

The Sign Permutation in the Rotation Matrix and the Formulation of the Collinearity and Coplanarity Equations

Tian-Yuan Shih

Abstract

Two basic variations arise in the formulation of the collinearity and coplanarity equations. These two variations are generally referred to as the diapositive equation and the negative equation. The relationship between these two formulations and the rotation matrices is investigated in this study. This study has revealed that the sign changes of principal distance in both the collinearity and coplanarity equations are coupled with the sign permutation of the elements in a rotation matrix.

Introduction

The collinearity condition states that the image point, the projective center, and the corresponding object point lie on the same line; meanwhile, the coplanarity condition describes a situation in which the object point and its corresponding image points on two overlapping photographs are located on the same plane with the base vector. This base vector joins the projective centers of these two photographs (Masry, 1977).

Two basic variations arise in the formulation of the collinearity and coplanarity equations. These two variations are generally referred to as the diapositive equations and the negative equations. As illustrated in Figure 1, the negative equations of the collinearity condition can be represented as Equation 1: i.e.,

$$x = c \frac{m_{11}(X - X_C) + m_{12}(Y - Y_C) + m_{13}(Z - Z_C)}{m_{31}(X - X_C) + m_{32}(Y - Y_C) + m_{33}(Z - Z_C)} \quad (1)$$

$$y = c \frac{m_{21}(X - X_C) + m_{22}(Y - Y_C) + m_{23}(Z - Z_C)}{m_{31}(X - X_C) + m_{32}(Y - Y_C) + m_{33}(Z - Z_C)}$$

The diapositive equations have rather similar forms to the negative equation, as shown in Equation 2: i.e.,

$$x = -c \frac{m_{11}(X - X_C) + m_{12}(Y - Y_C) + m_{13}(Z - Z_C)}{m_{31}(X - X_C) + m_{32}(Y - Y_C) + m_{33}(Z - Z_C)} \quad (2)$$

$$y = -c \frac{m_{21}(X - X_C) + m_{22}(Y - Y_C) + m_{23}(Z - Z_C)}{m_{31}(X - X_C) + m_{32}(Y - Y_C) + m_{33}(Z - Z_C)}$$

The only difference is the reversed sign of the principal distance c .

The base vector ($\mathbf{B}_X, \mathbf{B}_Y, \mathbf{B}_Z$) and the two corresponding image vectors ($\mathbf{U}, \mathbf{V}, \mathbf{W}$) and ($\mathbf{U}', \mathbf{V}', \mathbf{W}'$) form the coplanarity condition

$$\begin{vmatrix} B_X & B_Y & B_Z \\ U & V & W \\ U' & V' & W' \end{vmatrix} = 0 \quad (3)$$

where

$$\begin{vmatrix} U \\ V \\ W \end{vmatrix} = \begin{vmatrix} x_i - x_o + dx \\ y_i - y_o + dy \\ -c \end{vmatrix} = \begin{vmatrix} x \\ y \\ -c \end{vmatrix}, \quad \begin{vmatrix} U' \\ V' \\ W' \end{vmatrix} = \begin{vmatrix} x_i' - x_o' + dx' \\ y_i' - y_o' + dy' \\ -c' \end{vmatrix} = \begin{vmatrix} x' \\ y' \\ -c' \end{vmatrix}$$

$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{vmatrix} x' \\ y' \\ -c' \end{vmatrix} = \mathbf{R} \begin{vmatrix} x' \\ y' \\ -c' \end{vmatrix}$$

and \mathbf{R} is an orthogonal matrix.

Once again, Equation 3 is presented in the form of a diapositive equation. The signs of both c and c' are reversed in the negative equation. The relationship between the elements of the rotation matrix and the sign of the principal distance can be observed from Equations 1, 2, and 3. This relationship is further analyzed following a discussion on the rotation matrix and the sign permutation of its elements.

The Rotation Matrix

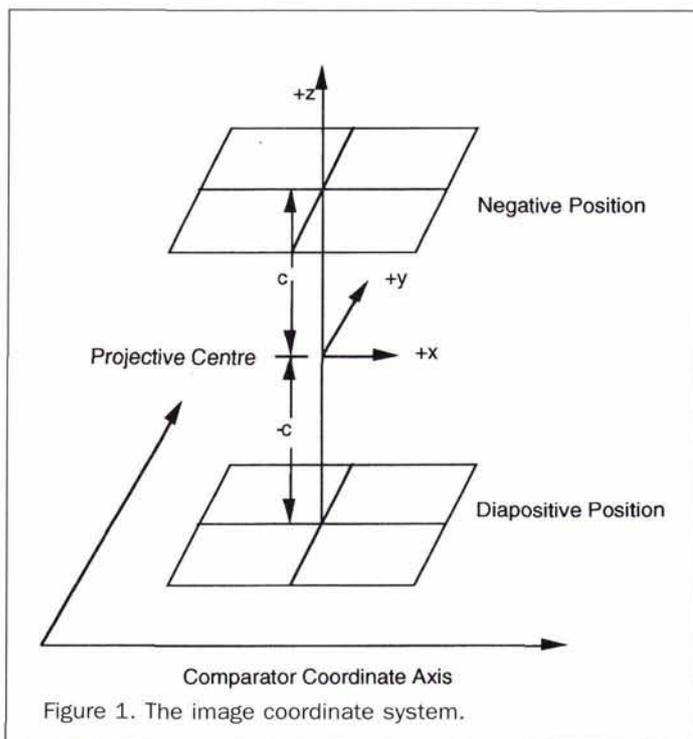
Realizing the rotation in a three-dimensional space with a cardan system, the three rotation elements are frequently symbolized as ω, ϕ, κ (Kraus, 1993; Schut, 1959; Shih, 1990; Thompson, 1969). Representing in matrix form, we obtain

$$\mathbf{R}_X = \mathbf{R}_\omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \omega & -\sin \omega \\ 0 & \sin \omega & \cos \omega \end{pmatrix}$$

$$\mathbf{R}_Y = \mathbf{R}_\phi = \begin{pmatrix} \cos \phi & 0 & \sin \phi \\ 0 & 1 & 0 \\ -\sin \phi & 0 & \cos \phi \end{pmatrix}$$

$$\mathbf{R}_Z = \mathbf{R}_\kappa = \begin{pmatrix} \cos \kappa & -\sin \kappa & 0 \\ \sin \kappa & \cos \kappa & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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changed from the original formulation. Notably, the signs discussed here are the signs relative to the original formulation, not the signs of the current cosine or sine value.

Examining the structure of the rotation matrix listed in Equation 4, the sign change can be illustrated as follows:

$$\begin{pmatrix} (+,-) & (+,-) & (+) \\ (+,-),(+,+,-) & (+,-),(+,+,-) & (+,+) \\ (+,-),(+,+,-) & (+,-),(+,+,-) & (+,+) \end{pmatrix}$$

where the (+,-) at the r_{11} position represents that $\cos \phi$ retains the same sign and that $\cos \kappa$ has a reversed sign. Therefore, the resulting r_{11} will have the same magnitude but with reversed signs as compared with the original formulation. The sign change for this case can be summarized as follows:

$$\begin{pmatrix} - & - & + \\ - & - & + \\ - & - & + \end{pmatrix}$$

This is equivalent to comparing the signs of the rotation matrix formulated with $(\omega, \phi, \pi+\kappa)$, and the one formulated with (ω, ϕ, κ) for the $\mathbf{R}_{\omega\phi\kappa}$ formulation. A numerical example is given below.

$$(\omega, \phi, \kappa) = (0.3, 0.4, 0.5)$$

$$\mathbf{R}_{\omega\phi\kappa} = \begin{pmatrix} 0.808307 & -0.441580 & 0.389418 \\ 0.559006 & 0.783214 & -0.272192 \\ -0.184803 & 0.437702 & 0.879923 \end{pmatrix}$$

$$(\omega, \phi, \kappa) = (0.3, 0.4, 3.641592) = (0.3, 0.4, \pi+0.5)$$

$$\mathbf{R}_{\omega\phi\kappa} = \begin{pmatrix} -0.808307 & 0.441580 & 0.389418 \\ -0.559006 & -0.783214 & -0.272192 \\ 0.184803 & -0.437702 & 0.879923 \end{pmatrix}$$

While the pattern of sign change is independent of the manner in which the rotation matrix is formulated, the values of the physical rotation elements are dependent on the rotation matrix formulation. This is observed in a comparison of Tables 1 and 2.

Notably, not all of these 64 combinations will generate rotation matrices with unchanged magnitude for every r_{ij} . Only 32 combinations actually satisfy this requirement. These 32 combinations subsequently result in 16 different sign patterns of the rotation matrix. All these 16 patterns and their corresponding combinations are summarized in Tables 1 and 2.

Rotation Elements and Principal Distance

Examining Equation 1 and 2, the negative sign before the principal distance c can be distributed and Equation 2 can be re-written into Equation 6: i.e.,

$$x = c \frac{(-m_{11})(X - X_c) + (-m_{12})(Y - Y_c) + (-m_{13})(Z - Z_c)}{m_{31}(X - X_c) + m_{32}(Y - Y_c) + m_{33}(Z - Z_c)} \quad (6)$$

$$y = c \frac{(-m_{21})(X - X_c) + (-m_{22})(Y - Y_c) + (-m_{23})(Z - X_c)}{m_{31}(X - X_c) + m_{32}(Y - Y_c) + m_{33}(Z - Z_c)}$$

This implies that the sign change of the principal distance can be absorbed by a corresponding pattern of sign changes for the elements of the rotation matrix r_{ij} . The pattern of sign changes in this particular case is

Applying matrix multiplication in the order $(\omega-\phi-\kappa)$ and $(\phi-\omega-\kappa)$, we obtain

$$\mathbf{R}_{\omega\phi\kappa} = \begin{pmatrix} \cos \phi \cos \kappa & & & \\ \cos \omega \sin \kappa + \sin \omega \sin \phi \cos \kappa & & & \\ \sin \omega \sin \kappa - \cos \omega \sin \phi \cos \kappa & & & \\ & -\cos \phi \sin \kappa & & \sin \phi \\ \cos \omega \cos \kappa - \sin \omega \sin \phi \sin \kappa & & -\sin \omega \cos \phi & \\ \sin \omega \cos \kappa + \cos \omega \sin \phi \sin \kappa & & \cos \omega \cos \phi & \end{pmatrix} \quad (4)$$

and

$$\mathbf{R}_{\phi\omega\kappa} = \begin{pmatrix} \cos \phi \cos \kappa + \sin \phi \sin \omega \sin \kappa & & & \\ \cos \omega \sin \kappa & & & \\ -\sin \phi \cos \kappa + \cos \phi \sin \omega \sin \kappa & & & \\ & -\cos \phi \sin \kappa + \sin \phi \sin \omega \cos \kappa & & \sin \phi \cos \omega \\ \cos \omega \cos \kappa & & & -\sin \omega \\ \sin \phi \sin \kappa + \cos \phi \sin \omega \cos \kappa & & \cos \omega \cos \phi & \end{pmatrix} \quad (5)$$

The above two rotation matrices are those for the case of rotating a point with respect to a fixed coordinate system.

The Permutation of Rotation Elements

Examining the components in Equation 4 and 5, these rotation matrices are composed of six different numerical elements, i.e., $\cos \omega$, $\sin \omega$, $\cos \phi$, $\sin \phi$, $\cos \kappa$, and $\sin \kappa$. If the magnitude of each element remains unchanged, two choices are available for the sign change, i.e., keep the sign or reverse the sign. Because there are six elements, 64 combinations can be constructed in total. For example, let $\cos \omega$, $\sin \omega$, $\cos \phi$, and $\sin \phi$ remain unchanged and the signs of $\cos \kappa$ and $\sin \kappa$ be reversed. That is,

$$\text{the sign change pattern of } \begin{pmatrix} \cos \omega & \sin \omega \\ \cos \phi & \sin \phi \\ \cos \kappa & \sin \kappa \end{pmatrix} = \begin{pmatrix} + & + \\ + & + \\ - & - \end{pmatrix},$$

and the composed rotation matrix will have exactly the same magnitude for each r_{ij} . However, the pattern of signs has

TABLE 1. THE SIGN CHANGE PATTERNS OF ROTATION MATRICES AND COSINE/SINE VALUES OF ROTATION ELEMENTS ($\mathbf{R}_{\text{rot}(x)}$)

	Rotation Matrix	Combination 1	Combination 2
1	$\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & + \end{pmatrix}$	$\begin{pmatrix} + & + \\ + & + \\ + & + \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & + \\ - & - \end{pmatrix}$
2	$\begin{pmatrix} - & - & + \\ - & - & + \\ - & - & + \end{pmatrix}$	$\begin{pmatrix} + & + \\ + & + \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & + \\ + & + \end{pmatrix}$
3	$\begin{pmatrix} + & - & - \\ - & + & + \\ - & + & + \end{pmatrix}$	$\begin{pmatrix} + & + \\ + & - \\ + & - \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & - \\ - & + \end{pmatrix}$
4	$\begin{pmatrix} - & + & - \\ + & - & + \\ + & - & + \end{pmatrix}$	$\begin{pmatrix} + & + \\ + & - \\ - & + \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & - \\ + & - \end{pmatrix}$
5	$\begin{pmatrix} - & - & + \\ + & + & - \\ + & + & - \end{pmatrix}$	$\begin{pmatrix} + & + \\ - & + \\ + & + \end{pmatrix}$	$\begin{pmatrix} - & - \\ + & + \\ - & - \end{pmatrix}$
6	$\begin{pmatrix} + & + & + \\ - & - & - \\ - & - & - \end{pmatrix}$	$\begin{pmatrix} + & + \\ - & + \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & - \\ + & + \\ + & + \end{pmatrix}$
7	$\begin{pmatrix} - & + & - \\ - & + & - \\ - & + & - \end{pmatrix}$	$\begin{pmatrix} + & + \\ - & - \\ + & - \end{pmatrix}$	$\begin{pmatrix} - & - \\ + & - \\ - & + \end{pmatrix}$
8	$\begin{pmatrix} + & - & - \\ + & - & - \\ + & - & - \end{pmatrix}$	$\begin{pmatrix} + & + \\ - & - \\ - & + \end{pmatrix}$	$\begin{pmatrix} - & - \\ + & - \\ + & - \end{pmatrix}$
9	$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ + & + \\ + & - \end{pmatrix}$	$\begin{pmatrix} - & + \\ - & + \\ - & + \end{pmatrix}$
10	$\begin{pmatrix} - & + & + \\ + & - & - \\ - & + & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ + & + \\ - & + \end{pmatrix}$	$\begin{pmatrix} - & + \\ - & + \\ + & - \end{pmatrix}$
11	$\begin{pmatrix} + & + & - \\ + & + & - \\ - & - & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ + & - \\ + & + \end{pmatrix}$	$\begin{pmatrix} - & + \\ - & - \\ - & - \end{pmatrix}$
12	$\begin{pmatrix} - & - & - \\ - & - & - \\ + & + & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ + & - \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & + \\ - & - \\ + & + \end{pmatrix}$
13	$\begin{pmatrix} - & + & + \\ - & + & + \\ + & - & - \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & + \\ + & - \end{pmatrix}$	$\begin{pmatrix} - & + \\ + & + \\ - & + \end{pmatrix}$
14	$\begin{pmatrix} + & - & + \\ + & - & + \\ - & + & - \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & + \\ - & + \end{pmatrix}$	$\begin{pmatrix} - & + \\ + & + \\ + & - \end{pmatrix}$

TABLE 1. CONTINUED

	Rotation Matrix	Combination 1	Combination 2
15	$\begin{pmatrix} - & - & - \\ + & + & + \\ - & - & - \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & - \\ + & + \end{pmatrix}$	$\begin{pmatrix} - & + \\ + & - \\ - & - \end{pmatrix}$
16	$\begin{pmatrix} + & + & - \\ - & - & + \\ + & + & - \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & + \\ - & + \end{pmatrix}$	$\begin{pmatrix} - & + \\ + & - \\ + & + \end{pmatrix}$
			$\begin{pmatrix} - & - & - \\ - & - & - \\ + & + & + \end{pmatrix}$

From Table 1, this is No. 12 in the list.

This phenomenon arises in the coplanarity equation as well. Expressing the elements composing the determinant value for the coplanarity equation, these six elements are

$$\begin{aligned}
 \mathbf{B}_x \mathbf{VW}' &= \mathbf{B}_x y(r_{31}x' + r_{32}y' - r_{33}c') \\
 \mathbf{B}_y \mathbf{WU}' &= \mathbf{B}_y (-c)(r_{11}x' + r_{12}y' - r_{13}c') \\
 \mathbf{B}_z \mathbf{UV}' &= \mathbf{B}_z x(r_{21}x' + r_{22}y' - r_{23}c') \\
 \mathbf{B}_x \mathbf{WV}' &= \mathbf{B}_x (-c)(r_{21}x' + r_{22}y' - r_{23}c') \\
 \mathbf{B}_y \mathbf{UW}' &= \mathbf{B}_y x(r_{31}x' \pm r_{32}y' - r_{33}c') \\
 \mathbf{B}_z \mathbf{VU}' &= \mathbf{B}_z y(r_{11}x' \pm r_{12}y' - r_{13}c')
 \end{aligned} \tag{7}$$

The sign change of the principal distances can then be compensated for with a corresponding sign change of r_{11} , r_{12} , r_{21} , r_{22} , r_{33} , together with a reversed sign of \mathbf{B}_z . The pattern of sign change can be represented as

$$\begin{pmatrix} - & - & + \\ - & - & + \\ + & + & - \end{pmatrix}$$

Checking through Table 1, this pattern does not exist. However, pattern No. 11 provides this pattern with a reversed sign for all r_j ; i.e.,

$$\begin{pmatrix} + & + & - \\ + & + & - \\ - & - & + \end{pmatrix}$$

The effect of this reversed sign is that the sign of all residuals computed from the coplanarity equations will be reversed. This situation still satisfies the coplanarity condition.

A Numerical Experiment with the Coplanarity Equation

The relationship between the sign of the principal distance and the elements of the rotation matrix is relatively straightforward. On the other hand, the situation with the coplanarity equation is more complicated. This complexity is caused because the sign of base components also influences the principal distance. A numerical experiment is then conducted. The image coordinates and principal distances are listed in Table 3. Using the same image coordinates, the sign of the principal distance is changed. The signs of rotations and base components computed from the coplanarity equation are listed in the Table 4.

Concluding Remarks

For both the collinearity and the coplanarity conditions, the negative and diapositive formulations can be derived from

TABLE 2. THE SIGN CHANGE PATTERNS OF ROTATION MATRICES AND COSINE/SINE VALUES OF ROTATION ELEMENTS ($R_{\omega\phi\kappa}$)

	Sign Pattern	Combination 1	Combination 2
1	$\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & + \end{pmatrix}$	$\begin{pmatrix} + & + \\ + & + \\ + & + \end{pmatrix}$	$\begin{pmatrix} - & + \\ - & - \\ - & - \end{pmatrix}$
2	$\begin{pmatrix} - & - & + \\ - & - & + \\ - & - & + \end{pmatrix}$	$\begin{pmatrix} + & + \\ + & + \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & + \\ - & - \\ + & + \end{pmatrix}$
3	$\begin{pmatrix} + & - & - \\ - & + & + \\ - & + & + \end{pmatrix}$	$\begin{pmatrix} + & + \\ + & - \\ + & - \end{pmatrix}$	$\begin{pmatrix} - & + \\ - & + \\ - & + \end{pmatrix}$
4	$\begin{pmatrix} - & + & - \\ + & - & + \\ + & - & + \end{pmatrix}$	$\begin{pmatrix} + & + \\ + & - \\ - & + \end{pmatrix}$	$\begin{pmatrix} - & + \\ - & + \\ + & - \end{pmatrix}$
5	$\begin{pmatrix} - & - & + \\ + & + & - \\ + & + & - \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & + \\ + & + \end{pmatrix}$	$\begin{pmatrix} - & - \\ + & - \\ - & - \end{pmatrix}$
6	$\begin{pmatrix} + & + & + \\ - & - & - \\ - & - & - \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & + \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & - \\ + & - \\ + & + \end{pmatrix}$
7	$\begin{pmatrix} - & + & - \\ - & + & - \\ - & + & - \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & - \\ + & - \end{pmatrix}$	$\begin{pmatrix} - & - \\ + & + \\ - & + \end{pmatrix}$
8	$\begin{pmatrix} + & - & - \\ + & - & - \\ + & - & - \end{pmatrix}$	$\begin{pmatrix} + & - \\ - & - \\ - & + \end{pmatrix}$	$\begin{pmatrix} - & - \\ + & + \\ + & - \end{pmatrix}$
9	$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ + & + \\ + & - \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & - \\ - & + \end{pmatrix}$
10	$\begin{pmatrix} - & + & + \\ + & - & - \\ - & + & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ + & + \\ - & + \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & - \\ + & - \end{pmatrix}$
11	$\begin{pmatrix} + & + & - \\ + & + & - \\ - & - & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ + & - \\ + & + \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & + \\ - & - \end{pmatrix}$
12	$\begin{pmatrix} - & - & - \\ - & - & - \\ + & + & + \end{pmatrix}$	$\begin{pmatrix} + & - \\ + & - \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & + \\ + & + \end{pmatrix}$
13	$\begin{pmatrix} - & + & + \\ - & + & + \\ + & - & - \end{pmatrix}$	$\begin{pmatrix} + & + \\ - & + \\ + & - \end{pmatrix}$	$\begin{pmatrix} - & + \\ + & - \\ - & + \end{pmatrix}$
14	$\begin{pmatrix} + & - & + \\ + & - & + \\ - & + & - \end{pmatrix}$	$\begin{pmatrix} + & + \\ - & + \\ - & + \end{pmatrix}$	$\begin{pmatrix} - & + \\ + & - \\ + & - \end{pmatrix}$

TABLE 2. CONTINUED

	Sign Pattern	Combination 1	Combination 2
15	$\begin{pmatrix} - & - & - \\ + & + & + \\ - & - & - \end{pmatrix}$	$\begin{pmatrix} + & + \\ - & - \\ + & + \end{pmatrix}$	$\begin{pmatrix} - & + \\ + & + \\ - & - \end{pmatrix}$
16	$\begin{pmatrix} + & + & - \\ - & - & + \\ + & + & - \end{pmatrix}$	$\begin{pmatrix} + & + \\ - & - \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & + \\ + & + \\ + & + \end{pmatrix}$

TABLE 3. THE IMAGE COORDINATES OF A STEREO-PAIR. PRINCIPAL DISTANCE OF IMAGE 1:55.264 MM. PRINCIPAL DISTANCE OF IMAGE 2:53.678 MM.

Point	x_1	y_1	x_2	y_2
1103	-1.662	-0.417	-5.568	-2.177
1104	-1.622	-3.127	-4.418	-4.267
1106	-1.552	-8.527	-2.038	-8.617
1403	6.328	-0.347	-0.538	2.033
1505	8.928	-5.587	4.062	-0.907
1601	11.398	4.933	0.592	9.393
1603	11.458	-0.307	3.282	5.163
1604	11.468	-2.897	4.682	2.933
1606	11.458	-8.117	7.612	-1.747

TABLE 4. THE ELEMENTS OF RELATIVE ORIENTATION COMPUTED FROM THE COPLANARITY EQUATION

	Positive P.D.	Negative P.D.
ω	0.36932	-0.36932
ϕ	0.60849	-0.60849
κ	-0.71705	-0.71705
B_y	-0.50300	-0.50300
B_z	-0.37491	0.37491
σ	0.61669	0.61669
$R_{\omega\phi\kappa}$	$\begin{pmatrix} 0.61845 & 0.53921 & 0.57162 \\ -0.45732 & 0.83852 & -0.29618 \\ -0.63903 & -0.07824 & 0.76518 \end{pmatrix}$	$\begin{pmatrix} 0.61845 & 0.53921 & -0.57162 \\ -0.45732 & 0.83852 & 0.29618 \\ 0.63903 & 0.07824 & 0.76518 \end{pmatrix}$

each other. The sign change of the principal distances can be compensated for with a different rotation matrix. In this case, the magnitudes of the elements of the rotation matrix remain the same; however, the signs change according to a specific pattern. In the coplanarity condition, the base components are also involved.

Although in both the collinearity and the coplanarity equations the reversed sign of the principal distances can be compensated for, the physical meanings are different. In the collinearity equation case, the geometry of the intersecting bundles does not change. In the coplanarity case, a mirror image of the object will be formed. This can be verified from Equation 8, i.e., an intersection equation. While the X, Y coordinates remain, the sign of Z is reversed: i.e.,

$$X - X_0 = (Z - Z_0) \begin{matrix} (+) & (+) & (-)(-) \\ \frac{m_{11}x + m_{21}y + m_{31}f}{m_{13}x + m_{23}y + m_{33}f} \\ (-) & (-) & (+)(-) \end{matrix} \quad (8)$$

These findings show why both positive and negative principal distances are acceptable in the same aerotriangulation package for the same image observations. Although the analytical nature remains to be studied further, this dual acceptability is often revealed when an aerotriangulation pack-

age with an automated initial value generation scheme is used, e.g., UNBASC2 (Moniwa, 1977). The confusing signs of the principal distances are experienced more often in close-range applications than in aerial projects, due to the irregularity of exterior orientation configurations frequently encountered in close-range projects.

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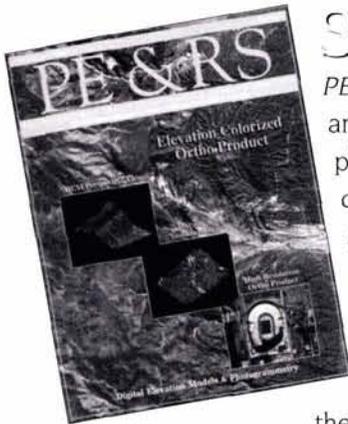
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