Automated Photogrammetric Network Design Using Genetic Algorithms

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Abstract

This work describes the use of genetic algorithms for automating the photogrammetric network design process. When planning a photogrammetric network, the cameras should be placed in order to satisfy a set of interrelated and competing constraints. Furthermore, when the object is three-dimensional, a combinatorial problem occurs. Genetic algorithms are stochastic optimization techniques, which have proved useful for solving computationally difficult problems with high combinatorial aspects. A system based on genetic algorithms, EPOCA (Evolving POsitions of CAmeras), was implemented using a three-dimensional CAD interface. The system provides the attitude of each camera in the network, taking into account the imaging geometry, as well as several major constraints such as visibility, convergence angle, and workspace constraint. EPOCA reproduces configurations reported in the photogrammetric literature. Moreover, the system can design networks for several adjoining planes and complex objects, opening interesting new research avenues.

Introduction

Photogrammetric network design is the process of placing cameras in order to perform photogrammetric tasks. An important aspect of any close-range photogrammetric system is to achieve an optimal spatial distribution of the cameras comprising the network. Planning an optimal photogrammetric network for some special purpose, such as for monitoring structural deformation or for determining the precise shape characteristics of an object, demands special attention from the quality of the network design. Previous approaches to photogrammetric network design have attempted to identify the main stages in the process. Following the widely accepted classification scheme of Grafarend (1974), network design has been divided into four design stages from which only the first three are used in close-range photogrammetry (Fraser, 1984):

- Zero-Order Design (ZOD): This stage attempts to define an optimal datum in order to obtain accurate object point coordinates and exterior orientation parameters.
- First-Order Design (FOD): This stage involves defining an optimal imaging geometry which, in turn, determines the accuracy of the system.
- Second-Order Design (SOD): This stage is concerned with adopting a suitable measurement precision for the image coordinates. It consists usually in taking multiple images from each camera station.
- Third-Order Design (TOD): This stage deals with the improvement of a network through the inclusion of additional points in a weak region.

Photogrammetric measurement operations attempt to satisfy, in an optimal manner, several objectives such as precision, reliability, and economy. The ZOD and SOD are greatly simplified in comparison to the geodetic networks for which the four stages were originally developed. Indeed FOD, the design of network configuration or the *sensor placement task*, needs to be comprehensively addressed for photogrammetric projects. This design stage must provide an optimal imaging geometry and convergence angle for each set of points placed over a complex object (Fraser, 1996). Photogrammetrists have acknowledged the degree of expertise needed to carry out a photogrammetric project. For example, Mason and Grün (1995) developed a work called CONSENS that follows the expert system approach and uses multiple cameras in combination with optical triangulation. It outlines a method of overcoming the set of constraints and objectives presented in camera station placement. The method is based on the theory of *generic networks*, which constitutes compiled expertise, describing an ideal configuration of four camera stations that can be employed to provide a strong imaging geometry for the class of planar network design problems. Complex objects are divided into planes; each plane is evaluated through one of these networks and then connected with some additional cameras with the purpose of establishing just one common datum. However, the expert system approach has shown it unlikely that full automation of the network design process will be achieved, due in large part to the human expert's extensive use of commonsense reasoning (Fraser, 1996). On the other hand, the Grafarend classification just presented serves the photogrammetric user by identifying what set of tasks needs to be implemented in designing a network.

Despite the progress that photogrammetrists have made in understanding this design problem, the photogrammetric measurement technique has rarely been applied by other than experienced photogrammetrists. Although its definition seems simple, it reaches a high complexity mainly due to the numerous constraints and design decisions that need to be made. Photogrammetric network design is also difficult to obtain due to the unknown number of configurations all having very similar accuracy, but with a very different imaging geometry. Consequently, photogrammetric network design in many machine vision applications is often conducted in a trial-and-error fashion or by using heuristic reasoning strategies (Mason, 1997). These strategies fail at solving the problem for the case of complex objects. Moreover, the main question, how to obtain an initial configuration with an optimal imaging geometry, is unsolved and left as the responsibility of the designer. The motivation of this research is to reduce the cost of vision system design and to equip autonomous inspection sys-

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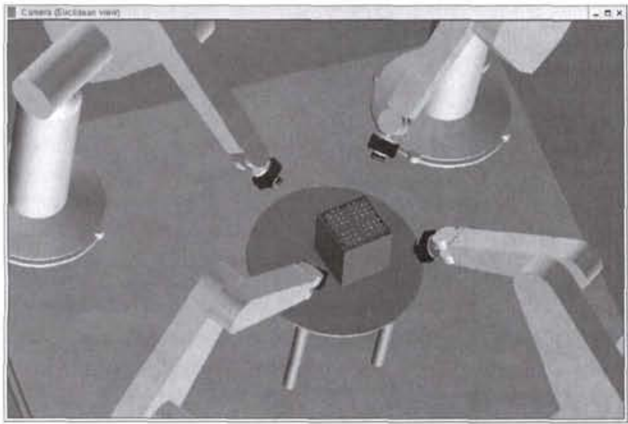


Figure 1. Photogrammetric network simulation of four robots. Each camera is mounted on the robot's hand, with the goal of measuring the box on the table.

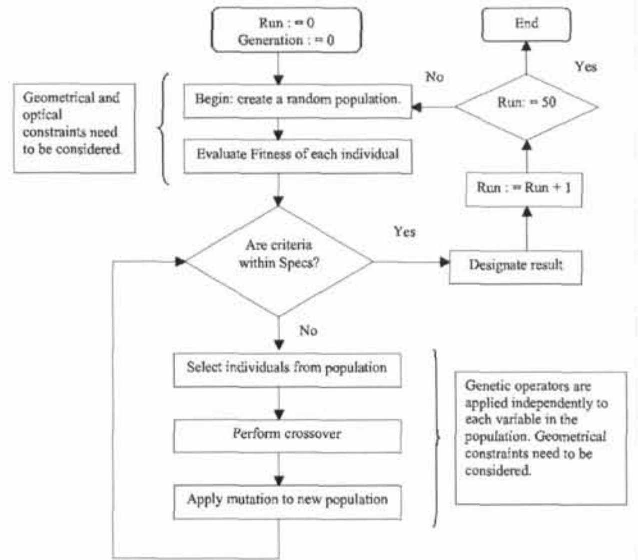


Figure 2. Flow diagram for photogrammetric network design using genetic algorithm's strategy.

tems with photogrammetric network capabilities, e.g., measurement robots used in flexible manufacturing (see Figure 1).

Expert photogrammetrists regard simulation as a viable strategy for the problem of photogrammetric network design (Fraser, 1996). Computer simulation of close-range photogrammetric networks has been successfully employed and, with the sophistication of computers, a considerable boost to interactive network design has been achieved. The process of photogrammetric network design optimization through computer simulation can follow a number of approaches. One traditional procedure is based on the ZOD, FOD, and SOD stages. Given the criteria related to required triangulation precision, the initial step is to adopt a suitable observation and measuring scheme (the FOD stage). This entails the selection of an appropriate camera format, focal length, and image measurement system, as well as a first approximation to suitable network geometry. Once this design stage is finished, the network is evaluated against the specified criteria. If the network fails to achieve the criteria, a new stage to diagnose and identify the problem is performed. The FOD or SOD will be applied to produce the new solution. If both corrections are insufficient, a completely new network will be proposed until a solution to the problem is achieved. In this way, network design is iterative in nature. The aim of this paper is to present a new simulation-based method for solving the most fundamental stage in network design, i.e., configuring an optimal imaging geometry. The problem is set in terms of a global optimization design (Olague, 1998), which is capable of managing the problem using an adaptive strategy. It explores the solution space using both non-continuous optimization and combinatorial search. The approach then is to minimize the uncertainty of the three-dimensional measurements using as a criterion the average variance of the 3D object points, presuming that the optimization satisfies a number of primary constraints. Emphasis in this paper is given to the optimization process using a genetic algorithm's strategy and how the primary constraints and design decisions are managed to overcome the computational burden. Figure 2 shows a flow diagram of the algorithm detailed in this paper.

This paper is organized as follows: first, the bundle adjustment, the mathematical model universally accepted by photogrammetrists, is reviewed in order to obtain a criterion useful to the optimization process. Later, the camera placement reasoning is introduced. Then, a brief summary of the constraints on network design is presented. The problem of photogrammetric network design in terms of a stochastic global optimization is described together with implementation details about visibility and occlusion constraints related to the complexity of the search space. Finally, results are presented followed by a conclusion.

Photogrammetric Network Modeling

Brown (1958) originally developed the bundle method in a fully general form. Today, the bundle method is recognized as a critical factor in exploiting the mensuration potential of photogrammetry and is almost exclusively used in applications requiring high accuracy. The method accords simultaneous consideration to all sets (or "bundles") of photogrammetric rays from all cameras. The bundle method is based on a mathematical camera model comprised of separate functional and stochastic models. The functional model describing the relationship between the desired and measured quantities consists of the well-known collinearity equations. The collinearity equations, derived from the perspective transformation, are based on the fundamental assumption that the perspective center, the ground point, and its corresponding image point all lie on a straight line. For each pair of image coordinates (x_{ij}, y_{ij}) observed on each image, the following pair of equations is written

$$
\begin{aligned}
F_x &= x_{ij} - x_p \\
&\quad + \mathrm{f}\left[\frac{m_{11}(X_j - X_i^c) + m_{12}(Y_j - Y_i^c) + m_{13}(Z_j - Z_i^c)}{m_{31}(X_j - X_i^c) + m_{32}(Y_j - Y_i^c) + m_{33}(Z_j - Z_i^c)}\right] = 0 \\
F_y &= y_{ij} - y_p \qquad (1) \\
&\quad + \mathrm{f}\left[\frac{m_{21}(X_j - X_i^c) + m_{22}(Y_j - Y_i^c) + m_{23}(Z_j - Z_i^c)}{m_{31}(X_j - X_i^c) + m_{32}(Y_j - Y_i^c) + m_{33}(Z_j - Z_i^c)}\right] = 0
\end{aligned}
$$

where (x_{ij}, y_{ij}) denote the coordinates of point j on photograph i, f and (x_p, y_p) are the camera constant and image coordinates of the principal point of the sensor defining the sensor's orientation, (X_j, Y_j, Z_j) are the object space coordinates of the corresponding point feature, (X_i^c, Y_i^c, Z_i^c) are the object space coordinates of the perspective center, and m_{kl} are elements of an orthogonal matrix which defines the rotation between the image and object coordinate systems. This system of equations assumes that light rays travel in straight lines, that all rays entering a camera lens system pass through a single point, and that the lens system is distortionless or, as is usual in highly accurate measurement, that distortion has been cancelled out after having been estimated. Due to the nature of the measurement process, observations are accompanied by errors. Because of random errors, as evidenced by the small differences between observations of the same quantity, observations can be regarded as random variables and their effects described by means of a stochastic model. Equation 1 can be linearized through the first-order development using the Taylor series. A functional model can be given as

$$\mathbf{v} = \mathbf{A}\mathbf{y} - \mathbf{l}$$

$$\mathbf{C}_1 = \sigma_0^2 \mathbf{P}^{-1}$$

where $\mathbf{l}$, $\mathbf{v}$, and $\mathbf{y}$ are the vectors of observations, residuals, and unknown parameters, respectively; $\mathbf{A}$ is the design or configuration matrix; $\mathbf{C}_1$ is the covariance matrix of observations; $\mathbf{P}$ is the weight matrix; and σ_0^2 is the variance factor. In situations where $\mathbf{A}$ is of full rank (i.e., where redundant or explicit minimal constraints are imposed), the parameter estimates $\hat{\mathbf{y}}$ and the corresponding cofactor matrix Q_y and covariance matrix C_y are obtained as

$$\hat{\mathbf{y}} = (\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}\mathbf{L} = \mathbf{Q}_y\mathbf{A}^T\mathbf{P}\mathbf{L} \tag{2}$$

and

$$\mathbf{C}y = \sigma_0^2 \mathbf{Q}_y.$$

The ultimate aim of any photogrammetric measurement is the determination of triangulated object point coordinates along with estimates for their precision. The bundle method is simplified by considering two groups of parameters in the vector $\hat{\mathbf{y}}$: $\hat{\mathbf{y}}_1$ comprising exterior orientation (self-calibration parameters were not considered for simplicity), and $\hat{\mathbf{y}}_2$ containing object coordinate corrections. Equation 2 then assumes the form

$$\begin{pmatrix} \hat{\mathbf{y}}_1 \\ \hat{\mathbf{y}}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 & \mathbf{A}_1^T\mathbf{P}\mathbf{A}_2 \\ \mathbf{A}_2^T\mathbf{P}\mathbf{A}_1 & \mathbf{A}_2^T\mathbf{P}\mathbf{A}_2 \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{A}^T\mathbf{P}\mathbf{L} \\ \mathbf{A}^T\mathbf{P}\mathbf{L} \end{pmatrix}$$

and the cofactor matrix $\mathbf{Q}_y$ can be written as

$$\mathbf{Q}_y = \begin{pmatrix} \mathbf{Q}_1 & \mathbf{Q}_{1,2} \\ \mathbf{Q}_{2,1} & \mathbf{Q}_2 \end{pmatrix}.$$

The design optimization goal for precision is to achieve an optimal form of $\mathbf{Q}_2$ and therefore the covariance matrix of object point coordinates (X_j, Y_j, Z_j), considering the applicable design constraints. The criterion used in the minimization process was the average variance along the covariance matrix σ_c^2: i.e.,

$$\sigma_c^2 = \sigma_0^2/3n(\text{trace } \mathbf{Q}_2).$$

Before dramatic improvements in computer processing power in recent years, a valid criticism of designing close-range networks by simulation was the computation time required for a bundle adjustment after each design-iteration even for relatively small networks. As shown in Brown (1980), the covariance matrix can be obtained through the equation

$$\mathbf{Q}_2 = \sigma_0^2 \, [(\mathbf{A}_2^T\mathbf{P}\mathbf{A}_2)^{-1} + \mathbf{K}]$$

where

$$\mathbf{K} = \mathbf{M}\mathbf{Q}_1\mathbf{M}^T$$

and

$$\mathbf{M} = (\mathbf{A}_2^T\mathbf{P}\mathbf{A}_2)^{-1}\,\mathbf{A}_2^T\mathbf{P}\mathbf{A}_1.$$

In this way, the determination of $\mathbf{Q}_2$ using this approach represents a rigorous approach that is termed *Total Error Propagation* (TEP). On the other hand, it has been demonstrated (Fraser, 1987) that, for a wide range of convergent photogrammetric networks, $\mathbf{K} = 0$. This consideration is non-rigorous in that it implicitly assumes that exterior orientation parameters exhibit no dispersion and is called *Limited Error Propagation* (LEP). The perspective parameters are assumed to be error free and the variances in object point coordinates arise solely from the propagation of random errors in the image coordinate measurements. What is remarkable from a network design standpoint is that, for strong networks (convergent networks), LEP is sufficiently accurate compared to TEP, causing considerable computation savings.

Representation for Camera Placement

Camera placement is considered a viewing direction problem. This section describes how the cameras are placed with respect to the object or target field. The viewing sphere model was selected for this task (see Figure 3a). This representation consists of a sphere positioned with its origin at the center of the object under inspection. This model provides convergent configurations, which give an improved object measurement precision compared to other network configurations (see Fraser (1996)). The viewing sphere model offers the advantage that the search space could be coded within two variables α and β. Therefore, the imaging geometry is decomposed in two main concepts: (1) distribution of camera stations and (2) inclination of a camera station with respect to the point group. Figure 3 serves to illustrate the effect on triangulation precision that is reflected by changes in the shape and size of error ellipsoids as a consequence of improving the distribution of cameras. This experiment considers a plane of 200 by 200 mm^2 to be inspected by a digital camera of 768 by 484 pixels in size and a nominal 12-mm lens. Initially, the cameras are separated 10° from each other, and their final error ellipsoids are depicted on Figure 3b. After a separation of 90°, the error ellipsoids are relatively homogeneous and isotropic. The simulated network agrees well with results reported in the literature.

Constraints on Network Design

Photogrammetric network design (PND) must deal with a series of constraints in order to propose an optimal camera distribution. The accuracy of the system is related to the imaging geometry (main objective in PND) as well as the convergence angle of each camera with respect to each object surface. In order to answer the most basic question of a favorable imaging geometry (FOD or the configuration problem), we must distinguish among the several constraints limiting the search space. Mason (1994) has proposed a set of constraints and objectives that we separate into two parts.

Main Objective and Primary Constraints

Considering the constraints limiting the search space, we identify the following main objective and three constraints due to the characteristics of the FOD problem:

- *Contribution to Intersection Angles or the Imaging Geometry.* Within a camera placement system the main objective is to

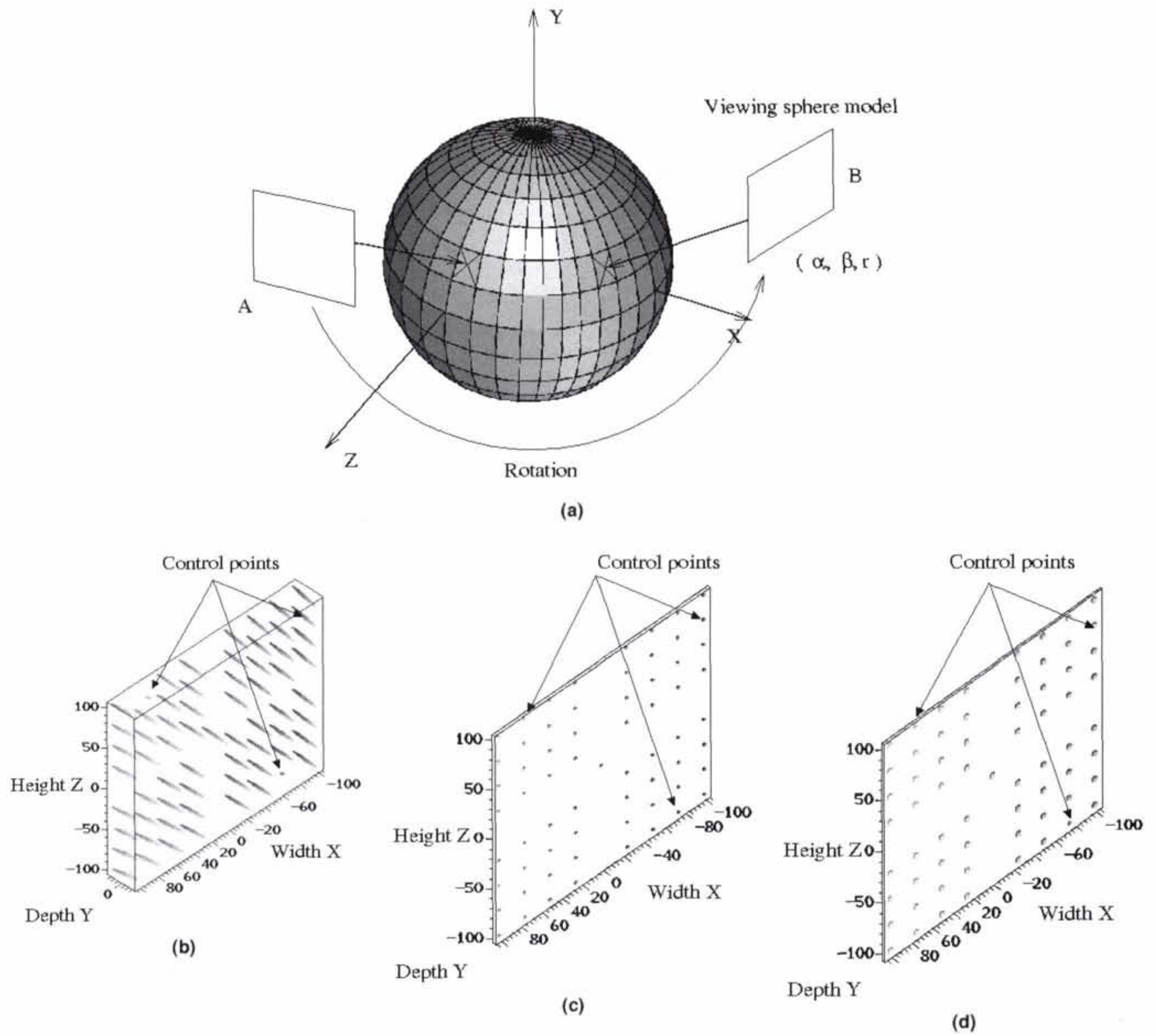


Figure 3. A simple network of two cameras is shown in order to exemplify the effect on triangulation precision in the shape and size of point error ellipsoids after increasing the angle of separation or distribution of camera stations. (a) Two cameras with an angle $\alpha = 45^\circ$ and increasing separation between cameras A and B. (b) Error ellipsoids at 10° of separation. (c) Error ellipsoids at 90° of separation. (d) Error ellipsoids at 140° of separation.

know the contribution of each camera with respect to the others. Two fundamental questions need to be answered: how many cameras will be needed and where should they be placed? However, before answering the first question, we need to answer the second one. Once we know where to place a given number of cameras, it is a trivial matter to decide on the number.

- *Convergence Angle*. The reliability of image measurements from directions close to coplanar are difficult and even impossible to obtain. The minimum allowable incidence angle is dependent on the type of feature, its geometry, and its material. The accuracy of the measurement with respect to the convergence angle is a function of the viewing direction and the surface normal at the feature. In the case of circular targets, the minimum convergence angle is about 20 to 30 degrees for the kind of retro-reflective targets that are normally used.
- *Working Space Constraint*. The workspace in which the photogrammetric survey is conducted can impose restrictions on the selection of an ideal imaging geometry. This constraint includes the walls of the room, any obstructions in the working environment, and the workspace of the robot where the camera could be mounted.
- *Visibility*. This constraint is related to the problem of obstructions in the environment. Viewpoints affected by occlusions caused by other objects in the workspace, or the object itself, should be avoided if possible. A ray tracing technique (POV-RAY, a free software package) was used in order to obtain visibility information of an object from different viewpoints. We created a database that was then used in our optimization process.

Secondary Constraints

Optical constraints such as field of view, depth of field, resolution, and image scale will not be taken into account when estimating a favorable imaging geometry. PND is mainly a function of the imaging geometry, as well as the convergence angle. Optical constraints lack significant importance once the camera observes the entire object. In this way, an optimum distance from the camera to the object can be defined *a priori* in order to measure the different object points. Thus, for the purpose here, all object points appear within the field-of-view, in focus, and at a given resolution and depth of field. In addition, in order to compute the exterior orientation parameters, photogrammetrists affirm that the total number of points is irrelevant once a sufficient number of points are used during the simulation (Fraser, 1984).

Network Design as an Optimization Problem

The problem of PND presents discontinuities mainly due to the occlusion of targets, leading to a combinatorial optimization process, which we have approached using a multi-cellular genetic algorithm (MGA). In Olague (2000), a solution to the PND problem based on genetic algorithms is presented. Genetic algorithms (GAs) are probabilistic parallel search techniques based on the mechanism of natural selection and natural genetics. Since their development in the late 1960s (Holland, 1992), genetic algorithms have been proven effective in searching large, non-linear, and poorly understood search spaces, where expert knowledge is scarce or difficult to encode and where traditional optimization techniques fail. Bhandari *et al.* (1996) suggest that an elitist genetic algorithm (fixed length strings) will converge to the optimal string as the number of iterations goes to infinity. Similar results have been obtained for other stochastic algorithms as simulated annealing but without proof for the general case. This convergence is independent of the choice of values for the algorithm parameters (N, Pc, Pm, etc.), although these parameter values do influence the rate of convergence. There is no theory to indicate the number of iterations necessary for convergence. Two popular heuristic stopping rules are (1) execute the process for a fixed number of iterations and report the best string found as the solution, or (2) execute the process until the fitness value does not show adequate improvement over a fixed number of iterations, and report the best string found as the solution. We use the second heuristic rule. GAs can be applied to a wide range of optimization problems with little adjustment. In the case of PND, the fitness function as well as the structure that represents a photogrammetric network were designed. Thus, it is possible to use the same basic algorithm with some modifications, for example, the way of managing the different constraints. These aspects will be explained next. Automation of PND with the goal of achieving highly accurate measurements is complex. Many decisions need to be made with the purpose of proposing one optimal configuration. The complexity of the problem becomes evident when we study complex objects using multiple cameras. Deterministic methods do not adapt very well because of the high number of design decisions in the form of thresholds. The system must take into account all the constraints in order to solve the numerical and combinatorial problem. Genetic algorithms provide a general framework useful in solving this problem. Moreover, it is necessary to note that PND is based on the very precise rules of imaging geometry and convergence angle. This allows us to differentiate a good configuration from a bad one. Genetic algorithm strategy is based only on the direct comparison of solutions. The set of solutions corresponding to the local minima does not have a significant difference with respect to the global minimum as photogrammetrists affirm (Mason and Grün, 1995). This set of solutions, called here *alternative solutions*, provides similar characteristics concerning the homogeneity of the ellipsoid of uncertainty. Consequently, we can conclude that these alternative solutions are of the same nature. However, all these alternative solutions present configurations that are very different with respect to the imaging geometry. Because deterministic methods follow a fixed set of steps using thresholds, they usually arrive at the same point in the space. Consequently, it presents only one solution depending on the initial point. Stochastic methods do not have this limitation. This is the main reason we have selected them. A process of combinatorial search like genetic algorithms must find the different topologies that a set of cameras presents with respect to an object, taking into account the constraints that limit the search space. Knowledge of these configurations is an important step towards PND.

Genetic Algorithms for Network Design

The PND can be achieved following genetic algorithm methodology. This methodology is composed of five major components:

- The definition of a structure, $A \in \alpha$, which represents a tentative solution to the problem. We represent the structure as a set of variables, which are grouped into just one common structure (see Figure 4).
- The environment, $E \in \varepsilon$, which limits the structure, is represented here as the set of geometrical and optical constraints.
- A measure μ_E of performance, i.e., the fitness of the structures for the environment, is represented here as the value σ_c^2.
- The adaptive plan, $\Gamma \in \tau$, whereby the system's structure is modified to effect improvements. This is the genetic algorithm detailed later on.
- The operator's set, $\Omega \in \omega$, which are used by the adaptive plan. This is represented here by the crossover and mutation operations.

These five main elements are fundamental to establish an evolutionary strategy for the PND problem. Definition of a structure in the genetic algorithm is especially important from the photogrammetric point of view. The search space using binary representation lets the algorithm converge around the multiple local minima, which are closest to the global minima (Goldberg, 1989). While binary representation is not the only alternative, it gives us the possibility of carrying out larger mutations automatically. Moreover, the coding size has no relation to the quality of the solution. Evaluation of a photogrammetric network is directly related to the angle of separation between each camera composing the network. Thus, once we have found an optimal solution (camera distribution), small displacements do not improve the accuracy of the system. Furthermore, the algorithm must take into account the spatial constraints, as well as the occlusion problems, in order to solve the numerical and combinatorial problem in the case of a multi-dimensional search space.

Complexity of the Search Space

Description of the search space includes the definition of forbidden zones for which the geometric constraints and visibility prohibit the camera placement. An example is the convergence angle, which limits the number of cameras observing a set of surfaces, creating sub configurations $\mathbf{C}_s$ for each surface. Decoding each tree structure produces several error computations, with respect to the number of surfaces comprising the object, as shown in Figure 5. After all these computations, we select as the evaluation of the structure the maximum mean variance $\boldsymbol{\sigma}_c^2$. In summary, interpretation of structure is realized dynamically, contrary to traditional genetic algorithms, because the solution of the problem presents combinatorial and numerical characteristics. Similar to genetic programming, decoding is carried out over a variable tree structure. Consequently, sub configurations $\mathbf{C}_s \in \mathbf{C}$ are produced with respect to the surfaces mainly due to the target's orientation in the following way:

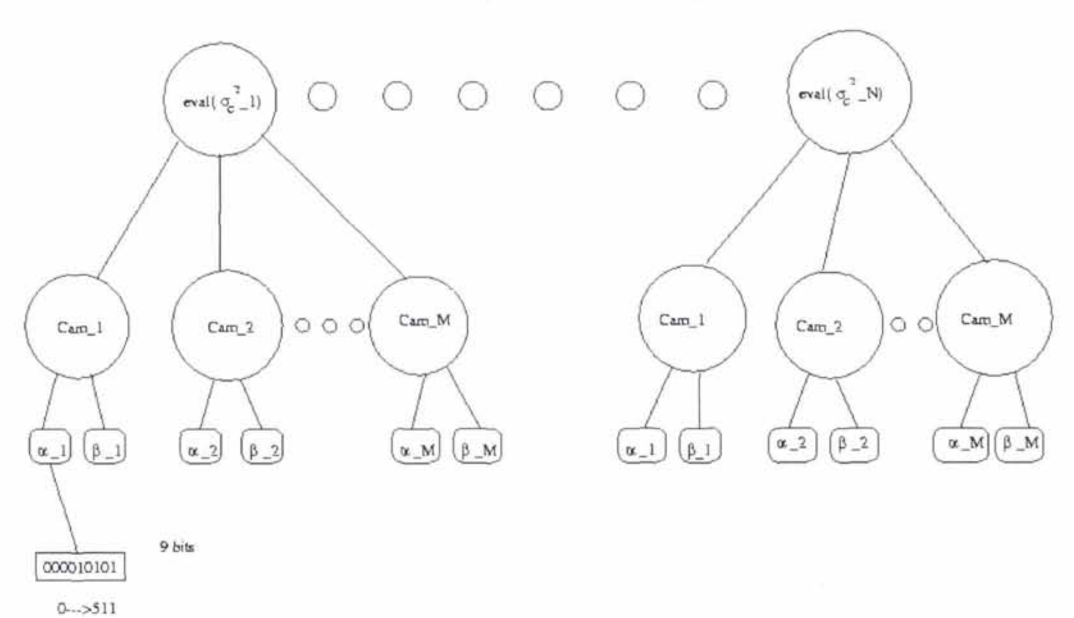


Figure 4. The multi-cellular genetic algorithm is represented by a tree structure composed of a main node where the evaluation process is stored and several leaves corresponding to each camera. All cameras are codified in two parameters (α,β), which correspond to the cells of an artificial being. Because the network evaluation uses only the cells that satisfy the visibility constraints, a combinatorial problem is then involved.

$$Cam_1 \in \mathbf{C_s} = \begin{cases} true & if\ \theta_1 < \theta_{\max} \quad where\ \theta\ is\ the\ convergence\ angle \\ false & otherwise \end{cases}$$

A camera belongs to several configurations due to the target's orientation. PND needs to optimize the distribution of each camera with respect to the others with the purpose of improving the convergence of the network. Therefore, the system needs to take into account the convergence of each camera with respect to the surfaces and at the same time to measure all targets with at least two cameras for each surface. Under these circumstances, our system must confront a combinatorial problem. Contrary to expert photogrammetrists or the strategy developed by Mason (1994), the solution to the problem can be obtained without reducing the complexity of the problem. In other words, we neither need to divide the object into several parts, with the purpose of solving each surface individually; nor do we need to insert new cameras with the goal of improving the system. The solution is obtained by using a specified number of cameras, as we will see in the examples below.

Search Space for Convex Objects

Within a stochastic optimization process such as the genetic algorithm we have used, the search space **C** must be taken into account in order to generate suitable camera positions. We need to consider, then, a genetic algorithm working under constraints. Strategies need to be implemented in order to avoid forbidden zones. Consequently, a rejection process has been developed in order to access the valid space. The search space in the case of a convex object is represented in Figure 5. This figure shows how the search space is divided using the viewing sphere model, where the cameras move on a sphere looking inwards towards a central point. In this example, we want to place a set of cameras in order to measure three contiguous surfaces; several sub configurations are defined as is represented in the Venn diagram of Figure 5. In this manner, the search

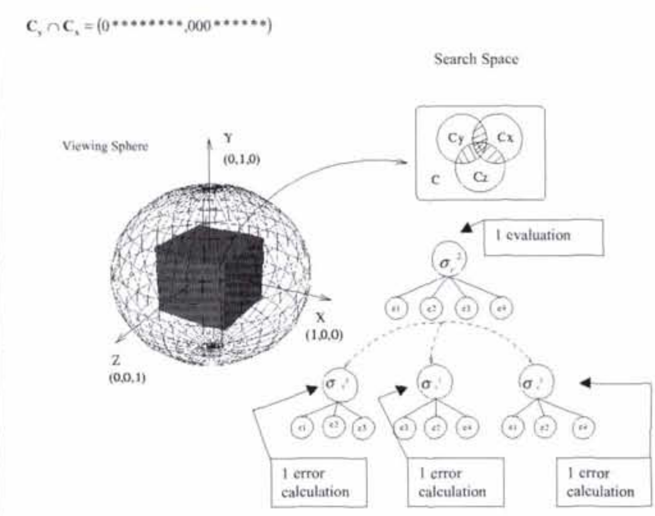


Figure 5. This Figure shows how we divide the search space containing the cube with respect to the (X, Y, Z) axes. The intersection of two schemas defines precisely with respect to axes (Y, X) a zone of the space, which lets both surfaces be observed at the same time. Similar considerations are used for the other zone of the space. The photogrammetric network needs to optimize the image distribution of each camera with respect to the others taking into account the convergence angle. It produces sub configurations (**Cx, Cy, Cz**) for each measured surface that we need to consider in our algorithm.

space is divided $S = 3$ in zones with respect to main axes X, Y, and Z of the interest surfaces composing the object. The set of regions

$$Sol\_Space = \bigcup_{i=1}^{S} \mathbf{C}_s$$

is called the valid solution space, which is constrained in our search. Within this space, a zone corresponding to the intersection of all the sub spaces designates the position space for which we can observe, at the same time, all interest surfaces of the cube. This space zone was the final goal searched by the systems devoted to the placement of only one camera (Cowan and Kovesi, 1988). However, for a system conceived for three-dimensional measurements, a spatial distribution of the camera network must be achieved over all the valid space in order to obtain an optimal configuration from the convergence viewpoint. Cameras must be separated within the space in order to improve the convergence of the system. A compromise is implied over the space zones designated by the disjunction of two neighboring surfaces: i.e.,

$$\mathbf{C}_y + \mathbf{C}_z\text{: } \mathbf{C}_z + \mathbf{C}_x\text{: } \mathbf{C}_x + \mathbf{C}_y$$

where $U + V = \{x \in U \, or \, x \in V\text{: } x \notin U \cap V\}$

Thus, these space regions do not allow each camera to observe all the surfaces of the object at the same time. A camera position can be defined using the following rule:

$$Cam_1(\alpha_1,\beta_1) \in (\mathbf{C}_x + \mathbf{C}_y)$$
$$= \begin{cases} true & if\ \theta_{x,y} < \theta_{\max} \quad for\ each\ \mathbf{S}_{x,y}| \\ & Cam_1(\alpha_1,\beta_1) \notin (\mathbf{C}_x \cap \mathbf{C}_y) \\ false & otherwise \end{cases}$$

Moreover, we can observe that within this space exists a privilege space:

$$(\mathbf{C}_x + \mathbf{C}_y) \cap \mathbf{C}_z$$

which represents the sub spaces where a camera can observe either surface $\mathbf{S}_x$ or $\mathbf{S}_y$ at the same time as surface $\mathbf{S}_z$. This implies a space zone where a better contribution to the triangulation of the system could be obtained with respect to the cameras placed within the region $\mathbf{C}_y \cap \mathbf{C}_x$. Knowledge can be extended to complex objects presenting convex and concave regions. A combinatorial explosion generated by these constraints increases as more cameras are added.

Occlusion Problems

A difficult aspect within the optimization process is related to the visibility of the targets. An occlusion problem implies new prohibited zones, which must be taken into account within the combinatorial search process. Thus, solution space is redefined as the visibility space. We need to avoid the camera placement where a camera cannot measure the targets. This constraint limits the space zones where we can observe all the interest surfaces. Consequently, the set of regions called the visibility space is defined as follows:

$$Vis\_Space = Sol\_Space$$
$$= \bigcup_{i=1}^{S} Cs \mid at\ least\ one\ part\ of\ the\ object\ is\ visible.$$

Division of the visibility space could be done for each surface studied in order to optimize the test of visibility. In this way, we approximate the size of each viewing direction in the sphere in order to obtain homogeneous regions. However, the visibility space just obtained increases the combinatorial problem. A question related to which sub set of cameras is needed in order to measure the surface S_x is implied. This problem is solved partially by a previous computation of the visibility space; to solve it, we use the ray tracing technique as many researchers have done for similar problems (Trucco *et al.* 1997). As a result of the division of the sphere, the test of visibility is carried out over each section of the sphere in order to know the visible targets for a given viewpoint. In this way, the global optimization process can use the information recorded previously into a database, in order to search within the combinatorial space, saving a great amount of computational time.

The Multi-Cellular Genetic Algorithm

The multi-cellular genetic algorithm (MGA) then proceeds as follows:

(1) An initial random population of $N = 30°$ convergent photogrammetric networks that satisfy the environment constraints is chosen and it is represented by (α,β), coded into a binary string representation.
(2) Next, we evaluate each photogrammetric network, and store the corresponding average variance along the covariance matrix $\boldsymbol{\sigma}_c^2$ for each tree structure. This corresponds to the fitness value which says how good the network is, compared with other solutions in the population $P(t)$ at generation t.
(3) Then, we select a population of "good" networks by *tournament selection*: two networks are selected from $P(t)$ and are compared selecting the best individual according to its fitness, yielding the population $P(t + 1)$.
(4) From this population, we recombine the binary strings (α_n,β_n) for each camera using the following operations[1]:
   (a) Crossover, with a probability Pc = 0.7. This operation was implemented using one-cut point. Actually, due to the classification of the MGA, this operation works like a *multiple-cut-point*. Let the two parents be

   $$\alpha_x = [\alpha_{x1},\alpha_{x2},\alpha_{x3},\alpha_{x4},\alpha_{x5},\alpha_{x6},\alpha_{x7},\alpha_{x8},\alpha_{x9}],$$
   $$\alpha_y = [\alpha_{y1},\alpha_{y2},\alpha_{y3},\alpha_{y4},\alpha_{y5},\alpha_{y6},\alpha_{y7},\alpha_{y8},\alpha_{y9}].$$

   If they are crossed after the random kth position = 4, the resulting offspring are

   $$\alpha'_x = [\alpha_{x1},\alpha_{x2},\alpha_{x3},\alpha_{x4},\alpha_{y5},\alpha_{y6},\alpha_{y7},\alpha_{y8},\alpha_{y9}],$$
   $$\alpha'_y = [\alpha_{y1},\alpha_{y2},\alpha_{y3},\alpha_{y4},\alpha_{x5},\alpha_{x6},\alpha_{x7},\alpha_{x8},\alpha_{x9}].$$

   (b) Mutation, with a probability Pm = 0.005. This operation alters one or more genes. Assume that the $\alpha_{y5} = 1$ gene of the chromosome α'_x is selected for a mutation. Because the gene is 1, it would be flipped into 0.

   These operations yield a new population, which we copy into $P(t)$.
(5) Steps 2, 3, and 4 are repeated until the optimization criterion stabilizes.

Finally, this algorithm minimizes the average variance along the covariance matrix $\boldsymbol{\sigma}_c^2$: i.e., $fitness = \min_{i=1,\ldots,N}(\max \boldsymbol{\sigma}_c^2)$.

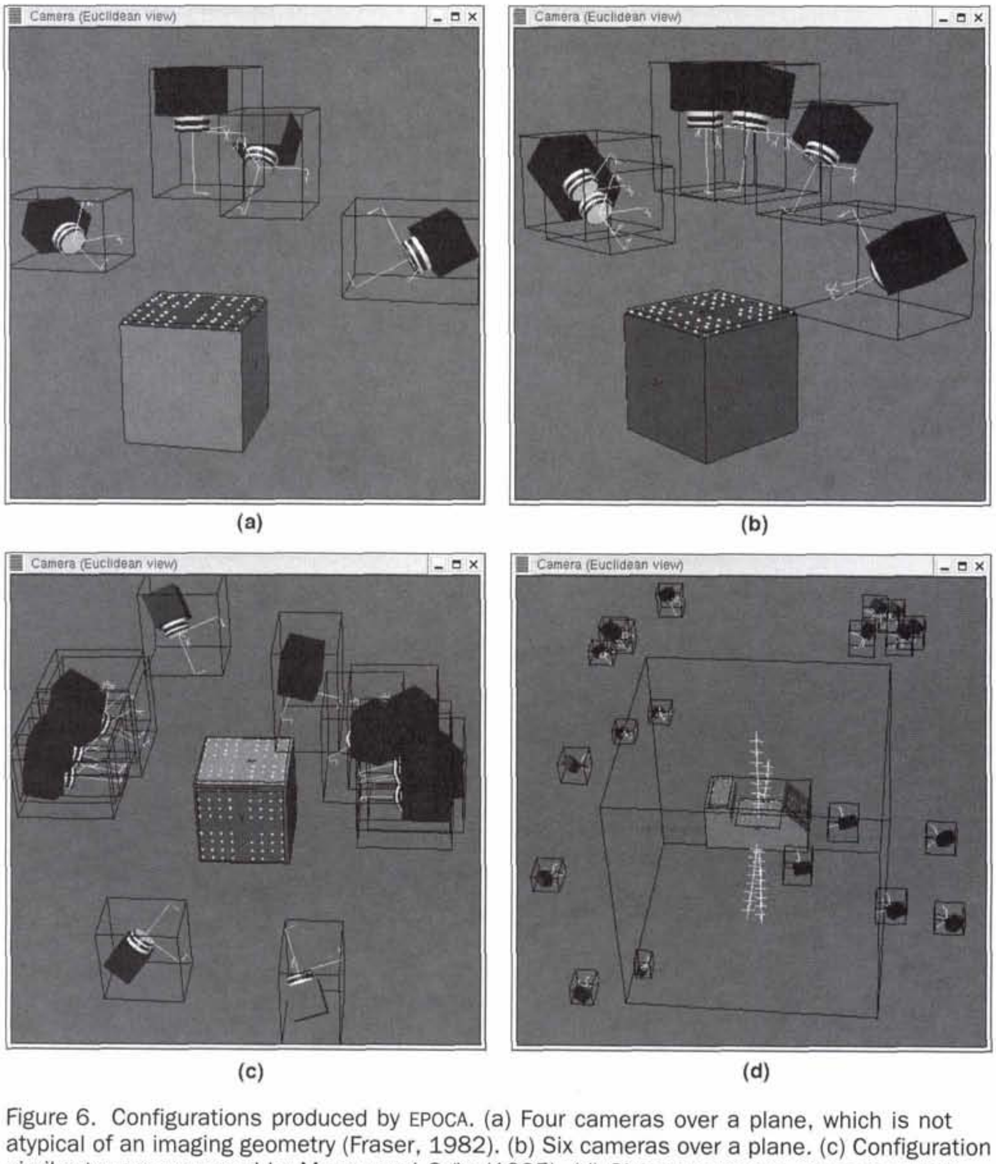
The camera placement M_i relative to the world coordinate frame is now optimized. Geometrically, each covariance matrix represents an ellipsoid, which changes its orientation and size as each sensor placement M_i changes. Thus, an optimal placement solution is proposed, where the combined uncertainty of all points is minimal.

Examples

We have run a series of experiments to show the usefulness of our approach. We present select results in Figures 6a through 6d, which show several configurations designed by EPOCA. The

[1]The threshold values associated to Pc and Pm were adopted from the literature (Mitchell, 1996).

cameras are looking at a set of targets represented by their error ellipsoids aligned in one or two planes, as well as over a complex object. These configurations are a product of our evolutionary system. In fact, within a stochastic optimization process we cannot arrive at conclusions from just one trial. In Olague and Mohr (2002), a statistical study in connection with this work was presented. Each configuration presented is the product of about 50 independent runs. Figure 6a illustrates a solution with four cameras looking at a planar surface. This solution is not the standard one used by the expert photogrammetrists; a photogrammetrist usually puts the four cameras at four-corners of a cube whose center contains the targets to be measured. In fact, Fraser (1982) has already discussed our configuration; he noticed that this configuration is not atypical. Our experiments confirm Fraser's statement; hence, the equivalence between both configurations (Mason, 1994). Figure 6b presents an interesting result, which corresponds to the *second-order design* (SOD) used by photogrammetrists. Cameras are placed automatically over the same places as Fraser's configuration. This operation requires the acquisition of multiple exposures from each camera composing the network. The SOD operation is normally used after having selected a basic configuration. The multiple exposures thus yield a good improvement in the mean standard error value over that obtained in the basic network. Figure 6c presents a configuration similar to one proposed by Mason and Grün (1995), for which the imaging geometry and convergence angle are distributed in an optimal way. In the case of Figure 6d, just 20 trials were necessary. This result shows how the object shapes constrain the search space where cameras could be placed. These results correspond well to the generic network theory. These examples demonstrate how our system is able to propose solutions used by expert photogrammetrists. Moreover, our system can contribute to finding solutions in the case of complex objects.



(a) (b) (c) (d)

Figure 6. Configurations produced by EPOCA. (a) Four cameras over a plane, which is not atypical of an imaging geometry (Fraser, 1982). (b) Six cameras over a plane. (c) Configuration similar to one proposed by Mason and Grün (1995). (d) Sixteen cameras over a complex object. Configurations reported in the literature (a) and (c) were reproduced by EPOCA. Figure (b) improves upon Fraser's solution due to SOD operation, which is automatically generated. Moreover, EPOCA can be used in the case of complex objects, as can be appreciated from (d).

Conclusions

In this paper we have presented a solution to the problem of configuring an optimal photogrammetric network (the FOD problem) with the goal of achieving highly accurate 3D measurements in terms of an optimization design. The problem has been divided into two main parts. The first was devoted to reviewing the bundle method in order to derive a mathematical criterion useful for solving the problem. The criterion we have chosen is the average variance σ_c^2 extracted from the covariance matrix of the object point coordinates. This criterion was chosen in order to simplify the stochastic optimization process. The second part was devoted to a global optimization method, which has minimized the criterion. Due to the occlusion of points, caused by the different constraints, the problem presents discontinuities, which leads to combinatorial aspects in the optimization process. These constraints are logically incorporated into the genetic algorithm strategy. The strategy proposed in this paper provides the ability to adapt the PND to produce a suitable multi-station configuration for each new inspection task. The optimization search strategy considers then the large number of competing considerations towards the optimal satisfaction of all placement constraints until the best acceptable configuration is found. At present, the computational time required for designing a photogrammetric network under the proposed methodology should not be considered a limitation in view of the continuing boost in computer technology. In summary, our work confirms Holland's (1992) results that genetic algorithms are able to solve efficiently such kinds of optimization problems with high combinatorial aspects. EPOCA successfully produces two- and three-camera network designs similar to those used by photogrammetrists. In the case of four cameras, a non-standard design was proposed which gives results similar to those obtained from the more classical networks. Moreover, the system can design networks for several adjoining planes and complex objects. All the configurations are good in terms of the camera distribution and ray inclination. This work has to be considered as an additional step towards automated photogrammetric network design. The strength of our approach is its generality. Consideration of alternative optimization processes working on a non-uniform criterion could easily extend this work. Of the three principal objectives in photogrammetry—precision, reliability, and economy—this paper has presented a solution to the FOD problem that is related to the precision of the system. Economy due to the decreasing cost of technology should not be considered as a scientific problem. Finally, reliability aspects were not considered in our study. Reliability is directly related to the number and distribution of points in the object. Such reliability relates to the accuracy of the system; to explore it is our goal for future research.

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