis System (ASCAS).
Michel Boulianne, Rock Santore, Paul-André Gagnon, and Clément Nollette, Floating Lines and Cones for Use as a GPS Mission Planning Aid.
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